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A note on (Strawson) entailment*

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Abstract It is argued that the notion of classical entailment faces two problems, the second argument projection problem and the P-to-Q problem, which arise because classical entailment is not designed to handle partial functions. It is shown that while the second argument projection problem can be solved either by flattening the syntactic tree or with naïve multi-valued logics, the P-to-Q problem cannot. Both problems are solved by introducing a new notion of entailment that is defined in terms of Strawson entailment (in the sense of von Fintel 1999, 2001).

Keywords: entailment, Strawson entailment, upward/downward-entailingness, presupposition, presupposition projection, determiners

1 The main claim

The goal of this paper is to scrutinize the formal notion of entailment as it is understood and used in the semantics literature, and to propose an

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alternative that is inspired by the notion of Strawson entailment (as it is understood and used in von Fintel 1999, 2001).

One of the goals of semantic theory is to explain speakers' intuitions regarding relations between sentences. An example of such a linguistic intuition is the one regarding (1), which is reflected by speakers' discomfort with *It's true that some student arrived early but it isn't true that some student arrived* (or [(1a) *but not* (1b)]).

(1) a. Some student arrived early.
 b. Some student arrived.

A widely held view is that the relation between (1a) and (1b) is that of classical entailment — or ⇒entailment — defined informally in (2) (where a *statement* is something that has a truth value, a *predicate* is something that takes arguments to yield a truth value, and a *type-relevant* x is an x of the type that can serve as an argument of P and of Q).

(2) **⇒entailment (classical entailment)**
 a. For any statements p and q, p *⇒entails* q *iff* p is false or q is true.
 b. For any predicates P and Q, P *⇒entails* Q *iff* for all type-relevant x, $P(x)$ ⇒entails $Q(x)$.

Assume that *arrive* and *arrive early* have no presuppositions. It follows from clause (2b) that *arrive early* ⇒entails *arrive*, because for any type-relevant x, it follows from clause (2a) that [x *arrived early*] ⇒entails [x *arrived*]. By similar reasoning, (1a) ⇒entails (1b).

The pair in (1) also illustrates the fact that the determiner *some* is upward-entailing with respect to its second argument. An informal definition of *upward-entailingness* is given in (3).

(3) O is *upward-entailing iff* for all type-relevant P and Q such that P ⇒entails Q, $O(P)$ ⇒entails $O(Q)$.

Indeed, for any type-relevant X (e.g., *student*), and any type-relevant P and Q such that P ⇒entails Q, [O P] is false or [O Q] is true, where O = *some* X.

However, on the view that presuppositions are encoded semantically, two problems arise with (2)–(3) when we consider cases where *some* takes presuppositional arguments such as *likes his mother*. The examples in (4)–(5)

illustrate the second argument projection problem and the example in (6) illustrates the *P*-to-*Q* problem.

(4) a. Some *French student* arrived.
 b. Some *student* arrived.

(5) a. Some *French student* likes his mother.
 b. Some *student* likes his mother.

(6) a. Some student *likes his mother*.
 b. Some student *likes someone*.

The second argument projection problem. Assume that *French student* and *student* have no presuppositions. By (2), *French student* $\Rightarrow$-entails *student* and (4a) $\Rightarrow$-entails (4b), but (5a) does not $\Rightarrow$-entail (5b), despite the fact that speakers reject [(5a) *but not* (5b)] just as they reject [(4a) *but not* (4b)]. The reason is, presumably, that when all the students are motherless, both (5a) and (5b) are neither true nor false. Accordingly, while (3) makes *some* upward-entailing with respect to its second argument (as we saw), it does not make it upward-entailing with respect to its first argument: it is impossible to establish that *Some P likes his mother* $\Rightarrow$-entails *Some Q likes his mother* for every type-relevant *P* and *Q* such that *P* $\Rightarrow$-entails *Q*. Consequently, we fail to capture the fact that speakers have the same reaction to [(4a) *but not* (4b)] and [(5a) *but not* (5b)].

The *P*-to-*Q* problem. Speakers reject [(6a) *but not* (6b)] just as they reject [(1a) *but not* (1b)]. We want to blame this on the fact that *some* is upward-entailing, by (3), with respect to its second argument. In other words, we want to be able to say that just like (1a) $\Rightarrow$-entails (1b) because *arrive early* $\Rightarrow$-entails *arrive*, (6a) $\Rightarrow$-entails (6b) because *likes his mother* $\Rightarrow$-entails *likes someone*. But we cannot say this. While for any type-relevant x, speakers reject ['x *likes his mother*' *but not* 'x *likes someone*'], *likes his mother* does not $\Rightarrow$-entail *likes someone* by (2): if x is motherless and doesn't like anyone, x *likes his mother* is neither true nor false and x *likes someone* is false.

To solve these problems, we introduce two new relations: $\overset{ST}{\Rightarrow}$-entailment, informally defined in (7), and $\Rrightarrow$-entailment, informally defined in (8). The notion of $\overset{ST}{\Rightarrow}$-entailment is inspired by the notion of Strawson entailment in von Fintel 1999, 2001 (the coined term "Strawson entailment" is von Fintel's tribute to Strawson (1952)). The notion of $\Rrightarrow$-entailment is defined in terms of Strawson entailment. We also propose that the definition of upward-entailingness in (3) be replaced with (9).

(7) **$\overset{ST}{\Rightarrow}$-entailment**

a. For any statements p and q, p *$\overset{ST}{\Rightarrow}$-entails* q *iff* p is false or q is true.

b. For any predicates P and Q, P *$\overset{ST}{\Rightarrow}$-entails* Q *iff* for all x such that the presuppositions of $P(x)$ and $Q(x)$ are satisfied, $P(x)$ $\overset{ST}{\Rightarrow}$-entails $Q(x)$.

(8) **$\Rrightarrow$-entailment**

For any P and Q, P *$\Rrightarrow$-entails* Q *iff*

a. P $\overset{ST}{\Rightarrow}$-entails Q; and

b. if P and Q are predicates, for all $n \geq 1$ and all $\langle x_1, x_2, \ldots, x_n \rangle$, if $P(\langle x_1, x_2, \ldots, x_n \rangle)$ is a truth value, satisfaction of the presuppositions of $P(\langle x_1, x_2, \ldots, x_n \rangle)$ guarantees satisfaction of the presuppositions of $Q(\langle x_1, x_2, \ldots, x_n \rangle)$.

(9) O is *upward-entailing iff* for all type-relevant P and Q such that P $\Rrightarrow$-entails Q and the presuppositions of $O(P)$ are satisfied, the presuppositions of $O(Q)$ are satisfied and $O(P)$ $\Rrightarrow$-entails $O(Q)$.

Both problems are solved. When x has a mother, x *likes his mother* is false or x *likes someone* is true. Consequently, *likes his mother* $\Rrightarrow$-entails *likes someone*. When some French student has a mother, some student has a mother, and (5a) is false or (5b) is true. When some student has a mother, (6a) is false or (6b) is true. Consequently, *some* comes out upward-entailing with respect to its first and second arguments. We contend that on the view that presuppositions are semantically encoded, the significant forms of entailment in natural language are $\overset{ST}{\Rightarrow}$-entailment and $\Rrightarrow$-entailment. Classical entailment — namely, $\Rightarrow$-entailment as defined in (2) — plays a very small role in semantics.

In Section 2 we discuss the second argument projection problem and the P-to-Q problem in some detail. In Section 3 we introduce and revise the notion of Strawson upward- and downward-entailingness in order to solve the second argument projection problem. In Section 4 we build on the proposal in Section 3 to solve the P-to-Q problem. Section 5 compares our proposal to a proposal that is based on a trivalent logic and a proposal that is based on a flattened clause structure.

2 Modeling upward- and downward-entailingness

2.1 The basics

In what follows, when we say that sentence A intuitively entails sentence B, we mean that whenever speakers judge A true, they automatically judge B true. For example, *Mary and Jane arrived* intuitively entails *Mary arrived*. When we say that one-place predicate α intuitively entails one-place predicate β, we mean that for any type-appropriate x, [x (is) α] intuitively entails [x (is) β]. For example, *arrived early* intuitively entails *arrived* and *French student* intuitively entails *student*. When we say that a determiner DET is intuitively upward-entailing on its first argument, we mean that for any type-appropriate α, β and γ such that α entails β, [DET α γ] intuitively entails [DET β γ]. When we say that DET is intuitively downward-entailing on its first argument, we mean that for any type-appropriate α, β and γ such that α entails β, [DET β γ] intuitively entails [DET α γ]. Similar conventions apply to "DET is upward/downward-entailing on its second argument". For example, the determiner *no* is intuitively downward-entailing on its first and second arguments, as evidenced by the fact that *No student arrived* intuitively entails both *No French student arrived* and *No student arrived early*. The determiner *some* is intuitively upward-entailing on its first and second arguments, as evidenced by the fact that *Some French student arrived* and *Some student arrived early* each intuitively entails *Some student arrived*.

A widely held view is that intuitive entailment is modeled on '$\Rightarrow$' (roughly, $\Rightarrow$-entailment in Section 1), which is defined recursively in (10) (see, for example, von Fintel 1999). Intuitive upward- and downward-entailingness are modeled on UE and DE defined in (11) and (12) respectively.

(10) a. **Cross-categorial $\Rightarrow$ (classical entailment)**

For all $p, q \in D_t$: $p \Rightarrow q$ if p = False or q = True,
$p \not\Rightarrow q$ if p = True and q = False.

For all $f, g \in D_{\langle\sigma,\tau\rangle}$: $f \Rightarrow g$ if for all $x \in D_\sigma$, $f(x) \Rightarrow g(x)$,
$f \not\Rightarrow g$ if for some $x \in D_\sigma$, $f(x) \not\Rightarrow g(x)$.

b. D_ρ is the domain of semantic objects of type ρ, where
(i) t is a type and $D_t = \{\text{True}, \text{False}\}$;
(ii) e is a type and D_e is the domain of individuals;
(iii) for any types σ and τ, $\langle\sigma,\tau\rangle$ is a type and $D_{\langle\sigma,\tau\rangle}$ is the domain of functions from D_σ to D_τ.[1]

1 We are only concerned here with the types of determiners and their arguments.

(11) A function $f \in D_{\langle\sigma,\tau\rangle}$ is **UE** *iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$, $f(P) \Rightarrow f(Q)$.

(12) A function $f \in D_{\langle\sigma,\tau\rangle}$ is **DE** *iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$, $f(Q) \Rightarrow f(P)$.

Let us illustrate how *no* comes out DE. We assume, as is standard in linguistic semantics, that semantic interpretation proceeds in a compositional fashion. The syntactic representation of *No student arrived* that serves as input to semantic interpretation — its LF — is $[_{\mathrm{S}}\ [_{\mathrm{NP}}$ *no student*$]\ [_{\mathrm{VP}}$ *arrived*$]]$, where *student* is the first argument of *no* and *arrived* is its second argument. The interpretation function $[\![\ ]\!]$ is relativized to (at least) the following two parameters: (i) w, an element of W (the set of all possible worlds), and (ii) C, a function that assigns to every w' in W a subset of D_e that includes all and only individuals that are relevant in w'. Assuming that for any C and w, $[\![\mathit{student}]\!]^{C,w}$ and $[\![\mathit{arrived}]\!]^{C,w}$ are (total) functions of type $\langle e, t\rangle$ and $[\![\mathit{no}]\!]^{C,w}$ is a (total) function of type $\langle\langle e,t\rangle, \langle\langle e,t\rangle, t\rangle\rangle$ as in (13), $[_{\mathrm{S}}\ [_{\mathrm{NP}}$ *no student*$]$ $[_{\mathrm{VP}}$ *arrived*$]]$ receives the interpretation in (14).[2]

(13) For any w and C, and any $Z, Y \in D_{\langle e,t\rangle}$, $[\![\mathit{no}]\!]^{C,w}(Z)(Y) = \text{True}$ *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \{y \in D_e \mid Y(y) = \text{True}\} = \varnothing$.

(14) $[\![[\mathit{no\ student}]\ \mathit{arrived}]\!]^{C,w} = [\![\mathit{no}]\!]^{C,w}([\![\mathit{student}]\!]^{C,w})([\![\mathit{arrived}]\!]^{C,w}) =$ True *iff* $\{y \in C(w) \mid [\![\mathit{student}]\!]^{C,w}(y) = \text{True}\} \cap$ $\{y \in D_e \mid [\![\mathit{arrived}]\!]^{C,w}(y) = \text{True}\} = \varnothing$.

