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Lessons from Sobel sequences*

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Abstract Folklore has it that Sobel sequences favor a variably strict analysis of conditionals over its plainly strict alternative. While recent discussions for or against the lore have focussed on Sobel sequences involving counterfactuals, this paper draws attention to the fact that indicative Sobel sequences are just as felicitous as are their counterfactual cousins. The fact, or so I shall argue here, disrupts the folklore: given minimal assumptions about the semantics and pragmatics of indicative conditionals, a textbook variably strict analysis fails to predict that indicative Sobel sequences are felicitous. The correct lesson to draw from Sobel sequences is that their felicity challenges classical implementations of the variably strict and of the plainly strict analysis alike. In response to this challenge I develop a dynamic strict analysis of conditionals that handles indicative Sobel sequences with grace while preserving intuitive constraints on the semantics and pragmatics of their members. A discussion of how such an analysis may handle the challenge from reverse Sobel sequences is provided.

Keywords: Conditionals, Sobel Sequences, Dynamic Semantics, Variably Strict Analysis

1 The plot

Folklore has it that Sobel sequences (see Sobel 1970) favor a *variably* strict analysis of conditionals over its plainly strict alternative. Consider the following example:

(1) (a) If Alice had come to the party, it would have been fun. (b) But if Alice and Bert had come, it would not have been fun. (c) But if Charles had come as well, it would have been fun...

The fact that sequences of this kind are felicitous causes trouble for an analysis of counterfactuals as strict material conditionals but — so the classical story from

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Lewis (1973) continues — lives happily with a variably strict interpretation on which the consequents of such conditionals are evaluated at the *closest* possible worlds at which their antecedents are true.[1] The goal of this paper is to show that the folklore is prone to disruption.

This is not the first critical reflection on the classical story about Sobel sequences: von Fintel (2001) and Gillies (2007) complain that a variably strict analysis of (1) fails to explain the infelicity of *reversing* the order of its members and advocate for an alternative interpretation on which counterfactuals are strict material conditionals over a set of possible worlds evolving dynamically as discourse proceeds.[2] But Moss (2012) argues that the issue is best handled by a pragmatic supplement to Lewis's story, and in any case one might wonder why discussions of the folklore focus so much on Sobel sequences involving counterfactual conditionals when their *indicative* cousins are just as much resistant to strengthening of the antecedent:

(2) (a) If Alice comes to the party, it will be fun. (b) But if Alice and Bert come, it will not be fun. (c) But if Charles comes as well, it will be fun...

It is overwhelmingly plausible to say that there is no noticeable difference in acceptability between the sequences in (1) and (2).[3] This is not a special feature of future-directed indicative conditionals since their past- and present-tensed variants are just as acceptable given suitable contexts of deliberation. The first of the following two sequences, for instance, is flawless in case we are wondering about how the party went, while the other is fine in case we are wondering whether Mary is or is not at some party that is happening right as we speak:

(3) (a) If Alice came to the party, it was fun. (b) But if Alice and Bert came, it was not fun. (c) But if Charles came as well, it was fun...

1 The *locus classicus* of a variably strict interpretation is the discussion by Stalnaker (1968), who assumes that there is always a unique closest possible world at which the antecedent is true. Lewis (1981) discusses the equivalence between an ordering semantics for counterfactual conditionals and the premise semantics from Kratzer (1979, 1981, 1991a, 2012). I have chosen a formulation of the variably strict interpretation that relies on the Limit Assumption: that there is always a set of antecedent-worlds most similar to the world of evaluation. Though Lewis (1973) officially rejects it and nothing said here depends on it, I will adopt this assumption to streamline the discussion.

2 The observation that the felicity of Sobel sequences is sensitive to the order of their members goes back to Heim and is reported by von Fintel (2001), who points to a seminar presentation by Heim at MIT in 1994 and to discussions by Warmbrōd (1981b,a, 1983) as precedents to his account.