By (12), for any C and w and any $Z \in D_{\langle e,t\rangle}$, $[\![\mathit{no}]\!]^{C,w}(Z)$ is a DE function. To see why, assume that $[\![\mathit{arrived\ early}]\!]^{C,w}$ is also of type $\langle e,t\rangle$ and that $[\![\mathit{arrived\ early}]\!]^{C,w} \Rightarrow [\![\mathit{arrived}]\!]^{C,w}$. Accordingly, for any $Z \in D_{\langle e,t\rangle}$, either

$$[\![\mathit{no}]\!]^{C,w}(Z)([\![\mathit{arrived}]\!]^{C,w}) = \text{False}$$

or

$$[\![\mathit{no}]\!]^{C,w}(Z)([\![\mathit{arrived\ early}]\!]^{C,w}) = \text{True},$$

2 *No* combines with *student*, and *no student* with *arrived*, via Functional Application (see, for example, Heim & Kratzer 1998):

(i) **Functional Application**: If α is a branching node and $\{\beta, \gamma\}$ is the set of its daughters, then for any w, C and assignment g, $[\![\alpha]\!]^{C,w,g}$ is defined if $[\![\beta]\!]^{C,w,g}$ and $[\![\gamma]\!]^{C,w,g}$ are defined and $[\![\gamma]\!]^{C,w,g} \in \mathrm{Dom}([\![\beta]\!]^{C,w,g})$. In that case, $[\![\alpha]\!]^{C,w,g} = [\![\beta]\!]^{C,w,g}([\![\gamma]\!]^{C,w,g})$.

so $[\![no]\!]^{C,w}(Z)([\![arrived]\!]^{C,w}) \Rightarrow [\![no]\!]^{C,w}(Z)([\![arrived\ early]\!]^{C,w})$ (see (I), Appendix). Likewise, $[\![no]\!]^{C,w}$ itself is also DE: on the assumption that $[\![French\ student]\!]^{C,w}$ is also of type $\langle e,t\rangle$ and that $[\![French\ student]\!]^{C,w} \Rightarrow [\![student]\!]^{C,w}$, for any $Z \in D_{\langle e,t\rangle}$, either

$$[\![no]\!]^{C,w}([\![student]\!]^{C,w})(Z) = \text{False}$$

or

$$[\![no]\!]^{C,w}([\![French\ student]\!]^{C,w})(Z) = \text{True},$$

so $[\![no]\!]^{C,w}([\![student]\!]^{C,w}) \Rightarrow [\![no]\!]^{C,w}([\![French\ student]\!]^{C,w})$.

We say that *no* is DE on both its first and second arguments. By this we mean that as defined in (13), $[\![no]\!]^{C,w}$ is DE for any C and w ("*no* is DE on its first argument") and that for any $Z \in D_{\langle e,t\rangle}$, $[\![no]\!]^{C,w}(Z)$ is DE for any C and w ("*no* is DE on its second argument"). The determiner *some*, with the meaning in (15), comes out UE on its second argument (see (II), Appendix). It also comes out UE on its first argument, as expected.

(15) For any w and C, and any $Z, Y \in D_{\langle e,t\rangle}$, $[\![some]\!]^{C,w}(Z)(Y) = \text{True}$ *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \{y \in D_e \mid Y(y) = \text{True}\} \neq \varnothing$.

Why is it important to classify *no* as downward-entailing and *some* as upward-entailing in a purely technical sense? It is often claimed that formal properties such as DE and UE are linguistically significant in the sense that certain linguistic rules explicitly refer to them (see, for example, Heim & Kratzer 1998, Chapter 6 and references cited there). For the sake of the discussion, we take it for granted that the following holds. There are linguistic rules that refer to DE functions and linguistic rules that refer to UE functions. Since *no* behaves as if it is referred to by the former and *some* behaves as if it is referred to by the latter, we expect *no* to come out DE and *some* UE.

As is well known, however, it is not obvious that the functions in (13) and (15) are indeed the meanings of *no* and *some*, because these determiners seem to carry presuppositions.

2.2 Presuppositional arguments

The sentence *John likes his first book*, with the definite description *his first book*, is judged odd when John doesn't have a first book (for discussion of related experimental evidence see, for example, Abrusán & Szendrői 2013). Likewise, *No professor who likes his first book arrived* is judged odd when no one has a first book, and *No professor likes his first book* is judged odd when no professor has a first book. One way to account for these facts, within the semantic approach to presuppositions, is the following. *Likes his first book* denotes a potentially partial function of type $\langle e,t\rangle$, as in (16), rather than the total function in (17) (we say that a function is of type $\langle \rho,\sigma\rangle$ even if its domain is a proper subset of D_ρ; see, for example, Heim & Kratzer 1998).[3] Likewise, *no* denotes a potentially partial function of type $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$, as in (18), rather than the total function in (13).

(16) For any C, w and $x \in D_e$:

a. $[\![$*likes his first book*$]\!]^{C,w}(x)$ is defined *iff* the cardinality of $\{y \in C(w) \mid y$ is a book of x in w and no $z \in \{z' \in C(w) \mid z'$ is a book of x in $w\}$ precedes y (relative to x) in $w\}$ is 1;

b. if defined, $[\![$*likes his first book*$]\!]^{C,w}(x)$ = True *iff* for all $y' \in \{y \in C(w) \mid y$ is a book of x in w and no $z \in \{z' \in D_e \mid z'$ is a book of x in $w\}$ precedes y (relative to x) in $w\}$, x likes y' in w.

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3 A more detailed LF of *likes his first book* might be [2 [[*the* [*first* [*book of he*$_2$]]] [3 [t_2 *likes* t_3]]]], where 2 and 3 are indices, *he*$_2$ is a pronoun of type e and t_2 and t_3 are traces of type e. In addition to Functional Application in Footnote 2, we assume:

(i) For any w and C, $[\![$*likes*$]\!]^{C,w}$ and $[\![$*of*$]\!]^{C,w}$ are of type $\langle e,\langle e,t\rangle\rangle$, $[\![$*book*$]\!]^{C,w}$ is of type $\langle e,t\rangle$, $[\![$*first*$]\!]^{C,w}$ is of type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ and:

a. if *likes his first book* is as in (16), then

$[\![$*the*$]\!]^{C,w}$ =

$\lambda P \in D_{\langle e,t\rangle}$: the cardinality of$\{y \in C(w) \mid P(y) = \text{True}\}$ is 1
$.\ \lambda Q \in D_{\langle e,t\rangle}$. for all $z \in \{y \in C(w) \mid P(y) = \text{True}\}, Q(z) = \text{True}$.

b. if *likes his first book* is as in (17), then $[\![$*likes*$]\!]^{C,w}$, $[\![$*of*$]\!]^{C,w}$, $[\![$*first*$]\!]^{C,w}$ and $[\![$*book*$]\!]^{C,w}$ are total, and

$[\![$*the*$]\!]^{C,w}$ =

$\lambda P \in D_{\langle e,t\rangle}.\lambda Q \in D_{\langle e,t\rangle}$. the cardinality of $\{y \in C(w) \mid P(y) = \text{True}\}$ is 1, and
for all $z \in \{y \in C(w) \mid P(y) = \text{True}\}$, $Q(z) = \text{True}$.

(ii) **Predicate Abstraction**: If α is a branching node and $\{n,\gamma\}$ is the set of its daughters (where n is a numerical index), then for any w, C and assignment g, $[\![\alpha]\!]^{C,w,g}$ = λx: $[\![\gamma]\!]^{C,w,g[n\to x]}$ is defined. $[\![\gamma]\!]^{C,w,g[n\to x]}$.

(iii) **Traces and Pronouns Rule**: If α is a pronoun or a trace and n a numerical index, then for any w, C and assignment g, $[\![\alpha_n]\!]^{C,w,g}$ is defined only if n is in the domain of g. When defined, $[\![\alpha_n]\!]^{C,w,g} = g(n)$.

(17) For any C, w and $x \in D_e$, $[\![$*likes his first book*$]\!]^{C,w}(x)$ = True *iff* the cardinality of $\{y \in C(w) \mid y$ is a book of x in w and no $z \in \{z' \in C(w) \mid z'$ is a book of x in $w\}$ precedes y (relative to x) in $w\}$ is 1, and for all $y' \in \{y \in C(w) \mid y$ is a book of x in w and no $z \in \{z' \in C(w) \mid z'$ is a book of x in $w\}$ precedes y (relative to x) in $w\}$, x likes y' in w.

(18) For any w, C and any $Z, Y \in D_{\langle e,t \rangle}$:

a. $[\![$*no*$]\!]^{C,w}(Z)$ is defined *iff* $C(w) \cap \mathrm{Dom}(Z) \neq \varnothing$;

b. when $[\![$*no*$]\!]^{C,w}(Z)$ is defined, then $[\![$*no*$]\!]^{C,w}(Z)(Y)$ is defined *iff* $\mathrm{Dom}(Y) \neq \varnothing$ and $\{y \in C(w) \mid Z(y) = \mathrm{True}\} \subseteq \mathrm{Dom}(Y)$;

c. when defined, $[\![$*no*$]\!]^{C,w}(Z)(Y)$ = True *iff* $\{y \in C(w) \mid Z(y) = \mathrm{True}\} \subseteq \{y \in \mathrm{Dom}(Y) \mid Y(y) = \mathrm{False}\}$.

Let us call the first presupposition of *no* ((a) in (18)) the first argument presupposition, and the second presupposition of *no* ((b) in (18)) the second argument presupposition. Presumably, all determiners have a first and a second argument presupposition, though there is some controversy regarding their exact formulation. Some scholars (e.g., Heim (1983)) argue that the second argument presupposition is always the *universal* $\{y \in C(w) \mid Z(y) = \mathrm{True}\} \subseteq \mathrm{Dom}(Y)$ in (18b). Other scholars (e.g., Beaver (2001)) have proposed an *existential* version of the second argument presupposition (at least for some determiners), namely, $\{y \in C(w) \mid Z(y) = \mathrm{True}\} \cap \mathrm{Dom}(Y) \neq \varnothing$. In addition, at least for some determiners, the first argument presupposition has been argued to be the *strong* $\{y \in C(w) \mid Z(y) = \mathrm{True}\} \neq \varnothing$ rather than the *weak* (18a).[4] Nothing we say in this section and in Section 3 hinges on which versions are adopted, and we assume different versions for different determiners. For example, we assume *some* has an existential second argument presupposition, though the reason for this will become apparent only in Section 4, where we consider a variation in predictions arising from the different versions of these presuppositions.

As it turns out, '$\Rightarrow$', as defined in (10a) — repeated below — is not explicit about the status of partial functions.

4 From now on for any X, '$\{y \in C(w) \mid X(y) = \mathrm{True}\}$' is shorthand for '$\{y \in D_e \mid y \in C(w) \cap \mathrm{Dom}(X)$ and $X(y) = \mathrm{True}\}$'.

(10) a. **Cross-categorial** $\Rightarrow$

For all $p, q \in D_t$: $p \Rightarrow q$ if p = False or q = True,
$p \not\Rightarrow q$ if p = True and q = False.

For all $f, g \in D_{\langle\sigma,\tau\rangle}$: $f \Rightarrow g$ if for all $x \in D_\sigma$, $f(x) \Rightarrow g(x)$,
$f \not\Rightarrow g$ if for some $x \in D_\sigma$, $f(x) \not\Rightarrow g(x)$.

This is problematic because by (10a), for any σ and $f \in D_{\langle\sigma,t\rangle}$ such that $\mathrm{Dom}(f) \subset D_\sigma$, there is at least one $x \in D_\sigma$ such that we cannot determine that $f(x)$ = True, nor can we determine that $f(x) \neq$ True (i.e., that $f(x)$ = False). We cannot determine that $f(x)$ = Undefined either, as Undefined is not a member of D_t in this system: by (10b), $D_t = \{\text{True}, \text{False}\}$. Rather, $f(x)$ is undefined. Therefore, for any σ, any $f, g \in D_{\langle\sigma,t\rangle}$, and any $x \in D_\sigma$ such that

i. $x \notin \mathrm{Dom}(f)$ and

ii. $x \notin \mathrm{Dom}(g)$ or $g(x)$ = False,

we cannot determine by (10a) either that $f \Rightarrow g$ or that $f \not\Rightarrow g$. As a result, the following two problems arise: the second argument projection problem, illustrated by (19a) and (19b), and the P-to-Q problem, illustrated by (19c) and (19d).

(19) a. No *professor* likes his first book.
b. No *French professor* likes his first book.
c. Some professor *likes his first book*.
d. Some professor *likes something*.

If *no* indeed denotes a partial function, (19a) and (19b) sometimes lack a truth value. Consequently, we cannot establish that *no* is DE on its first argument, despite the fact that (19a) intuitively entails (19b). If *likes his first book* indeed denotes a partial function, we cannot establish that $[\![$*likes his first book*$]\!]^{C,w} \Rightarrow [\![$*likes something*$]\!]^{C,w}$ for all C and w. Therefore, we cannot blame the fact that (19c) intuitively entails (19d) on the claim that $[\![$*likes his first book*$]\!]^{C,w} \Rightarrow [\![$*likes something*$]\!]^{C,w}$ for all C and w. Let us discuss this in some more detail, starting with the second argument projection problem.

No determiner that has a second argument presupposition — "existential" or "universal" — comes out either DE or UE on its first argument. For example, according to (18), we cannot establish that for any C and w and any P and Q

such that $P \Rightarrow Q$,

$$[\![no]\!]^{C,w}(Q)([\![\textit{likes his first book}]\!]^{C,w}) \Rightarrow [\![no]\!]^{C,w}(P)([\![\textit{likes his first book}]\!]^{C,w}).$$

To see why, assume as before that $[\![\textit{French professor}]\!]^{C,w} \Rightarrow [\![\textit{professor}]\!]^{C,w}$; it follows from the definition of '$\Rightarrow$' that $\mathrm{Dom}([\![\textit{French professor}]\!]^{C,w}) = \mathrm{Dom}([\![\textit{professor}]\!]^{C,w}) = D_e$ (given that we are now allowing partial functions, the domains of these functions could, in principle, be smaller; we come back to this issue in Section 4.1). But there is at least one C and w such that $C(w)$ has French and non-French professors, but none of them has in w a first book in $C(w)$, so $[\![no]\!]^{C,w}([\![\textit{professor}]\!]^{C,w})([\![\textit{likes his first book}]\!]^{C,w})$ and $[\![no]\!]^{C,w}([\![\textit{French professor}]\!]^{C,w})([\![\textit{likes his first book}]\!]^{C,w})$ are both undefined. Therefore, we cannot establish that $[\![no]\!]^{C,w}(Q) \Rightarrow [\![no]\!]^{C,w}(P)$ for all C, w, P and Q such that $P \Rightarrow Q$. (We also cannot establish that for some C, w, P and Q such that $P \Rightarrow Q$, $[\![no]\!]^{C,w}(Q) \not\Rightarrow [\![no]\!]^{C,w}(P)$.)

Similarly, within the semantic approach to presuppositions, (20) — and not (15) — is probably the proper meaning of *some*.

(20) For any w and C, and any $Z, Y \in D_{\langle e,t\rangle}$:
  a. $[\![some]\!]^{C,w}(Z)$ is defined *iff* $\{y \in C(w) \mid Z(y) = \mathrm{True}\} \neq \varnothing$;
  b. when $[\![some]\!]^{C,w}(Z)$ is defined, $[\![some]\!]^{C,w}(Z)(Y)$ is defined *iff* $\{y \in C(w) \mid Z(y) = \mathrm{True}\} \cap \mathrm{Dom}(Y) \neq \varnothing$;
  c. when defined, $[\![some]\!]^{C,w}(Z)(Y) = \mathrm{True}$ *iff* $\{y \in C(w) \mid Z(y) = \mathrm{True}\} \cap \{y \in \mathrm{Dom}(Y) \mid Y(y) = \mathrm{True}\} \neq \varnothing$.

Because of its second argument presupposition, *some* is not UE on its first argument.

As we saw in Section 2.1, *no* and *some* do come out DE and UE respectively on their first argument if we assume that they are total (and that *likes his first book* is total). If we don't want to give up on partiality completely, we may try to avoid the second argument projection problem by giving up only the second argument presupposition. Suppose we say that *No professor likes his first book* is (at least optionally) interpreted as 'No professor who has a first book likes his first book', via the local accommodation of the presupposition of *likes his first book* into the first argument position of *no* (along the lines of Berman 1991). As argued in von Fintel 2008, this is not a viable option, for the following reason (see also Beaver 2004). If local presupposition accommodation were an available process in the grammar, a locally accommodated presupposition would not be perceived as a presupposition at all, but rather as an embedded assertion. For example, (21a) and (21b) would both be acceptable. In point of fact, only (21a) is; (21b) sounds contradictory.

(21) a. No student who has a mother wrote to his mother, yet the students who don't have a mother wished they had a mother to write to.

b. #No student wrote to his mother, yet the students who don't have a mother wished they had a mother to write to.

So if *no* and *some* are indeed partial, they have a second argument presupposition (as in (18) and (20)), which prevents them from being DE/UE on their first argument.[5,6]

Note that neither the first argument presupposition nor the second argument presupposition prevents any determiner from being DE or UE on its *second* argument. For example, when we check whether *no* is DE on its second argument — that is, when we check whether $[\![no]\!]^{C,w}(Z)(Q) \Rightarrow [\![no]\!]^{C,w}(Z)(P)$ — by the definition of "DE" we only consider combinations of *no* and Z such that $[\![no]\!]^{C,w}(Z)$ is a function (i.e., is defined; so the presuppositions of Z, whatever they are, are satisfied), and only pairs $\langle P, Q\rangle$ such that $P \Rightarrow Q$ (whose domains, by the definition of '⇒', are D_e). This implies that when we say that *no* is DE on its second argument, we mean that for any Z, C and w such that $[\![no]\!]^{C,w}(Z)$ is defined, $[\![no]\!]^{C,w}(Z)$ is DE; when we say that *some* is UE on its second argument, we mean that for any Z, C and w such that $[\![some]\!]^{C,w}(Z)$ is defined, $[\![some]\!]^{C,w}(Z)$ is UE (see (III)–(IV), Appendix).

On to the P-to-Q problem. Notice that we cannot prove that the informal statement in (22) is valid. Yet it seems reasonable to take its validity for granted, given intuitions regarding *some*.

5 Global presupposition accommodation is certainly possible, and explains why *No professor likes his first book* is fine, while *No student likes his first book* sounds odd. If we don't know that the professors have written at least one book, we are willing to revise our beliefs upon hearing *No professor likes his first book*. But global accommodation of the presupposition that the students have written at least one book is usually harder (given that students are usually too young to have written a book).

6 It is worth noting that we only expect hard presuppositions (in the sense of Abusch 2002, 2010), but not soft presuppositions (such as the presupposition of *stop smoking*), to give rise to the second argument projection problem because, presumably, only the former are semantically encoded. Indeed, *No student stopped smoking, yet the students who smoke envy those who have never smoked* is not odd (cf. (21b)).

(22) For any type-appropriate α and β: (a) *iff* (b).

a. For any type-appropriate γ: [*Some* γ α] intuitively entails [*Some* γ β].

b. α intuitively entails β.

On the additional (reasonable) assumption that "intuitively entails" is modeled on some well-defined notion of formal entailment, we expect (23) to be a (provably) valid statement.

(23) For any type-appropriate α and β: (a) *iff* (b).

a. For any $Z \in D_{\langle e,t\rangle}$, C and w such that $[\![\textit{some}]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w})$ is defined: $[\![\textit{some}]\!]^{C,w}(Z)([\![\beta]\!]^{C,w})$ is defined and $[\![\textit{some}]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w})$ formally entails $[\![\textit{some}]\!]^{C,w}(Z)([\![\beta]\!]^{C,w})$.

b. For any C and w, $[\![\alpha]\!]^{C,w}$ formally entails $[\![\beta]\!]^{C,w}$.

A natural candidate for "formally entails" is '$\Rightarrow$', so let us replace all occurrences of "formally entails" in (23) with '$\Rightarrow$'. We get (24), which is valid on the assumption that all natural language functions are total.

(24) For and type-appropriate α and β: (a) *iff* (b).

a. For any $Z \in D_{\langle e,t\rangle}$, C and w such that $[\![\textit{some}]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w})$ is defined: $[\![\textit{some}]\!]^{C,w}(Z)([\![\beta]\!]^{C,w})$ is defined and $[\![\textit{some}]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w})$ $\Rightarrow [\![\textit{some}]\!]^{C,w}(Z)([\![\beta]\!]^{C,w})$.

b. For any C and w, $[\![\alpha]\!]^{C,w} \Rightarrow [\![\beta]\!]^{C,w}$.

Indeed, if all natural language functions are total, then for any α and β that denote functions of type $\langle e,t\rangle$, α and β denote total functions; and $[\![\alpha]\!]^{C,w} \Rightarrow [\![\beta]\!]^{C,w}$ for all C and w *iff* $[\![\textit{some}]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w}) \Rightarrow [\![\textit{some}]\!]^{C,w}(Z)([\![\beta]\!]^{C,w})$ for all $Z \in D_{\langle e,t\rangle}$, C and w.[7] But if natural language functions are potentially partial, we cannot prove that (24) is valid (or that it is not valid). For example, *likes his first book* and *likes something* denote functions of type $\langle e,t\rangle$, but if *likes his first book* is partial (as in (16)), we cannot establish that $[\![\textit{likes his first book}]\!]^{C,w} \Rightarrow [\![\textit{likes something}]\!]^{C,w}$ for all C and w: there are worlds w and individuals x such that x does not have in w a first book in

7 Why is that? It follows from the definition of UE and the proven claim that *some* is UE on its second argument that if for all C and w, $[\![\alpha]\!]^{C,w} \Rightarrow [\![\beta]\!]^{C,w}$, then for all Z, C and w, $[\![\textit{some}]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w}) \Rightarrow [\![\textit{some}]\!]^{C,w}(Z)([\![\beta]\!]^{C,w})$. In addition, if for some x, C and w, $[\![\alpha]\!]^{C,w}(x) = \text{True}$ and $[\![\beta]\!]^{C,w}(x) = \text{False}$, it follows from the semantics of *some* that for some C and w, $[\![\textit{some}]\!]^{C,w}(\lambda x \in D_e.\, [\![\beta]\!]^{C,w}(x) = \text{False})([\![\alpha]\!]^{C,w}) = \text{True}$ and $[\![\textit{some}]\!]^{C,w}(\lambda x \in D_e.\, [\![\beta]\!]^{C,w}(x) = \text{False})([\![\beta]\!]^{C,w}) = \text{False}$.

$C(w)$ and x likes nothing in w; that is, $[\![$*likes his first book*$]\!]^{C,w}(x)$ is undefined and $[\![$*likes something*$]\!]^{C,w}(x)$ = False.[8] (Nor can we establish that there are a C and a w such that $[\![$*likes his first book*$]\!]^{C,w} \not\Rrightarrow [\![$*likes something*$]\!]^{C,w}$.) For advocates of the semantic approach to presuppositions, this implies that "formally entails" in (23) is not '$\Rightarrow$'. Given the (assumed) validity of (22), we have to come up with an alternative definition of "formally entails" that would make (23) a valid statement.

Note that advocates of a purely pragmatic approach to presuppositions will probably consider these problems to be support for the pragmatic approach. After all, the second argument projection problem and the P-to-Q problem are byproducts of the assumption that the only way to account for the emergence of presuppositions is by treating natural language functions as potentially partial. We do not take issue with this position. Indeed, solving the second argument projection problem and the P-to-Q problem is the burden of advocates of the semantic approach. Of course, advocates of the pragmatic approach have to provide an alternative explanation for the emergence of presuppositions; this is a nontrivial task which we do not discuss any further.

In Section 3 we explore a semantic solution to the second argument projection problem that is based on Strawson entailment — formally, '$\overset{\text{ST}}{\Rightarrow}$' (roughly, $\overset{\text{ST}}{\Rightarrow}$-entailment in Section 1) — which is weaker than '$\Rightarrow$'. Accordingly, *no* and *some* come out formally Strawson downward- and upward- entailing, respectively, on their first argument. In Section 4 we solve the P-to-Q problem by modeling intuitive entailment on '$\Rrightarrow$' (roughly, $\Rrightarrow$-entailment in Section 1), which is defined in terms of '$\overset{\text{ST}}{\Rightarrow}$' and is stronger than '$\overset{\text{ST}}{\Rightarrow}$'. As we show, (23) is valid when "formally entails" is replaced with '$\Rrightarrow$'. One might wonder whether exploiting a trivalent logic, or a flattened LF, would offer other — perhaps simpler — semantic solutions to both these problems. In Section 5 we explore and reject two such alternatives.