3 Lycan (1993, 2001), Weatherson (2001), and Williams (2008) all pay attention to indicative Sobel sequences, but their lessons differ substantially from the one I intend to draw here. The usual disclaimer: categorizing conditionals as "indicative" and "counterfactual" is flawed. But speaking of "indicative" versus "subjunctive" conditionals is problematic too, and so I will stick with the labels that Lewis (1973) employs.

(4) (a) If Alice is at the party, Mary is at the party. (b) But if Alice and Bert are at the party, Mary is not at the party. (c) But if Charles is there too, Mary is at the party…

So there is every reason to think that indicative conditionals live happily in Sobel sequences given suitable contexts, and every reason to demand that whatever explains why counterfactual Sobel sequences are unremarkable must also account for why their indicative cousins are no less felicitous.[4] This turns out to be nontrivial for the folklore since its strategy for handling the classical case of counterfactual Sobel sequences does not generalize so that it applies to indicative Sobel sequences without negative side effects.

In brief, while a textbook variably strict analysis has no trouble predicting that indicative Sobel sequences are just as consistent as their counterfactual cousins, this alone is by no means sufficient to explain why they are felicitous. For the elements of an indicative Sobel sequence presuppose that their antecedents pertain to possibilities that are *open* in context and this presupposition — given minimal semantic and pragmatic assumptions — will inevitably be violated if they receive a variably strict interpretation. Predicting that indicative Sobel sequences suffer from presupposition failures, however, would be just as bad as is failing to predict that they are consistent, and so we have good reason to think that the folklore does not get the general import of Sobel sequences quite right: their felicity poses challenges to classical implementations of the variably strict and of the plainly strict analysis of conditionals alike (Section 2).

Section 3 explores in detail one specific way of overcoming the observed challenges. If elaborated with a dynamic spin, a strict analysis of indicative conditionals handles Sobel sequences with grace while preserving those minimal constraints on the semantics and pragmatics of conditionals that lead to all the trouble in Section 2. On the proposed analysis, indicative conditionals are strict material conditionals over contextual possibilities, yet they also have the potential to expand the domain of quantification by bringing open but hitherto ignored possibilities into view (cf. von Fintel 2001 and Gillies 2007 on counterfactuals). What accounts for the consistency of Sobel sequences is not that the consequents of their members are evaluated with respect to disjoint modal domains — as the folklore would have it — but rather with respect to a single dynamically evolving modal domain.

4 The claim is not that every indicative Sobel sequence will strike the ear as felicitous in every scenario — context matters. But that is also true of counterfactual Sobel sequences, and by everyone's admission: specifically, on a variably strict analysis, context must provide a suitable similarity relation between worlds for the sequence under consideration to run smoothly. The point is that indicative Sobel sequences are in general *just as acceptable as* their counterfactual cousins given suitable contexts, and that this fact is in need of an explanation.

Section 4 elaborates the framework developed in the previous section so that it addresses a few additional pressing issues. First, it discusses the phenomenon of reverse Sobel sequences. Moss (2012) argues that reverse Sobel sequences are not always infelicitous and that this presents a problem for dynamic strict analyses, which treat all such sequences as inconsistent by design. I show that the problem can be resolved given plausible pragmatic assumptions (Section 4.1). Second, I explain how a dynamic story can address Stalnaker's (2011) recent puzzle about conditionals, disjunction, and the direct argument (Section 4.2). A brief sketch of how the proposal for indicatives can be extended to counterfactuals — explained in more detail in the Appendix — is provided as well (Section 4.3).

Section 5 concludes the discussion by briefly contextualizing the story told here in light of alternative reactions to the challenge from Sobel sequences.