3 Solving the second argument projection problem with Strawson entailment

Strawson entailment was introduced by von Fintel (1999, 2001) as part of an attempt to account for the distribution of weak NPIs (negative polarity items such as *any* and *ever*).[9] The notion of Strawson entailment has since

8 A detailed LF of *likes something* is [2 [*something* [3 [t_2 *likes* t_3]]]] (cf. Footnote3).

9 The distribution of other types of NPIs is a more complex matter (see, for example, Zwarts 1998).

proven useful not only for capturing generalizations regarding weak NPIs in various constructions (see, for example, Condoravdi 2010), but also for capturing generalizations regarding other linguistic phenomena such as modification by temporal adverbials (see Csirmaz 2008), covert reciprocity (see Schwarz 2006) and scalar implicatures (see Gajewski & Sharvit 2012). It is not our purpose here to provide new arguments for the claim that Strawson entailment underlies these linguistic phenomena (other theories are, of course, conceivable). We merely solve the second argument projection problem using Strawson entailment.

A well known theory of weak NPIs, due to Fauconnier (1978) and Ladusaw (1979), says that they are licensed in the scope of functions that are DE, such as sentential negation. (Given the definition of "DE" assumed here, the condition should be: An NPI is licensed in the scope of an α such that $[\![\alpha]\!]^{C,w}$ is DE for any C and w.) As is also well known (at least since Ladusaw 1979), there are expressions that license weak NPIs but are not even intuitively downward-entailing. *Only* is a typical example: *Only John has ever visited Paris* is grammatical, yet (25a) may be judged true when John arrived late.

(25) a. Only John arrived.
b. Only John arrived early.

Rather, (25a) intuitively Strawson-entails (25b). We say "(25a) intuitively Strawson-entails (25b)" to mean that the truth of (25a) and the presupposition of (25b) — *John arrived early* — intuitively guarantee the truth of (25b). More generally, whenever we say "Sentence A intuitively Strawson-entails sentence B", we mean that whenever A and the presuppositions of B are judged true, B is judged true. A similar behavior is exhibited by *sorry* and *longest*.[10]

Given intuitions regarding *only* and similar NPI-licensers, the suggestion in von Fintel 1999, 2001 is that weak NPIs are licensed in the scope of functions that are Strawson DE — or SDE — as defined in (26). The term "SUE", the "upward" counterpart of "SDE", is defined in (27).

(26) A function $f \in D_{\langle\sigma,\tau\rangle}$ is *SDE iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$ and $f(P)$ and $f(Q)$ are defined: $f(Q) \Rightarrow f(P)$.[11]

10 For example, *sorry* licenses NPIs (as in *I am sorry I ever met you*), but *Jon is sorry Jim left* does not intuitively entail *Jon is sorry Jim left early*. Rather, *Jon is sorry Bill left and he knows Bill left early* intuitively entails *Jon is sorry Jim left early* (because, presumably, *know p* is presupposed by *be sorry that p*).

11 Formulation (26) is not entirely faithful to von Fintel (1999), whose definition of SDE says "...such that $P \Rightarrow Q$ and $f(P)$ is defined: $f(Q) \Rightarrow f(P)$". This minor difference has no effect

(27) A function $f \in D_{\langle\sigma,\tau\rangle}$ is *SUE iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$ and $f(P)$ and $f(Q)$ are defined: $f(P) \Rightarrow f(Q)$.

Indeed, on the assumption that *only* denotes a partial function of type $\langle e, \langle\langle e,t\rangle, t\rangle\rangle$ as in, say, (28), it does not come out DE, but it comes out SDE on its second argument. Thus, the meaning of *Only John arrived* is derived as in (29). (Strictly for simplicity, we assume that *only x* is a VP-level operator, and not that *only* is a sentence-level operator.)

(28) For any C and w, $Z \in D_{\langle e,t\rangle}$ and $x \in D_e$:

a. $[\![only]\!]^{C,w}(x)$ is defined *iff* $x \in C(w)$;

b. when $[\![only]\!]^{C,w}(x)$ is defined, $[\![only]\!]^{C,w}(x)(Z)$ is defined *iff* $Z(x)$ is defined and $Z(x) = \text{True}$; and

c. when defined, $[\![only]\!]^{C,w}(x)(Z) = \text{True}$ *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} = \{x\}$.

(29) When defined,

$$\begin{aligned}&[\![[\textit{only John}]\ [\textit{arrived}]]\!]^{C,w}\\ &\quad = [\![only]\!]^{C,w}([\![John]\!]^{C,w})([\![arrived]\!]^{C,w})\\ &\quad = \text{True } \textit{iff } \{y \in C(w) \mid [\![arrived]\!]^{C,w}(y) = \text{True}\} = \{\text{John}\}.\end{aligned}$$

We say that *only* is SDE on its second argument because for any C and w, and any $x \in D_e$ such that $[\![only]\!]^{C,w}(x)$ is defined, $[\![only]\!]^{C,w}(x)$ is, by (26), SDE (see (V), Appendix). For example, for any C and w, it follows from the (nontrivial) assumption that $[\![\textit{arrived early}]\!]^{C,w} \Rightarrow [\![arrived]\!]^{C,w}$ that

$$\text{Dom}([\![\textit{arrived early}]\!]^{C,w}) = \text{Dom}([\![arrived]\!]^{C,w}) = D_e;$$

and it follows from (28) that when $[\![only]\!]^{C,w}([\![John]\!]^{C,w})([\![arrived]\!]^{C,w})$ and $[\![only]\!]^{C,w}([\![John]\!]^{C,w})([\![\textit{arrived early}]\!]^{C,w})$ are defined,

$$[\![only]\!]^{C,w}([\![John]\!]^{C,w})([\![arrived]\!]^{C,w}) = \text{False}$$

or

$$[\![only]\!]^{C,w}([\![John]\!]^{C,w})([\![\textit{arrived early}]\!]^{C,w}) = \text{True}.$$

In general, any DE function is also SDE, but the reverse does not hold; likewise, any function that is UE is also SUE, but the reverse does not hold.[12]

on the point made here, and we may ignore it. Also, the new condition on NPI licensing is: An NPI is licensed in the scope of an α such that $[\![\alpha]\!]^{C,w}$ is SDE for any C and w.

12 For discussion of challenges to the (S)DE theory of weak NPIs see, for example, Linebarger 1987, Rothschild 2002, Wagner 2006, Guerzoni & Sharvit 2007, Homer 2008, Crnič 2011.

Thus, Strawson upward/downward-entailingness provides us with a way to model properties of partial functions, and it seems promising to try to solve the second argument projection problem by appealing to it. The idea would be to say that *no* and *some*, as the partial functions in (18) and (20), are merely SDE and SUE respectively on their first argument (though they are DE and UE respectively on their second argument, as we saw in Section 2.2).

Some readers may be uncomfortable with this move, for the following reason. It relies on speakers' intuitions about Strawson entailment (e.g., on the claim that *No professor likes his first book* intuitively Strawson-entails *No French professor likes his first book*). To test such intuitions, we present speakers with a task that seems very strange; we ask them to decide whether the truth of A plus the presuppositions of B guarantees the truth of B. In their everyday life, speakers often ask themselves whether B follows from A, but rarely (probably never) whether B follows from "A plus the presuppositions of B". But notice that what we actually ask speakers to decide is whether the truth of "A and P" guarantees the truth of B, where P is the conjunction of the presuppositions of B which we explicitly spell out for them. What precisely the presuppositions of B are is determined independently. Granted, determining what a sentence presupposes is often subject to considerable theoretical debate, but that is an independent issue. Crucially, the task of figuring out whether B follows from "A and P" is no different from the task of figuring out whether B follows from A. And indeed, *No French professor likes his first book* follows from *No professor likes his first book and any and all French professors have a first book*, just like *Only John left early* follows from *Only John left and John left early*.

As we now show, the second argument presupposition prevents *no* and *some* from being SDE/SUE on their first argument, just like it prevents them from being DE/UE on their first argument. Fortunately, replacing SDE/SUE with a different notion of Strawson downward/upward-entailingness solves the problem.

Because of the second argument presupposition in (18), for any C and w with French and non-French professors in $C(w)$ that have in w no first book in $C(w)$, both $[\![no]\!]^{C,w}([\![\textit{French professor}]\!]^{C,w})([\![\textit{likes his first book}]\!]^{C,w})$ and $[\![no]\!]^{C,w}([\![\textit{professor}]\!]^{C,w})([\![\textit{likes his first book}]\!]^{C,w})$ are undefined. Therefore, it cannot be established that

$$[\![no]\!]^{C,w}([\![\textit{professor}]\!]^{C,w}) \Rightarrow [\![no]\!]^{C,w}([\![\textit{French professor}]\!]^{C,w})$$

and consequently, "*No* is SDE on its first argument" cannot be established. Similarly, "*Some* is SUE on its first argument" cannot be established. How does *only* avoid this problem and come out SDE on its second argument? The second argument of *only* is also its last argument: once all arguments have been fed, all the presuppositions associated with them are satisfied, and there is no additional source for potential presupposition failure. On the other hand, feeding the first argument of *no/some* does not guarantee satisfaction of the second argument presupposition; there can still be elements of $D_{\langle e,t\rangle}$ that do not satisfy it (as we saw).

As another illustration of this, consider *every*. For many speakers, (30b) presupposes that there are French students. For all speakers, (30a) intuitively Strawson-entails (30b), and (30c) intuitively entails (30a).

(30) a. Every student arrived.
b. Every French student arrived.
c. Every student arrived early.

For any C and w, $[\![every]\!]^{C,w}$ should come out SDE, and $[\![every]\!]^{C,w}(Z)$ should come out UE for any Z such that $[\![every]\!]^{C,w}(Z)$ is defined. Suppose *every* has the semantics in (31), with a "universal" second argument presupposition, and a "strong" first argument presupposition.

(31) For any w and C, and any $Z, Y \in D_{\langle e,t\rangle}$:
a. $[\![every]\!]^{C,w}(Z)$ is defined *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \neq \varnothing$;
b. when $[\![every]\!]^{C,w}(Z)$ is defined, $[\![every]\!]^{C,w}(Z)(Y)$ is defined *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \subseteq \text{Dom}(Y)$; and
c. when defined, $[\![every]\!]^{C,w}(Z)(Y) = \text{True}$ *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \subseteq \{y \in \text{Dom}(Y) \mid Y(y) = \text{True}\}$.

For any w and C, it follows from the assumption that $[\![arrived\ early]\!]^{C,w} \Rightarrow [\![arrived]\!]^{C,w}$ that $\text{Dom}([\![arrived\ early]\!]^{C,w}) = \text{Dom}([\![arrived]\!]^{C,w}) = D_e$; therefore, as long as there are students in $C(w)$ (or in other words, as long as $[\![every]\!]^{C,w}([\![student]\!]^{C,w})$ is defined),

$$[\![every]\!]^{C,w}([\![student]\!]^{C,w})([\![arrived\ early]\!]^{C,w}) = \text{False}$$

or

$$[\![every]\!]^{C,w}([\![student]\!]^{C,w})([\![arrived]\!]^{C,w}) = \text{True}.$$

Thus, *every* is indeed UE on its second argument (see (VI), Appendix). But when, for some C and w, there are French and non-French students in $C(w)$

but none of them has in w a (unique) mother in $C(w)$,

$$[\![every]\!]^{C,w}([\![student]\!]^{C,w})([\![likes\ his\ mother]\!]^{C,w})$$

and

$$[\![every]\!]^{C,w}([\![French\ student]\!]^{C,w})([\![likes\ his\ mother]\!]^{C,w})$$

are both undefined, and "*Every* is SDE on its first argument" cannot be established (even if we weaken the first argument presupposition of *every*).

Fortunately, a slightly different notion of Strawson downward/upward-entailingness does solve the second argument projection problem. Specifically, we propose SDE$^{\mathrm{ST}}$ and SUE$^{\mathrm{ST}}$, in (32) and (33) respectively, as alternatives to SDE in (26) and SUE in (27). Here, P and Q are constrained by '$\Rightarrow$' (as in (26) and (27)), but a new relation — '$\overset{\mathrm{ST}}{\Rightarrow}$' — relates $f(P)$ to $f(Q)$. This relation '$\overset{\mathrm{ST}}{\Rightarrow}$' is the Strawsonian counterpart of '$\Rightarrow$' defined in (34) (see Herdan & Sharvit 2006 and Gajewski 2007).

(32) A function $f \in D_{\langle\sigma,\tau\rangle}$ is *SDEST iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$ and $f(P)$ and $f(Q)$ are defined: $f(Q) \overset{\mathrm{ST}}{\Rightarrow} f(P)$.

(33) A function $f \in D_{\langle\sigma,\tau\rangle}$ is *SUEST iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$ and $f(P)$ and $f(Q)$ are defined: $f(P) \overset{\mathrm{ST}}{\Rightarrow} f(Q)$.

(34) **Cross-categorial $\overset{\mathrm{ST}}{\Rightarrow}$**

For all $p, q \in D_t$: $p \overset{\mathrm{ST}}{\Rightarrow} q$ if p = False or q = True,
$p \overset{\mathrm{ST}}{\nRightarrow} q$ if p = True and q = False.

For all $f, g \in D_{\langle\sigma,\tau\rangle}$: $f \overset{\mathrm{ST}}{\Rightarrow} g$ if for all $x \in \mathrm{Dom}(f) \cap \mathrm{Dom}(g)$, $f(x) \overset{\mathrm{ST}}{\Rightarrow} g(x)$,
$f \overset{\mathrm{ST}}{\nRightarrow} g$ if for some $x \in \mathrm{Dom}(f) \cap \mathrm{Dom}(g)$, $f(x) \overset{\mathrm{ST}}{\nRightarrow} g(x)$.

The relation '$\overset{\mathrm{ST}}{\Rightarrow}$' is explicit about the status of partial functions in the following way: for any pair of functions, it disregards anything that is not in the intersection of their domains. Otherwise, '$\overset{\mathrm{ST}}{\Rightarrow}$' is just like '$\Rightarrow$'.[13,14]

13 The definition of '$\overset{\mathrm{ST}}{\Rightarrow}$' in (34) fleshes out von Fintel's (1999) notion of Strawson validity ("an inference $p_1, \ldots, p_n \therefore q$ is *Strawson valid* iff the inference $p_1, \ldots, p_n, S \therefore q$ is classically valid, where S is a premise stating that the presuppositions of all the statements involved are satisfied").

14 While there is at least one w and C such that $[\![likes\ his\ mother]\!]^{C,W} \overset{\mathrm{ST}}{\Rightarrow} [\![likes\ his\ father]\!]^{C,W}$ holds vacuously, this is not the case for all w and C (specifically, those w and C where someone in $C(w)$ has a mother and a father). Notice that we could alternatively define '$\overset{\mathrm{ST}}{\Rightarrow}$' so that it is undefined for pairs $\langle f, g\rangle$ with nonintersecting domains. For current purposes, the differences between such a definition and (34) are insignificant.

The second argument projection problem is thus solved. For example, *no*, as defined in (18), comes out SDEST on its first argument (see (VII), Appendix). By (34), for any C, w and f such that

$$f \in \mathrm{Dom}(\llbracket no \rrbracket^{C,w}(\llbracket French\ professor \rrbracket^{C,w})) \cap \mathrm{Dom}(\llbracket no \rrbracket^{C,w}(\llbracket professor \rrbracket^{C,w})),$$

it is the case that

$$\llbracket no \rrbracket^{C,w}(\llbracket professor \rrbracket^{C,w})(f) \overset{\mathrm{ST}}{\Rightarrow} \llbracket no \rrbracket^{C,w}(\llbracket French\ professor \rrbracket^{C,w})(f);$$

therefore,

$$\llbracket no \rrbracket^{C,w}(\llbracket professor \rrbracket^{C,w}) \overset{\mathrm{ST}}{\Rightarrow} \llbracket no \rrbracket^{C,w}(\llbracket French\ professor \rrbracket^{C,w}).$$

If some professors in $C(w)$ lack a first book, $\llbracket$*likes his first book*$\rrbracket^{C,w}$ is ignored because it is not in

$$\mathrm{Dom}(\llbracket no \rrbracket^{C,w}(\llbracket French\ professor \rrbracket^{C,w})) \cap \mathrm{Dom}(\llbracket no \rrbracket^{C,w}(\llbracket professor \rrbracket^{C,w})).$$

By similar reasoning, *some* comes out SUEST on its first argument, and *every* SDEST on its first argument (see (VIII)–(IX), Appendix). Since *only* is SDE (see (V), Appendix), it is trivially SDEST.

This solution to the second argument projection problem is somewhat disappointing. For example, it makes *some* SUEST on its first argument, but it is intuitively upward-entailing, and not merely intuitively Strawson upward-entailing, on its first argument. As we will now see, the solution to the P-to-Q problem also solves the second argument projection problem in a more satisfactory manner.

4 Solving the *P*-to-*Q* problem with Strawson entailment

4.1 The scope of the problem

Before offering a solution to the P-to-Q problem, let us remind ourselves of what it is and expand its scope a bit. Given our intuitions about *some*, we expect (23), repeated below, to be valid.

(23) For any type-appropriate α and β: (a) *iff* (b).

a. For any $Z \in D_{\langle e,t \rangle}$, C and w such that $[\![some]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w})$ is defined: $[\![some]\!]^{C,w}(Z)([\![\beta]\!]^{C,w})$ is defined and $[\![some]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w})$ formally entails $[\![some]\!]^{C,w}(Z)([\![\beta]\!]^{C,w})$.

b. For any C and w, $[\![\alpha]\!]^{C,w}$ formally entails $[\![\beta]\!]^{C,w}$.

As we saw in Section 2.2, the problem is that "formally entails" cannot mean '⇒', within the semantic approach to presuppositions, because of pairs such as ⟨*likes his first book*, *likes something*⟩.

There are other generalizations that show that "formally entails" is not '⇒'; one of them also shows that the solution in Section 3 to the second argument projection problem is not satisfactory as far as *some* is concerned. Recall that our definitions make *some* merely SUE$^{\mathrm{ST}}$ and *every* merely SDE$^{\mathrm{ST}}$ on their first argument. But *some* is intuitively upward-entailing on its first argument, as shown by the fact that *Some student who likes his first book arrived* intuitively entails — and not merely intuitively Strawson-entails — *Some student who likes something arrived*. This means that we expect (35) to be valid just as much as we expect (23) to be valid.

(35) For any type-appropriate α and β: (a) *iff* (b).

a. For any C and w such that $[\![some]\!]^{C,w}([\![\alpha]\!]^{C,w})$ is defined: $[\![some]\!]^{C,w}([\![\beta]\!]^{C,w})$ is defined and $[\![some]\!]^{C,w}([\![\alpha]\!]^{C,w})$ formally entails $[\![some]\!]^{C,w}([\![\beta]\!]^{C,w})$.

b. For any C and w, $[\![\alpha]\!]^{C,w}$ formally entails $[\![\beta]\!]^{C,w}$.

On the other hand, given that for many speakers *Every student who likes someone arrived* merely intuitively Strawson-entails *Every student who likes his mother arrived*, there is no problem with classifying *every* as merely formally Strawson downward-entailing. However, given that (36a) merely intuitively Strawson-entails (36b) (it is possible that all the students are not book-destroyers, yet none of them has a unique first book), *no* should come out formally Strawson downward-entailing on its second argument and not DE, as it is classified now, just as it comes out formally Strawson downward-entailing on its first argument.

(36) a. No student destroyed a book.

b. No student destroyed his first book.

It is also important to acknowledge that the P-to-Q problem is not confined to pairs such as ⟨*likes his first book*, *likes something*⟩. So far we have assumed, for convenience, that for any C and w, $[\![\textit{French student}]\!]^{C,w} \Rightarrow [\![\textit{student}]\!]^{C,w}$. But in fact, these functions, too, may be partial; that is to say, even innocent-looking predicates such as *student* may carry nontrivial presuppositions (see Magidor 2013). For example, there might be individuals, such as the chair I'm sitting on, that cannot, *in principle*, be students in at least one possible world. We find ourselves, then, in a rather strange meta-theoretical situation. We can easily formally prove that *no* is DE and *some* UE on their second arguments (see (III)–(IV), Appendix), but if all world-dependent predicates are potentially partial, no world-dependent pair of predicates can actually illustrate these properties of *some* and *no*. For example, on the assumption that *John*, *be* and *self-identical* are rigid designators, $[\![\textit{be John}]\!]^{C,w} \Rightarrow [\![\textit{be self-identical}]\!]^{C,w}$ (for all C and w); and indeed *Some student is John* intuitively entails *Some student is self-identical*. However, *Some boy is a French student* intuitively entails *Some boy is a student*, yet we cannot establish that $[\![\textit{French student}]\!]^{C,w} \Rightarrow [\![\textit{student}]\!]^{C,w}$ (for all C and w).

4.2 The solution

4.2.1 Step 1: Constraining P and Q with $\Rrightarrow$

We introduce '$\Rrightarrow$' as a new formal notion of intuitive entailment, to replace all occurrences of '$\Rightarrow$' in the definitions of (Strawson) upward- and downward-entailingness. Informally, $f \Rrightarrow g$ iff (i) f Strawson-entails g, and (ii) satisfaction of the presuppositions of f guarantees the satisfaction of the presuppositions of g. More formally, we define '$\Rrightarrow$' as in (37).

(37) **Cross-categorial $\Rrightarrow$**

if $f, g \in D_t$, then $f \Rrightarrow g$ if $f \overset{\text{ST}}{\Rightarrow} g$

$f \not\Rrightarrow g$ if $f \overset{\text{ST}}{\not\Rightarrow} g$

if $f, g \in D_{\langle\sigma,\tau\rangle}$, then $f \Rrightarrow g$ if
for all $x \in \text{Dom}(f) \cap \text{Dom}(g)$, $f(x) \Rrightarrow g(x)$,
and $\text{Dom}(f) \subseteq \text{Dom}(g)$

$f \not\Rrightarrow g$ if
for some $x \in \text{Dom}(f) \cap \text{Dom}(g)$, $f(x) \not\Rrightarrow g(x)$,
or $\text{Dom}(f) \nsubseteq \text{Dom}(g)$

An example of a pair of predicates that satisfies '$\Rrightarrow$' in all C and w is $\langle$*likes his first book*, *likes something*$\rangle$. This is because for any C and w,

i. $[\![\textit{likes his first book}]\!]^{C,w} \stackrel{\text{ST}}{\Rightarrow} [\![\textit{likes something}]\!]^{C,w}$ (that is, if x satisfies the presuppositions of both, $[\![\textit{likes his first book}]\!]^{C,w}(x) = \text{False}$ or $[\![\textit{likes something}]\!]^{C,w}(x) = \text{True}$), and

ii. if x satisfies the presuppositions of $[\![\textit{likes his first book}]\!]^{C,w}$ (x has a first book in w and x is capable, *in principle*, of liking his first book), x automatically satisfies the presuppositions of $[\![\textit{likes something}]\!]^{C,w}$ (x is capable, *in principle*, of liking something).

Other examples are the pairs ⟨*French student*, *student*⟩ and ⟨*be John*, *be self-identical*⟩. An example of a pair of predicates that satisfies '$\stackrel{\text{ST}}{\Rightarrow}$' but does not satisfy '$\Rrightarrow$' in all C and w is ⟨*stabbed all his siblings*, *stabbed all his younger siblings*⟩. This is because

i. for any C and w,

$$[\![\textit{stabbed all of his siblings}]\!]^{C,w} \stackrel{\text{ST}}{\Rightarrow} [\![\textit{stabbed all of his younger siblings}]\!]^{C,w}$$

(if x stabbed in w all of x's siblings in $C(w)$ and x has in w younger siblings in $C(w)$, x stabbed in w all of x's younger siblings in $C(w)$), but

ii. for some x, C and w, x has in w siblings in $C(w)$ (satisfying the presupposition of $[\![\textit{stabbed all of his siblings}]\!]^{C,w}$), but none of them are younger than x in w (failing to satisfy the presupposition of $[\![\textit{stabbed all of his younger siblings}]\!]^{C,w}$).

The pair

⟨*has siblings that he likes*, *doesn't hate all of his siblings*⟩

does not satisfy '$\Rrightarrow$', but it satisfies Strawson equivalence, whereas

⟨*likes his first book*, *likes something*⟩,
⟨*French student*, *student*⟩, and
⟨*stabbed all of his siblings*, *stabbed all of his younger siblings*⟩

do not.

We define $\text{DE}^{\Rrightarrow}$ and $\text{UE}^{\Rrightarrow}$ as in (38), where P and Q are constrained by '$\Rrightarrow$', and the relation between $f(P)$ and $f(Q)$ is also '$\Rrightarrow$'. They are the new formal notions of downward- and upward-entailingness.

(38) **Downward/upward-entailingness**

a. A function $f \in D_{\langle\sigma,\tau\rangle}$ is **DE**$^{\Rightarrow}$ *iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$ and $f(Q)$ is defined: $f(P)$ is defined and $f(Q) \Rightarrow f(P)$.

b. A function $f \in D_{\langle\sigma,\tau\rangle}$ is **UE**$^{\Rightarrow}$ *iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$ and $f(P)$ is defined: $f(Q)$ is defined and $f(P) \Rightarrow f(Q)$.