2 The trouble

The hallmark of a variably strict analysis is that a conditional is true at some world w just in case its consequent is true throughout the set of possible worlds at which its antecedent holds but that otherwise differ minimally from the world of evaluation. Take a connected and transitive relation $\leq$ that is provided by some context c and keeps track of relative similarity or closeness between worlds, and define:

$$\min_{\leq,w}(\phi) = \{w' \colon\ w' \in [\![\phi]\!]^c \text{ and for all } w'' \colon\ \text{if } w'' \in [\![\phi]\!]^c, \text{ then } w' \leq_w w''\}$$

To say that conditionals are variably strict is to treat them as universal quantifiers over the $\leq_w$-minimal worlds at which their antecedent is true:

$$[\![(\text{if } \phi)(\psi)]\!]^{c,w} \text{ is true iff } \min_{\leq,w}(\phi) \subseteq [\![\psi]\!]^c$$

The simple observation then is that the closest possible worlds at which ϕ is true need not intersect with the closest possible worlds at which $\ulcorner \phi \wedge \psi \urcorner$ is true, and so if conditionals are variably strict their resistance to strengthening of the antecedent is just what we expect. In contrast, treating them as plain strict material conditionals does seem to license antecedent strengthening since whenever all ϕ-worlds are χ-worlds, so must be all $\ulcorner \phi \wedge \psi \urcorner$-worlds. So much for the rationale behind the folklore.

Lewis limits his discussion to counterfactual conditionals and subsequent arguments for or against the folklore have followed suit. This is surprising since *indicative* Sobel sequences are just as unremarkable as Lewis's original cases, and it is hard if not impossible to see how any analysis of conditionals could take heart from counterfactual Sobel sequences if it did not deliver a satisfying explanation of why sequences like (2) are just as fine. And of course, if the analysis of indicative conditionals is at its heart variably strict too — as Stalnaker (1968) holds — there is no trouble seeing how indicatives could resist strengthening of the antecedent as much as their counterfactual cousins do. But coming up with an interpretation of indicative conditionals that leaves room for all members of (2) to be true is *not* enough to explain why the sequence is felicitous, the reason being that indicatives impose distinct constraints on the discourse context to be felicitous:

(5) a. #Mary is not in New York. If she is in New York, she will meet Alex.
b. ✓Mary is not in New York. If she were in New York, she would meet Alex.

The sequence in (5a) is marked even if — as a variably strict semantics would have it — its members are consistent. Once we have explained why this is so and added a minimal semantic assumption to the mix we will see that, if the variably strict analysis described here were the last word, (2) would exhibit the same deficit as does (5a). And this is just the wrong prediction since (2) leaves nothing to be desired. Section 2.1 states the case in more detail. Section 2.2 helps the message stick by critically discussing some possible ways of avoiding the tension in a classical setting.

2.1 The case

Start with the textbook explanation for why (5a) is marked. An utterance of the first member of the sequence, so the story goes, expresses the proposition that Mary is not in New York and eliminates from the context set — the set of possible worlds compatible with what is common ground in discourse — all possible worlds at which Mary is in New York. But for an utterance of an indicative conditional to be felicitous, the context set must contain at least one possible world at which its antecedent is true, and so it is not surprising that (5a) is marked. (Since utterances of counterfactual conditionals do not come with such felicity constraints, we expect (5b) to be fine.)

What underlies the textbook explanation are two assumptions that are widely shared in the literature. Both of them appeal to the notion of the common ground, which intuitively keeps track of what is taken for granted in discourse and, following Stalnaker (2002), can be made more precise thus:

> It is common ground that ϕ in a group if all members accept (for the purpose of the conversation) that ϕ, and all believe that all accept that ϕ, and all believe that all believe that all accept that ϕ, etc. (p. 716)

The common ground is thus a body of information. Exactly *how* to model this information is a non-trivial question, but at a minimum we are interested in the context set understood as the set of possible states of the world that are compatible with what is common ground. To introduce a bit of notation, think of c as keeping track of everything we want to know about context: then s_c is the *context set* representing the set of possible states of the world that are compatible with the contextual information. The two assumptions underlying the textbook explanation can then be made more precise as follows:

Assertion If ϕ expresses a proposition $[\![\phi]\!]^c$ in context c, then the result of asserting ϕ in c, $c+\phi$, is such that $s_{c+\phi} = s_c \cap [\![\phi]\!]^c$.

Presupposition An utterance of an indicative conditional of the form $\ulcorner(\text{if } \phi)(\psi)\urcorner$ in context c presupposes that $s_{c+\phi} \neq \emptyset$.