For example, *some* is UE$^{\Rightarrow}$ on its second argument, as illustrated by the fact that for any C and w, $[\![\textit{likes his mother}]\!]^{C,w} \Rightarrow [\![\textit{likes someone}]\!]^{C,w}$ and if

$$[\![\textit{some}]\!]^{C,w}([\![\textit{student}]\!]^{C,w})([\![\textit{likes his mother}]\!]^{C,w})$$

is defined, then

$$[\![\textit{some}]\!]^{C,w}([\![\textit{student}]\!]^{C,w})([\![\textit{likes someone}]\!]^{C,w})$$

is defined and

$$[\![\textit{some}]\!]^{C,w}([\![\textit{student}]\!]^{C,w})([\![\textit{likes his mother}]\!]^{C,w}) \Rightarrow [\![\textit{some}]\!]^{C,w}([\![\textit{student}]\!]^{C,w})([\![\textit{likes someone}]\!]^{C,w}).$$

Likewise, *some* is UE$^{\Rightarrow}$ on its first argument and *every* is UE$^{\Rightarrow}$ on its second argument.

No is not DE$^{\Rightarrow}$ on either argument, as suggested by the fact that *No student destroyed a book* does not intuitively entail *No student destroyed his first book* (though it intuitively Strawson-entails it): it is possible that all the students are not book-destroyers, yet none of them has a (unique) first book. Likewise, *only* is not DE$^{\Rightarrow}$ on its second argument, and *every* is not DE$^{\Rightarrow}$ on its first argument.

We also define SDE and SUE as in (39a)–(39b), where P and Q are constrained by '$\Rightarrow$' and the relation between $f(P)$ and $f(Q)$ is '$\overset{\text{ST}}{\Rightarrow}$'. They are the new formal notions of Strawson upward- and downward-entailingness.

(39) **Strawson downward/upward-entailingness**

a. A function $f \in D_{\langle\sigma,\tau\rangle}$ is **SDE**$^{\Rightarrow}$ *iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$, and $f(P)$ and $f(Q)$ are defined: $f(Q) \overset{\text{ST}}{\Rightarrow} f(P)$.

b. A function $f \in D_{\langle\sigma,\tau\rangle}$ is **SUE**$^{\Rightarrow}$ *iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$, and $f(P)$ and $f(Q)$ are defined: $f(P) \overset{\text{ST}}{\Rightarrow} f(Q)$.

Being UE$^{\Rightarrow}$, *some* is also SUE$^{\Rightarrow}$ on both its arguments. *Only* is merely SDE$^{\Rightarrow}$ on its second argument, *no* is merely SDE$^{\Rightarrow}$ on both its arguments and *every* is merely SDE$^{\Rightarrow}$ on its first argument and SUE$^{\Rightarrow}$ on its second. Some of the results are summarized in (40).

(40) a. **First argument properties**

	$\text{DE}^{\Rightarrow}$	$\text{UE}^{\Rightarrow}$	$\text{SDE}^{\Rightarrow}$	$\text{SUE}^{\Rightarrow}$
some	−	+	−	+
no	−	−	+	−
every	−	−	+	−

b. **Second argument properties**

	$\text{DE}^{\Rightarrow}$	$\text{UE}^{\Rightarrow}$	$\text{SDE}^{\Rightarrow}$	$\text{SUE}^{\Rightarrow}$
some	−	+	−	+
no	−	−	+	−
every	−	+	−	+

4.2.2 Step 2: Constraining P and Q with $\overset{\text{ST}}{\Rightarrow}$

In the definitions of (S)DE$^{\Rightarrow}$ and (S)UE$^{\Rightarrow}$ above, P and Q are constrained by '$\Rightarrow$'. The question that naturally arises is whether corresponding properties where P and Q are constrained by '$\overset{\text{ST}}{\Rightarrow}$' are linguistically significant. In other words, are there natural language functions that are SDE$^{\overset{\text{ST}}{\Rightarrow}}$ or SUE$^{\overset{\text{ST}}{\Rightarrow}}$, as these terms are defined in (41), where P and Q are constrained by '$\overset{\text{ST}}{\Rightarrow}$' and the relation between $f(P)$ and $f(Q)$ is $\overset{\text{ST}}{\Rightarrow}$? Also, are there natural language functions that are DE$^{\overset{\text{ST}}{\Rightarrow}}$ or UE$^{\overset{\text{ST}}{\Rightarrow}}$, as these terms are defined in (42), where P and Q are constrained by '$\overset{\text{ST}}{\Rightarrow}$' and the relation between $f(P)$ and $f(Q)$ is '$\Rightarrow$'?

(41) a. A function $f \in D_{\langle\sigma,\tau\rangle}$ is **SDE**$^{\overset{\text{ST}}{\Rightarrow}}$ *iff* for any $P, Q \in D_\sigma$ such that $P \overset{\text{ST}}{\Rightarrow} Q$, and $f(P)$ and $f(Q)$ are defined: $f(Q) \overset{\text{ST}}{\Rightarrow} f(P)$.

b. A function $f \in D_{\langle\sigma,\tau\rangle}$ is **SUE**$^{\overset{\text{ST}}{\Rightarrow}}$ *iff* for any $P, Q \in D_\sigma$ such that $P \overset{\text{ST}}{\Rightarrow} Q$, and $f(P)$ and $f(Q)$ are defined: $f(P) \overset{\text{ST}}{\Rightarrow} f(Q)$.

(42) a. A function $f \in D_{\langle\sigma,\tau\rangle}$ is **DE**$^{\overset{\text{ST}}{\Rightarrow}}$ *iff* for any $P, Q \in D_\sigma$ such that $P \overset{\text{ST}}{\Rightarrow} Q$ and $f(Q)$ is defined: $f(P)$ is defined and $f(Q) \Rightarrow f(P)$.

b. A function $f \in D_{\langle\sigma,\tau\rangle}$ is **UE**$^{\overset{\text{ST}}{\Rightarrow}}$ *iff* for any $P, Q \in D_\sigma$ such that $P \overset{\text{ST}}{\Rightarrow} Q$ and $f(P)$ is defined: $f(Q)$ is defined and $f(P) \Rightarrow f(Q)$.

The answer to the first question is not obvious, and the answer to the second question is clearly "no". To see this, let us discuss some concrete cases.

If *every* is SDE$^{\overset{\text{ST}}{\Rightarrow}}$ on its first argument, we expect (43a) to intuitively Strawson entail (43b).

(43) a. Every student who stabbed all of his younger siblings went to jail.

b. Every student who stabbed all of his siblings went to jail.

Recall from Section 3 that when we check whether A intuitively Strawson-entails B, we have to decide what B presupposes, conjoin those presuppositions with A and check whether B follows. There doesn't seem to be any reason to say that (43a)–(43b) presuppose more than is implied by (31), repeated below.

(31) For any w and C, and any $Z, Y \in D_{\langle e,t \rangle}$:
 a. $[\![every]\!]^{C,w}(Z)$ is defined *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \neq \varnothing$;
 b. when $[\![every]\!]^{C,w}(Z)$ is defined, $[\![every]\!]^{C,w}(Z)(Y)$ is defined *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \subseteq \text{Dom}(Y)$; and
 c. when defined, $[\![every]\!]^{C,w}(Z)(Y) = \text{True}$ *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \subseteq \{y \in \text{Dom}(Y) \mid Y(y) = \text{True}\}$.

Suppose, then, that there are students who stabbed all of their younger siblings and students who stabbed all of their siblings, and that they all can, in principle, go to jail. In such a state of affairs, (43a) does not intuitively Strawson-entail (43b): when some student has only older siblings, and he stabbed them all and didn't go to jail, (43b) is intuitively false, but (43a) may be intuitively true. This implies that *every* is not SDE$^{\overset{ST}{\Rightarrow}}$ on its first argument, but merely SDE$^{\Rightarrow}$. Notice, though, that if *every* had a super-strong first argument presupposition — namely, $\{y \in C(w) \mid Z(y) = \text{True}\} \neq \varnothing$ and $C(w) \subseteq \text{Dom}(Z)$ — then (43a) *would* intuitively Strawson-entail (43b). We would expect speakers who have such a semantics for *every* to find both (44) and (45) incoherent.

(44) Every prisoner who had stabbed all of his siblings was denied an early hearing. Prisoner John never had any siblings and got an early hearing.

(45) Every prisoner who was denied an early hearing had stabbed all of his siblings. Prisoner John appealed the denial of his early hearing because he never had any siblings.

In an informal survey we conducted of fifteen speakers, all participants found (45) to be incoherent, but only two found (44) to be incoherent. This suggests the following:

i. For those speakers who found (44) incoherent, *every* might have a super-strong first argument presupposition, requiring John to have had at least one sibling at some point.

ii. For those speakers who found (44) coherent, *every* seems not to have a super-strong first argument presupposition — so John need not have had any siblings ever.

iii. No speaker excluded John from $C(w)$; for if they had, they would have found both (44) and (45) coherent.

It seems, then, that for (ii)-type speakers (the majority), *every* has the first argument presupposition in (31), and is SDE$^{\Rightarrow}$ — not SDE$\overset{\text{ST}}{\Rightarrow}$ — on its first argument. As the readers can verify, *some* is SUE$^{\Rightarrow}$ — not SUE$\overset{\text{ST}}{\Rightarrow}$ — on its first argument

Next, if *every* is SUE$\overset{\text{ST}}{\Rightarrow}$ on its second argument, we expect (46a) to intuitively Strawson-entail (46b), and if *some* is SUE$\overset{\text{ST}}{\Rightarrow}$ on its second argument, we expect (47a) to intuitively Strawson-entail (47a).

(46) a. Every student stabbed all of his siblings.
b. Every student stabbed all of his younger siblings.

(47) a. Some student stabbed all of her siblings.
b. Some student stabbed all of her younger siblings.

When all the students stabbed all their siblings, but not all students have younger siblings, not all students are younger-sibling-stabbers; this means that (46a) intuitively Strawson-entails (46b) only if *every* indeed has a "universal" second argument presupposition, as in (31). As mentioned in Section 3, there is no agreement among scholars about the quantificational force of the second argument presupposition. It has been suggested (see Chemla 2009) that the quantificational force of the second argument presupposition is predictable from the quantificational force of the assertion of the determiner. And indeed, *some* — which has an "existential" assertion as in (20), repeated below — seems to have an "existential" second argument presupposition.

(20) For any w and C, and any $Z, Y \in D_{\langle e,t\rangle}$:
a. $[\![\textit{some}]\!]^{C,w}(Z)$ is defined *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \neq \varnothing$;
b. when $[\![\textit{some}]\!]^{C,w}(Z)$ is defined, $[\![\textit{some}]\!]^{C,w}(Z)(Y)$ is defined *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \text{Dom}(Y) \neq \varnothing$;
c. when defined, $[\![\textit{some}]\!]^{C,w}(Z)(Y) = \text{True}$ *iff* $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \{y \in \text{Dom}(Y) \mid Y(y) = \text{True}\} \neq \varnothing$.

Indeed, when one student has younger siblings, (47a) could be intuitively true while (47b) is intuitively false (rather than a presupposition failure). This is further supported by the coherence of (48), confirming that *some* is not '$\text{SUE}^{\overset{\text{ST}}{\Rightarrow}}$' on its second argument.

(48) Some student likes her mother, but not all students have a mother.

On the other hand, since it seems that *every* has a "universal" assertion and a "universal" second argument presupposition, it may very well be $\text{SUE}^{\overset{\text{ST}}{\Rightarrow}}$ on its second argument. It is worth noting that if *some* had a "universal" second argument presupposition, it would still be $\text{UE}^{\Rrightarrow}$ on its second argument, but merely $\text{SUE}^{\Rrightarrow}$ on its first argument.

Finally, no determiner that we are aware of is either $\text{UE}^{\overset{\text{ST}}{\Rightarrow}}$ or $\text{DE}^{\overset{\text{ST}}{\Rightarrow}}$. For example, if *some* were $\text{UE}^{\overset{\text{ST}}{\Rightarrow}}$ on its second argument, we would expect *Some student stabbed all of her siblings* to intuitively entail *Some student stabbed all of her younger siblings*, but it does not. Similarly, *some* is not $\text{UE}^{\overset{\text{ST}}{\Rightarrow}}$ on its first argument and *every* is not $\text{UE}^{\overset{\text{ST}}{\Rightarrow}}$ on its second argument. *No* is trivially not $\text{DE}^{\overset{\text{ST}}{\Rightarrow}}$ (on either argument), since it is not $\text{DE}^{\Rrightarrow}$ (on either argument). Likewise, *every* is trivially not $\text{DE}^{\overset{\text{ST}}{\Rightarrow}}$ on its first argument.