The first principle goes back to Stalnaker's (1978) observation that context-content interaction is not a one way street — context affects what is said but what is said in turn affects the context in light of which subsequent utterances are interpreted — while the second one goes back to Stalnaker's (1975) discussion of the difference between indicative and subjunctive conditionals.[5] Taken together, they immediately predict that the sequence in (5a), even if consistent, is marked due to presupposition failure, as desired. The observation about the variably strict analysis is that, once we add a minimal semantic assumption about conditionals to the mix, it wrongly predicts that indicative Sobel sequences suffer just as much from presupposition failures as does (5a).

I assume here — as Stalnaker (1968) and Lewis (1973) do — that modus ponens is valid and thus that conditionals are bounded from below by the material conditional in the following sense:

Modus Ponens $(\text{if } \phi)(\psi) \vDash \phi \supset \psi$

In a variably strict setting, modus ponens is enforced by the — independently plausible — requirement that the contextually provided accessibility relation $\leq$ be *weakly centered*: for all w and w', $w \leq_w w'$, which just means that no possible world is more similar to w than w itself. As a consequence, we will not find a possible world at

5 The first principle is a bit weaker than what Stalnaker (1978) actually says in that it does not assume that *all* assertions express propositions — this hedge does not matter for current purposes but will become relevant once we look at its dynamic alternative. In the recent literature, the second principle is explicitly endorsed by Gillies (2009, 2010) and Starr (2014a,b), among others (Stalnaker, to be precise, thinks that speakers do the presupposing, not utterances).

which $\ulcorner(\text{if } \phi)(\psi)\urcorner$ is true but at which $\ulcorner\phi \wedge \neg\psi\urcorner$ is true as well. This is all we need to see the trouble.

Consider the Sobel sequence in (2) again: the relevant observation is that it is possible to assert (2c) without fuss after having asserted (2a) and (2b). The claim is that the folklore fails to leave room for this observation given our three assumptions. To see this, consider first the result of asserting (2a) in some context c, that is $c' = c + (\text{if } A)(F)$. The variably strict analysis has it that utterances of (2a) express some proposition in context and so by Stalnaker's principle about context-content interaction, an utterance of (2a) will eliminate from c all possible worlds at which the relevant proposition is false. Since an indicative conditional is assumed to be bounded from below by the material conditional, it follows that $s_{c'}$ does not contain any $A \wedge \neg F$-world. In other words: all worlds in $s_{c'}$ at which Alice comes to party are also worlds at which the party is fun.

For parallel reasons an assertion of (2b) eliminates from a context set all possible worlds at which Alice and Bert come to the party and it is fun. So if c'' is the context resulting from asserting (2a) and (2b) in c, then $s_{c''}$ does not contain any $(A \wedge B) \wedge F$-world. So we have:

> Every possible world in $s_{c''}$ at which Alice and Bert come to the party is a world at which the party is not fun.

But since $s_{c''}$ is a subset of $s_{c'}$ and we know that $s_{c'}$ contains no $A \wedge \neg F$-worlds, and hence no $(A \wedge B) \wedge \neg F$-worlds, we also have:

> Every possible world in $s_{c''}$ at which Alice and Bert come to the party is a world at which the party is fun.

It follows that $s_{c''}$ cannot contain an $A \wedge B$-world, as that world would need to be an $F \wedge \neg F$-world.

The upshot: there is no possible world in $s_{c''}$ at which the antecedent of (2c) is true, which is just to say that it is *not* possible to continue the sequence with (2c) without fuss. Quite to the contrary, and in light of our second assumption, we predict that an utterance of (2c) is marked because of presupposition failure.[6] So given modest semantic and pragmatic assumptions, the folklore predicts that the sequence in (2) exhibits the same kind of defect as does (5a), which is just the wrong result.

6 For parallel reasons, after accepting (2a) and (2b) into the common ground no discourse participant could felicitously flag his or her agreement like this:

> Yeah, if they both come, they'll get into a fight and ruin everything.

But as an anonymous reviewer observes, such a continuation of (2a) and (2b) seems perfectly alright.

2.2 Rejoinders and replies

Here I will discuss three possible escapes routes from the trouble for the folklore. All of them, or so I shall argue, turn out to be wanting.

2.2.1 Accommodation

One possible reaction to the problem just presented is to say that an utterance of (2c) triggers an *expansion* of the context set so that it includes some possible worlds at which the antecedent is true: presuppositions, after all, are commonly accommodated.[7] But if accommodation were at play in (2) it should also be at play in (5a) — in both cases do we have an indicative conditional whose antecedent is incompatible with the common ground — and so we would not expect there to be any difference between the two sequences.

In addition, there is a general reason to believe that a simple presupposition accommodation strategy does not work. A presupposition that ϕ may be accommodated, but this is generally so only in case the negation of ϕ fails to be common ground.[8] Consider:

(6) #Bob does not have children. His children are bald.

The second sentence presupposes that Bob has children. While this presupposition can be accommodated, this is not so if it is common ground that Bob does not have children, as (6) indicates. But now observe that whenever ϕ is not compatible with the common ground, it is common ground that ϕ is not compatible with the common ground (remember Stalnaker's (2002) definition of the common ground). So it is not straightforward to see how a presupposition that ϕ is compatible with the common ground could be accommodated without fuss if $\ulcorner \neg\phi \urcorner$ is already common ground. This closes off the option of appealing to off-the-shelf presupposition accommodation in accounting for the observation that (2c) is assertable without fuss once (2a) and (2b) have been uttered.

2.2.2 Assertion

In response to the problems with a simple accommodation strategy, one might try to tweak Stalnaker's principle about assertion. Start with the idea that the elements

7 This rejoinder (as well as the next one) owes some inspiration to Weatherson's (2001) discussion, who credits McCawley (1996) as his source of inspiration, and to Williams's (2008) proposal, but it is not clear that any of them would endorse the idea that indicative conditionals may expand the context set in case the antecedent is incompatible with what is common ground.

8 See, for instance, the framework for presupposition and accommodation by van der Sandt (1992), which makes exactly this prediction.

of a context set need not be created equal but ranked in terms of their plausibility, leading to a system of spheres centered on the set of possible worlds that are not merely open in discourse but also appear most plausible in light of what is taken for granted. It could then very well be that assertions of conditionals only eliminate elements from the minimal sphere in the context set (rather than affecting the entire context set) while their presuppositions may trigger an expansion of the minimal sphere to include possible worlds at which their antecedent is true. The antecedent of (2c) would then no longer be guaranteed to be incompatible with the common ground once (2a) and (2b) have been processed.

The spirit of this proposal has a lot going for it, but it is of no help for folklore. First, it relies on the nothing but *ad hoc* stipulation that assertions of conditionals differ in their pragmatic effects from other proposition expressing assertions. This is because the textbook explanation of why (5a) is marked crucially relies on the assumption that an assertion of "Mary is not in New York" eliminates *each* possible world at which the proposition expressed is false from the context set, rather than only pertaining to those most plausible in the context set — otherwise, there would be no reason to think that the subsequent utterance of "If Mary is in New York, she will meet Alex" is infelicitous due to presupposition failure. But if this is so, it is unclear why assertions of conditionals — which, after all, express propositions as well on the variably strict analysis — should not have the same pragmatic profile.[9]

Second, relying on a plausibility ordering in discourse, if coupled with a variably strict analysis, is to appeal to an ordering relation twice over and, what is more, the one that figures in the semantics becomes explanatorily superfluous when we look at Sobel sequences. For of course we may now just as well say that conditionals are strict material conditionals over the minimal sphere in a context set and explain the felicity of Sobel sequences in terms of how this sphere may expand as discourse proceeds. The strategy of appealing to a dynamically evolving plausibility ordering undermines the key rationale behind a variably strict reaction to the data about Sobel sequences.

2.2.3 Ignorance

Another possible response on behalf of the folklore is that uttering (2a) and (2b) indeed results in a context violating the presupposition carried by (2c), but the presupposition violation may be overlooked or ignored in discourse and so an utterance of (2c) may pass even though it is, strictly speaking, defective in context.