4.2.3 Step 3: Constraining *P* and *Q* with $\dashrightarrow$

None of the definitions of the determiner properties we have discussed relies on '$\Rightarrow$'. What, then, is its status? As already mentioned in Section 4.1, '$\Rightarrow$' doesn't even apply to ⟨*French student*, *student*⟩, because there might be individuals that cannot be a student or be French in principle. We also noted that '$\Rightarrow$' is applicable in all possible worlds only to some world-independent $\langle e, t\rangle$ pairs (e.g., ⟨*be John*, *be self-identical*⟩). Importantly, '$\Rrightarrow$' is applicable to those pairs as well, so there is no need to use '$\Rightarrow$' in any of the definitions of $\text{DE}^{\Rrightarrow}$, $\text{UE}^{\Rrightarrow}$, $\text{SDE}^{\Rrightarrow}$ or $\text{SUE}^{\Rrightarrow}$. But we can and should make more subtle distinctions. For example, we want to distinguish between ⟨*French student*, *student*⟩ and ⟨*likes his mother*, *likes someone*⟩. '$\Rrightarrow$' is applicable to both of them in all possible worlds, but they do differ from each other: in the first pair, both members have the same presuppositions (it seems that being able to be a French student, *in principle*, and being able to be a student, *in principle*, amount to the same set of requirements, whatever they are). To capture this, we define '$\dashrightarrow$', which is applicable to ⟨*French student*, *student*⟩ but not to ⟨*likes his mother*, *likes someone*⟩.

(49) **Cross-categorial** $\dashrightarrow$
if $f, g \in D_t$, then $f \dashrightarrow g$ if $f \Rightarrow g$
$f \not\dashrightarrow g$ if $f \not\Rightarrow g$
if $f, g \in D_{\langle\sigma,\tau\rangle}$, then $f \dashrightarrow g$ if
for all $x \in \mathrm{Dom}(f) \cap \mathrm{Dom}(g)$, $f(x) \dashrightarrow g(x)$,
and $\mathrm{Dom}(f) = \mathrm{Dom}(g)$
$f \not\dashrightarrow g$ if
for some $x \in \mathrm{Dom}(f) \cap \mathrm{Dom}(g)$, $f(x) \not\dashrightarrow g(x)$,
or $\mathrm{Dom}(f) \neq \mathrm{Dom}(g)$

The contrast below suggests that '$\dashrightarrow$' is indeed linguistically significant: (50a) does not intuitively entail (50b), it merely intuitively Strawson-entails it (because (50a) may be intuitively true while the students have no first book), but (51a) does intuitively entail (51b).

(50) a. No student destroyed a book.
b. No student destroyed his first book.

(51) a. No student stabbed a professor.
b. No student stabbed a French professor.

We established in Section 4.2.1 that *no* is not $\mathrm{DE}^{\Rightarrow}$; this is consistent with the judgments regarding (50). But judgments regarding (51) indicate that *no* is $\mathrm{DE}^{\dashrightarrow}$, as defined in (52).

(52) A function $f \in D_{\langle\sigma,\tau\rangle}$ is $\mathbf{DE}^{\dashrightarrow}$ *iff* for any $P, Q \in D_\sigma$ such that $P \dashrightarrow Q$ and $f(Q)$ is defined: $f(P)$ is defined and $f(Q) \Rightarrow f(P)$.

Classical entailment doesn't seem to be linguistically significant. It doesn't seem that any linguistically significant property of determiners is defined exclusively in terms of '$\Rightarrow$'.

We are now in a position to solve the P-to-Q problem. Let us weaken the first argument presupposition of *some* in (20a) to $C(w) \cap \mathrm{Dom}(Z) \neq \varnothing$. *Some* is still $\mathrm{UE}^{\Rightarrow}$ on its first and second arguments; moreover, (53a)–(53b) are both valid (weakening (20a) guarantees that Strawson-equivalent pairs such as $\langle$*has siblings that he likes*, *does not hate all of his siblings*$\rangle$ do not invalidate (53b)). In addition, (54a) is valid (for those speakers who do not have a super-strong first argument presupposition for *every*) and (54b) is also valid.

(53) a. For any type-appropriate α and β: (i) *iff* (ii).

(i) For any $Z \in D_{\langle e,t \rangle}$, C and w such that $[\![some]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w})$ is defined: $[\![some]\!]^{C,w}(Z)([\![\beta]\!]^{C,w})$ is defined and

$$[\![some]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w}) \Rightarrow [\![some]\!]^{C,w}(Z)([\![\beta]\!]^{C,w}).$$

(ii) For any C and w, $[\![\alpha]\!]^{C,w} \Rightarrow [\![\beta]\!]^{C,w}$.

b. For any type-appropriate α and β: (i) *iff* (ii).

(i) For any C and w such that $[\![some]\!]^{C,w}([\![\alpha]\!]^{C,w})$ is defined: $[\![some]\!]^{C,w}([\![\beta]\!]^{C,w})$ is defined and

$$[\![some]\!]^{C,w}([\![\alpha]\!]^{C,w}) \Rightarrow [\![some]\!]^{C,w}([\![\beta]\!]^{C,w}).$$

(ii) For any C and w, $[\![\alpha]\!]^{C,w} \Rightarrow [\![\beta]\!]^{C,w}$.

(54) a. For any type-appropriate α and β that are not Strawson-equivalent, (i) *iff* (ii).

(i) For any C and w such that both $[\![every]\!]^{C,w}([\![\alpha]\!]^{C,w})$ and $[\![every]\!]^{C,w}([\![\beta]\!]^{C,w})$ are defined:

$$[\![every]\!]^{C,w}([\![\alpha]\!]^{C,w}) \overset{\text{ST}}{\Rightarrow} [\![every]\!]^{C,w}([\![\beta]\!]^{C,w}).$$

(ii) For any C and w, $[\![\beta]\!]^{C,w} \Rightarrow [\![\alpha]\!]^{C,w}$.

b. For any type-appropriate α and β that are not Strawson-equivalent, (i) *iff* (ii).

(i) For any $Z \in D_{\langle e,t \rangle}$, C and w such that $[\![no]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w})$ is defined: $[\![no]\!]^{C,w}(Z)([\![\beta]\!]^{C,w})$ is defined and

$$[\![no]\!]^{C,w}(Z)([\![\alpha]\!]^{C,w}) \dashrightarrow [\![no]\!]^{C,w}(Z)([\![\beta]\!]^{C,w}).$$

(ii) For any C and w, $[\![\beta]\!]^{C,w} \dashrightarrow [\![\alpha]\!]^{C,w}$.

5 Alternative solutions

To recap, ignoring the P-to-Q problem, we solved the second argument projection problem in Section 3 by classifying determiners that are intuitively upward-entailing on their first argument as SUE$^{\text{ST}}$ on their first argument, as this term is defined in (33), repeated below (a similar move takes care of determiners that are intuitively downward-entailing on their first argument).

(33) A function $f \in D_{\langle \sigma,\tau \rangle}$ is **SUE**$^{\textbf{ST}}$ *iff* for any $P, Q \in D_\sigma$ such that $P \Rightarrow Q$ and $f(P)$ and $f(Q)$ are defined: $f(P) \overset{\text{ST}}{\Rightarrow} f(Q)$.

To also solve the P-to-Q problem, in Section 4 we disowned '$\Rightarrow$' and replaced it with '$\Rrightarrow$', itself defined in terms of '$\overset{\mathrm{ST}}{\Rightarrow}$' (see the valid (53) and (54) in Section 4.2.3). As we now show, it is easy to solve the second argument projection problem without appealing to '$\overset{\mathrm{ST}}{\Rightarrow}$'. But solving the P-to-Q problem without appealing to '$\overset{\mathrm{ST}}{\Rightarrow}$' is harder.

One alternative that easily solves the second argument projection problem without appealing to '$\overset{\mathrm{ST}}{\Rightarrow}$' involves expressing Strawson entailingness in Schönfinkelized terms (effectively flattening the tree structure). Accordingly, we may define SUE$^{\mathrm{Sch}}$ as follows (cf. Heim & Kratzer 1998: 157).

(55) a. f is SUE$^{\mathrm{Sch}}$ on its first argument *iff* for all sets S, S', S'': if $S \subseteq S'$, $F_f(S, S'') = \text{True}$ and $\langle S', S'' \rangle \in \mathrm{Dom}(F_f)$, then $F_f(S', S'') = \text{True}$.

b. f is SUE$^{\mathrm{Sch}}$ on its second argument *iff* for all sets S, S', S'': if $S \subseteq S'$, $F_f(S'', S) = \text{True}$ and $\langle S'', S' \rangle \in \mathrm{Dom}(F_f)$, then $F_f(S'', S') = \text{True}$.

For example, for any C and w, $F^{C,w}_{[\![some]\!]}$ — the Schönfinkel-counterpart of $[\![some]\!]^{C,w}$ — is that function g such that

i. $$\mathrm{Dom}(g) = \Big\{\langle A, B\rangle \;\Big|\; \begin{array}{l}\text{there is a } P \in \mathrm{Dom}([\![\mathit{some}]\!]^{C,w}) \\ \text{and a } Q \in \mathrm{Dom}([\![\mathit{some}]\!]^{C,w}(P)) \\ \text{such that } A = \{x \mid P(x) = \text{True}\} \\ \text{and } B = \{x \mid Q(x) = \text{True}\}\end{array}\Big\}$$

ii. $\mathrm{Ran}(g) \subseteq D_t$, and

iii. for all $\langle X, Y\rangle \in \mathrm{Dom}(g)$, $g(X, Y) = \text{True}$ *iff* $C(w) \cap X \cap Y \neq \varnothing$.

Unlike $[\![some]\!]^{C,w}$, $F^{C,w}_{[\![some]\!]}$ applies to its two arguments at the same time, so the second argument projection problem does not arise, and by (55), $[\![some]\!]^{C,w}$ is SUE$^{\mathrm{Sch}}$ on its first and second arguments.[15]

Another alternative that easily solves the second argument projection problem involves introducing a third truth value. Let D_t be $\{\text{True}, \text{False}, \text{Und}\}$, and let '$\overset{\mathrm{Ł}}{\Rightarrow}$' be as in (56)–(57) (this definition is based on Łukasiewicz 1920; a definition based on Kleene 1952 would be at least as problematic).

(56) For any $p, q \in \{\text{True}, \text{False}, \text{Und}\}$:

15 Another alternative that belongs to the same family of alternatives is the one that says that linguistic rules do not refer to (S)DE/(S)UE functions, but rather to (S)DE/(S)UE environments (see Gajewski 2007, Homer 2011). This solution, too, avoids the second argument projection problem by effectively flattening the syntactic tree.

a. $p \overset{L}{\Rightarrow} q$ if p = False or one of (i)–(iii) holds:
   (i) p = True and q = True,
   (ii) p = Und and q = True,
   (iii) p = Und and q = Und.
b. $p \overset{L}{\not\Rightarrow} q$ if p = True and q = False.
c. Otherwise, '$\overset{L}{\Rightarrow}$' is not defined for $\langle p, q\rangle$.

(57) For any functions h, h':
a. $h \overset{L}{\Rightarrow} h'$ if for all type-appropriate x, $h(x) \overset{L}{\Rightarrow} h'(x)$.
b. $h \overset{L}{\not\Rightarrow} h'$ if for all type-appropriate x, '$\overset{L}{\Rightarrow}$' is defined for the pair $\langle h(x), h'(x)\rangle$, and for some type-appropriate x, $h(x) \overset{L}{\not\Rightarrow} h'(x)$.
c. Otherwise, '$\overset{L}{\Rightarrow}$' is not defined for $\langle h, h'\rangle$.

We define 'SUE^{L}' in (58) as an alternative to (33), where P and Q are constrained by '$\Rightarrow$', but the relation between $f(P)$ and $f(Q)$ is '$\overset{L}{\Rightarrow}$'.

(58) f is **SUE^{L}** *iff* for all P, Q such that $P \Rightarrow Q$ and '$\overset{L}{\Rightarrow}$' is defined for $f(P)$ and $f(Q)$: $f(P) \overset{L}{\Rightarrow} f(Q)$.