9 For sure, it is conceivable that there is a systematic story to be told explaining why assertions of modal propositions differ in their pragmatic profile from assertions of non-modal propositions. The point is that, to my knowledge, no such story has been told and that it must differ substantially from the classical story about the semantics and pragmatics of modal expressions.

But if this were right, then the fan of a plainly strict interpretation would be just as justified to hold that, for sure, no two adjacent stages of a Sobel sequence can be (non-vacuously) true but add that *this* fact may be overlooked or ignored in discourse as well and so a Sobel sequence may pass as unremarkable even though, strictly speaking, it is not. In other words, it is hard to find a legitimate principle that allows one to dismiss the failure of a variably strict semantics to predict the felicity of indicative Sobel sequences as harmless while holding up to the original line of reasoning that motivated the analysis in the first place: that a plain strict interpretation licenses weakening of the antecedent and thus fails to predict the felicity of counterfactual Sobel sequences.

2.3 Conclusion and outlook

So far I have argued that data about Sobel sequences fail to support a variably strict semantics since it is not enough to only look at their subjunctive incarnations. Indicative Sobel sequences are unremarkable as well, and this fact, given minimal semantic and pragmatic assumptions anyway, is unexpected if the analysis were the last word.

To be clear, the point is not that when it comes to accounting for the felicity of indicative Sobel sequences, the variably strict analysis is in worse shape than its classical alternatives, given minimal semantic and pragmatic assumptions. *Any* conditional analysis, if couched in a classical setting, will have trouble doing so: treating conditionals as bounded from below by the material conditional, after all, seems to require that $\ulcorner(\text{if } \phi)(\chi)\urcorner$ and $\ulcorner(\text{if } \phi \wedge \psi)(\neg\chi)\urcorner$ jointly entail $\ulcorner\neg(\phi \wedge \psi)\urcorner$, and so it is hard to see how the result of adding these conditionals to the common ground could not carry that very same commitment. But this just stresses the point that Sobel sequences — in their indicative incarnations anyway — raise a very general puzzle about the semantics and pragmatics of conditionals. It is simply not trivial that the solution to the puzzle is bound to have a variably strict semantics at its core.

To make the last point more concrete, I will present an analysis of conditionals that is especially noteworthy since it analyzes indicative conditionals as strict, treats them as bounded from below by the material conditional (thus preserving the semantic validity of modus ponens), is faithful to everything else we said about assertion and presupposition, and leaves room for indicative Sobel sequences to be felicitous. I do not claim that the path I am about to take here is without alternatives, or that its revisions to the classical way of thinking about conditionals — most notably its dynamic semantic approach — are unavoidable. But since the story I am about to tell will give us everything one might ask for in that it accounts for the data *and* satisfies all the semantic and pragmatic constraints brought into play earlier, everyone should agree that it is an attractive option that cannot be immediately dismissed. Its viability drives home the point that the question of which semantic and pragmatic conclusion to draw from Sobel sequences — and in particular the question of whether they favor a variably strict over a plainly strict analysis — is far more complex than the lore suggests.

3 The proposal

The goal of this section is to tell a semantic story about indicative conditionals that accounts for the felicity of indicative Sobel sequences while preserving the intuitive semantic and pragmatic constraints articulated earlier. My plan for this section is as follows. I will first outline the proposal in informal terms (Section 3.1) and then make the proposal a bit more precise (Section 3.2). The final part of this section demonstrates that the resulting framework handles indicative Sobel sequences such as (2) with grace while preserving modus ponens as well as everything else we said about assertion and presupposition (Section 3.3).

3.1 Basics

Taking inspiration from von Fintel's (2001) and Gillies's (2007) analyses of counterfactuals, I treat indicative conditionals as strict material conditionals over a domain that evolves as discourse proceeds.[10] While the basic idea could be articulated in different ways, I choose here a dynamic semantic implementation that construes the meaning of a sentence in terms of its context change potential. Remember Stalnaker's view on context-content interaction: assertions express propositions in context and affect the context by adding the proposition expressed to the common ground. This view is truth-conditional at heart since context change is always mediated by truth-conditional content. But, as Dever (2006) and von Fintel & Gillies (2008) observe, it also motivates a *change* in perspective: instead of being all about truth-conditions, a semantics may be all about how assertions relate an input context (the context in which the sentence is uttered) to an output context (the context posterior to that utterance). Ordinary factual statements continue to affect the conversational score by adding a proposition expressed to the common ground, but in addition we leave room for the possibility of context change that is not mediated by propositional content. Whenever this possibility is realized we have content that fails to reduce to classical truth-conditional content.

10 In addition to von Fintel's and Gillies's proposals, see also Gillies 2004 and Starr 2014a,b. All of these proposals are similar in spirit to the one developed here, but there are differences at crucial moments of detail that matter for the data that interest us here, some of which are highlighted by Willer (2013b).

Needless to say, context plays a prominent role in dynamic semantics, and so it is mandatory to explain what a context is supposed to be. The details depend on one's specific theoretical interests, and here I will think of meanings as operating on abstract representations of the common ground. To suit my purposes, these representations will have to be more fine-grained than Stalnaker's context sets, and the required additional complexities can be motivated in two steps. First, I am interested in models of the common ground that draw a distinction between possibilities that are merely compatible with what is taken for granted in discourse and those that are "live" in the sense that they are explicitly treated as relevant. This distinction is already implicit in what Stalnaker (2002) says about the common ground (repeated):

> It is common ground that ϕ in a group if all members accept (for the purpose of the conversation) that ϕ, and all believe that all accept that ϕ, and all believe that all believe that all accept that ϕ, etc. (p. 716)

Defining what is common ground in such a way leaves room for two senses in which a proposition p is possible. The first, familiar one, is this: p may be possible in the sense that $\neg p$ fails to be common ground, which is just to say that p is compatible with what is mutually presupposed in discourse. The second, stronger one, is this: p may be possible in the sense that it is common ground that it might be that p, which we will here interpret as saying that the discourse participants agree to treat p as a relevant possibility in discourse. This, to be sure, is more demanding than saying that p is possible just because $\neg p$ fails to be common ground: for p to be a relevant possibility in discourse, the question of whether or not p must have been raised in discussion, but this is not a requirement for p to be compatible with the common ground.[11]

It strikes me as uncontroversial that the distinction between possibilities that are merely compatible with the common ground (which I will sometimes call "plain" possibilities) and those that are live in discourse is real. It is also — as Swanson (2006), Willer (2013a), Yablo (2011), and Yalcin (2011) all observe — of theoretical significance for the semantics and pragmatics of epistemic modals. Here it suffices to highlight the following observation: epistemic *might* is frequently used in discourse to highlight the theoretical and practical significance of certain possibilities. Consider the following exchange:

11 Since the common ground is a body of information, it should not be surprising that the distinction I have in mind here finds a natural analogue in the realm of knowledge: p may be compatible with what one knows without one knowing that p is compatible with what one knows.

(7) Mary I can't find Bob. Do you know where he is?
Alex: He might be at home.
Mary: Oh, OK. I'll call his home and check.

In this simple discourse, Alex tells Mary that Bob might be at home and, as a result, there is a non-trivial change in what is common ground between the discourse participants. And intuitively, what has happened is that the possibility of Bob's being at home has changed from a possibility that was previously only compatible with what is common ground to one that Alex and Mary now explicitly recognize as relevant in discourse.

Willer (2013a) uses observations such as the previous one as an inspiration for a dynamic semantic analysis of epistemic *might*: *might*-statements are designed to affect the context by transforming plain possibilities into live possibilities — possibilities that are compatible with the context and treated as relevant in discourse or reasoning. The hypothesis that I will pursue here is that conditional antecedents have the potential of expanding the set of possibilities that are treated as relevant in discourse and reasoning as well. Specifically, the suggestion is that indicative conditionals carry the presupposition that their antecedent *might* be the case. Combined with the observation that presuppositions are accommodated — whenever this is possible and within certain limits — this just means that indicative conditionals will sometimes transform possibilities that are merely compatible with what is taken for granted into live possibilities.