We adjust the meaning of *some* so that it is a total function such that for any Z, Y of type $\langle e, t\rangle$ and any C and w, if

$$\{y \in C(w) \mid Z(y) = \text{True}\} \cap \{y \mid Y(y) \in \{\text{True}, \text{False}\}\} = \varnothing,$$

then $[\![some]\!]^{C,w}(Z)(Y)$ = Und; otherwise $[\![some]\!]^{C,w}$ is as in (20). We derive that for any P and Q of type $\langle e, t\rangle$ such that $P \Rightarrow Q$ and '$\overset{L}{\Rightarrow}$' is defined for $[\![some]\!]^{C,w}(P)$ and $[\![some]\!]^{C,w}(Q)$, $[\![some]\!]^{C,w}(P) \overset{L}{\Rightarrow} [\![some]\!]^{C,w}(Q)$ for all C and w. For example, when the French and non-French professors are motherless in w,

$$\begin{aligned}[\![\textit{some}]\!]^{C,w}&([\![\textit{French professor}]\!]^{C,w})([\![\textit{likes his mother}]\!]^{C,w})\\ &= [\![\textit{some}]\!]^{C,w}([\![\textit{professor}]\!]^{C,w})([\![\textit{likes his mother}]\!]^{C,w})\\ &= \text{Und},\end{aligned}$$

so $[\![some]\!]^{C,w}([\![French\ professor]\!]^{C,w}) \overset{L}{\Rightarrow} [\![some]\!]^{C,w}([\![professor]\!]^{C,w})$. *Some* indeed comes out SUE^{L} on its first argument.

To solve the P-to-Q problem, however, we must find two appropriate alternatives to '$\Rightarrow$': one that makes *some* formally upward-entailing (relative to ⟨*likes his mother*, *likes someone*⟩; see (38)/(53)), and one that makes *no* formally downward-entailing (relative to ⟨*French student*, *student*⟩, but not relative to ⟨*likes his mother*, *likes someone*⟩; see (52)/(54)). This is not easily achieved without appealing to '$\overset{\text{ST}}{\Rightarrow}$', because on the one hand, ⟨*likes his mother*, *likes someone*⟩ doesn't satisfy '$\overset{\text{L}}{\Rightarrow}$', and, on the other hand, ⟨*French student*, *student*⟩ and ⟨*likes his mother*, *likes someone*⟩ both satisfy intuitive entailment. There might be less naïve multivalent logics that afford a better solution (and there probably are). Our point is this: *merely* switching to a multivalent logic cannot address the P-to-Q problem satisfactorily.

6 Conclusion

We have proposed formal notions of entailment, Strawson entailment, upward/downward-entailingness and Strawson upward/downward-entailingness stated in terms of '$\overset{\text{ST}}{\Rightarrow}$' and other relations that are themselves defined in terms of '$\overset{\text{ST}}{\Rightarrow}$'. We contend that if at least some determiners and some nouns and verbs denote partial functions, '$\overset{\text{ST}}{\Rightarrow}$' is a more useful basis for describing properties of natural language determiners, compared to '$\Rightarrow$' or '$\overset{\text{L}}{\Rightarrow}$'.

Appendix

More formal versions of some of the informal proofs given in the text are provided below (where C is any function from W to $\{X \mid X \subseteq D_e\}$ and w is any element of W).

On the assumption that all natural language functions are total, and by the definition of '$\Rightarrow$' in (10), (I)–(II) hold for any $P, Q, Z \in D_{\langle e,t\rangle}$ such that $P \Rightarrow Q$.

(I) a. $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \{y \in D_e \mid Q(y) = \text{True}\} \neq \varnothing$ or $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \{y \in D_e \mid P(y) = \text{True}\} = \varnothing$;

b. (i) by (a) and the meaning of *no* in (13): $[\![no]\!]^{C,w}(Z)(Q) = \text{False}$ or $[\![no]\!]^{C,w}(Z)(P) = \text{True}$,

(ii) by (i) and the definition of '$\Rightarrow$' in (10): $[\![no]\!]^{C,w}(Z)(Q) \Rightarrow [\![no]\!]^{C,w}(Z)(P)$.

(II) a. $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \{y \in D_e \mid P(y) = \text{True}\} = \varnothing$ or $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \{y \in D_e \mid Q(y) = \text{True}\} \neq \varnothing$;

b. (i) by (a) and the meaning of *some* in (15): $[\![some]\!]^{C,w}(Z)(P) = \text{False}$ or $[\![some]\!]^{C,w}(Z)(Q) = \text{True}$,

(ii) by (i) and the definition of '$\Rightarrow$' in (10): $[\![some]\!]^{C,w}(Z)(P) \Rightarrow [\![some]\!]^{C,w}(Z)(Q)$.

Therefore, by the definitions of ‘DE’ and ‘UE’ in (12) and (11), respectively, for any $Z \in D_{\langle e,t \rangle}$: $[\![no]\!]^{C,w}(Z)$ is DE and $[\![some]\!]^{C,w}(Z)$ is UE.

It follows from the definition of ‘$\Rightarrow$’ in (10) that for any $P, Q \in D_{\langle e,t \rangle}$ such that $P \Rightarrow Q$, $\mathrm{Dom}(P) = \mathrm{Dom}(Q) = D_e$. Thus, (III)–(IX) hold for any $P, Q \in D_{\langle e,t \rangle}$ such that $P \Rightarrow Q$, even if natural language functions can be partial in principle.

(III) for any $Z \in D_{\langle e,t \rangle}$ such that $C(w) \cap \mathrm{Dom}(Z) \neq \varnothing$:

a. $\{y \in C(w) \mid Z(y) = \text{True}\} \nsubseteq \{y \in \mathrm{Dom}(Q) \mid Q(y) = \text{False}\}$ or $\{y \in C(w) \mid Z(y) = \text{True}\} \subseteq \{y \in \mathrm{Dom}(P) \mid P(y) = \text{False}\}$;

b. (i) by (a) and the meaning of *no* in (18): $[\![no]\!]^{C,w}(Z)(Q) = \text{False}$ or $[\![no]\!]^{C,w}(Z)(P) = \text{True}$,

(ii) by (i) and the definition of ‘$\Rightarrow$’ in (10): $[\![no]\!]^{C,w}(Z)(Q) \Rightarrow [\![no]\!]^{C,w}(Z)(P)$.

(IV) for any $Z \in D_{\langle e,t \rangle}$ such that $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \mathrm{Dom}(P) \neq \varnothing$:

a. $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \{y \in \mathrm{Dom}(P) \mid P(y) = \text{True}\} = \varnothing$ or $\{y \in C(w) \mid Z(y) = \text{True}\} \cap \{y \in \mathrm{Dom}(Q) \mid Q(y) = \text{True}\} \neq \varnothing$;

b. (i) by (a) and the meaning of *some* in (20): $[\![some]\!]^{C,w}(Z)(P) = \text{False}$ or $[\![some]\!]^{C,w}(Z)(Q) = \text{True}$,

(ii) by (i) and the definition of ‘$\Rightarrow$’ in (10): $[\![some]\!]^{C,w}(Z)(P) \Rightarrow [\![some]\!]^{C,w}(Z)(Q)$.

(V) for any $x \in C(w)$ such that $x \in C(w)$ and $P(x) = \text{True}$:

a. $\{y \in C(w) \mid Q(y) = \text{True}\} \neq \{x\}$ or $\{y \in C(w) \mid P(y) = \text{True}\} = \{x\}$;

b. (i) by (a) and the meaning of *only* in (28): $[\![only]\!]^{C,w}(x)(Q) = \text{False}$ or $[\![only]\!]^{C,w}(x)(P) = \text{True}$,

(ii) by (i) and the definition of ‘$\Rightarrow$’ in (10): $[\![only]\!]^{C,w}(x)(Q) \Rightarrow [\![only]\!]^{C,w}(x)(P)$.

(VI) for any $Z \in D_{\langle e,t \rangle}$ such that $\{y \in C(w) \mid Z(y) = \text{True}\} \neq \varnothing$:

a. $\{y \in C(w) \mid Z(y) = \text{True}\} \nsubseteq \{y \in \mathrm{Dom}(P) \mid P(y) = \text{True}\}$ or $\{y \in C(w) \mid Z(y) = \text{True}\} \subseteq \{y \in \mathrm{Dom}(Q) \mid Q(y) = \text{True}\}$;

b. (i) by (a) and the meaning of *every* in (31): $[\![every]\!]^{C,w}(Z)(P) = \text{False}$ or $[\![every]\!]^{C,w}(Z)(Q) = \text{True}$,

(ii) by (i) and the definition of ‘$\Rightarrow$’ in (10): $[\![every]\!]^{C,w}(Z)(P) \Rightarrow [\![every]\!]^{C,w}(Z)(Q)$.

(VII) for any $Y \in D_{\langle e,t\rangle}$ such that $\{y \in C(w) \mid Q(y) = \text{True}\} \subseteq \text{Dom}(Y)$, if $\text{Dom}(Y) \neq \varnothing$ and $C(w) \cap \text{Dom}(P) \neq \varnothing$:

a. $\{y \in C(w) \mid Q(y) = \text{True}\} \nsubseteq \{y \in \text{Dom}(Y) \mid Y(y) = \text{False}\}$ or $\{y \in C(w) \mid P(y) = \text{True}\} \subseteq \{y \in \text{Dom}(Y) \mid Y(y) = \text{False}\}$;

b. by (a) and the meaning of *no* in (18): $[\![no]\!]^{C,w}(Q)(Y) = \text{False}$ or $[\![no]\!]^{C,w}(P)(Y) = \text{True}$;

c. by (b) and the definition of '$\overset{\text{ST}}{\Rightarrow}$' in (34), $[\![no]\!]^{C,w}(Q) \overset{\text{ST}}{\Rightarrow} [\![no]\!]^{C,w}(P)$.

(VIII) for any $Y \in D_{\langle e,t\rangle}$ such that $\{y \in C(w) \mid P(y) = \text{True}\} \cap \text{Dom}(Y) \neq \varnothing$:

a. $\{y \in C(w) \mid P(y) = \text{True}\} \cap \{y \in \text{Dom}(Y) \mid Y(y) = \text{True}\} = \varnothing$ or $\{y \in C(w) \mid Q(y) = \text{True}\} \cap \{y \in \text{Dom}(Y) \mid Y(y) = \text{True}\} \neq \varnothing$;

b. by (a) and the meaning of *some* in (20): $[\![some]\!]^{C,w}(P)(Y) = \text{False}$ or $[\![some]\!]^{C,w}(Q)(Y) = \text{True}$;

c. by (b) and the definition of '$\overset{\text{ST}}{\Rightarrow}$' in (34), $[\![some]\!]^{C,w}(P) \overset{\text{ST}}{\Rightarrow} [\![some]\!]^{C,w}(Q)$.

(IX) for any $Y \in D_{\langle e,t\rangle}$ such that $\{y \in C(w) \mid Q(y) = \text{True}\} \subseteq \text{Dom}(Y)$, if $\{y \in C(w) \mid P(y) = \text{True}\} \neq \varnothing$, then:

a. $\{y \in C(w) \mid Q(y) = \text{True}\} \nsubseteq \{y \in \text{Dom}(Y) \mid Y(y) = \text{True}\}$ or $\{y \in C(w) \mid P(y) = \text{True}\} \subseteq \{y \in \text{Dom}(Y) \mid Y(y) = \text{True}\}$;

b. by (a) and the meaning of *every* in (31): $[\![every]\!]^{C,w}(Q)(Y) = \text{False}$ or $[\![every]\!]^{C,w}(P)(Y) = \text{True}$;

c. by (b) and the definition of '$\overset{\text{ST}}{\Rightarrow}$' in (34), $[\![every]\!]^{C,w}(Q) \overset{\text{ST}}{\Rightarrow} [\![every]\!]^{C,w}(P)$.

Therefore, by the definitions of 'DE' and 'UE' in (12) and (11), respectively; the definition of 'SDE' in (26); and the definitions of 'SDE^{ST}' and 'SUE^{ST}' in (32) and (33), respectively:

(a) for any $Z \in D_{\langle e,t\rangle}$ such that $[\![no]\!]^{C,w}(Z)$ is defined, $[\![no]\!]^{C,w}(Z)$ is DE;

(b) for any $Z \in D_{\langle e,t\rangle}$ such that $[\![some]\!]^{C,w}(Z)$ is defined, $[\![some]\!]^{C,w}(Z)$ is UE;

(c) for any $Z \in D_{\langle e,t\rangle}$ such that $[\![every]\!]^{C,w}(Z)$ is defined, $[\![every]\!]^{C,w}(Z)$ is UE;

(d) for any $x \in D_e$ such that $[\![only]\!]^{C,w}(x)$ is defined, $[\![only]\!]^{C,w}(x)$ is SDE and SDE^{ST};

(e) $[\![no]\!]^{C,w}$ and $[\![every]\!]^{C,w}$ are SDE^{ST} and $[\![some]\!]^{C,w}$ is SUE^{ST}.

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