The first bit of the suggestion then is that indicative conditionals potentially affect the common ground by bringing hitherto ignored possibilities into view, and that they do so in virtue of their presupposed content. Accordingly, our model of the common ground cannot simply be a context set, since in such a model all possibilities are created equal. Instead, we will appeal to the set of sets of possible worlds satisfying everything that is common ground in discourse. Intuitively, a set of possible worlds supports the claim that p might be the case just in case it contains at least one possible world at which p is true, and we may then capture the difference between what is compatible with the common ground and what is explicitly recognized in discourse in a supervaluationist fashion (following Willer 2013a, though see Franke & de Jager 2011, de Jager 2009, Swanson 2006, and Yalcin 2011 for alternative awareness models).[12] If p is compatible with the common ground, $\neg p$ fails to be common ground and so there is at least one set of possible worlds satisfying everything that is

12 Applications of supervaluationist techniques to various philosophical topics can be found, for instance, in Fine 1975, van Fraassen 1966, Kamp 1975, and Thomason 1970. Beaver (2001: Chapter 9), recognizes the importance of supervaluationist techniques in modeling the common ground, though his motivation is different from mine: his interest is to account for the possibility that participants in a discourse may sometimes not know what the common ground is. See also Rothschild 2012 and Yalcin 2012 for recent semantic applications of supervaluationist techniques.

common ground and that contains a p-world. So p is a possibility in the common ground just in case *some* set of possible worlds satisfying everything that is common ground contains a p-world. If p is a live possibility — that is, if it is common ground that p might be the case — then *every* set of possible worlds satisfying everything that is common ground supports the claim that p might be the case, which is just to say that every such set contains a p-world.

Our supervaluationist model draws the distinction we need while preserving the familiar approach of modeling the common ground using possible worlds. It also allows us to elaborate the second bit of the suggestion, which addresses the question what indicative conditionals *assert*. Start with Ramsey's (1931) suggestion that evaluating a conditional should proceed by evaluating its consequent under the assumption of the antecedent: to see whether a conditional $\ulcorner(\text{if } \phi)(\psi)\urcorner$ is acceptable in light of one's current state of belief B, one needs to determine whether ψ is accepted in light of the derived belief state got by hypothetically strengthening B with the information carried by ϕ.

One obvious proposal for the semantics of indicative conditionals that is inspired by the Ramsey test — at least if one is already in a dynamic mood — then is that they are designed to modify the context so that the consequent is accepted in light of the common ground got by adding the information carried by the antecedent. Care needs to be taken, however, when it comes to analyzing the notion of acceptance: for suppose we say that ψ is accepted in light of the common ground just in case ψ is true at every possible world in the context set. The indicative "If Alice comes to the party, it will be fun" then requires that the party will be fun at every possible world in the context set at which Alice comes. But that is just to say that there is no possible world in the context set at which Alice and Bert come to the party and it will not be fun, and so a context satisfying the constraints imposed by (2a) is guaranteed to violate those imposed by (2b). In short, on certain conceptions of what it takes for a sentence to be accepted in light of the common ground, the Ramsey-inspired proposal about the semantics of indicatives does not to leave room for weakening of the antecedent to fail.

The alternative to the previous analysis of the notion of acceptance starts with the observation that what an agent accepts — at least on one intuitive use of the word — depends on the possibilities treated as *irrelevant* in discourse and reasoning. Some possibilities are more relevant than others, and inquiring agents frequently ignore irrelevant possibilities to focus on those that accord better with their expectations about the normal course of events. So for instance, an agent who sees an equid with black and white stripes and who wonders what kind of animal he or she sees will, in most cases, ignore the possibility that the animal is a cleverly disguised mule and thus see no reason not to accept the answer that the animal is a zebra on the basis of its outer appearance. But if the agent treats the possibility that the animal is a cleverly disguised mule as relevant, then the agent will not accept the answer that the animal is a zebra until he or she has eliminated the possibility that it is a cleverly disguised mule.

The suggestion then is that our dynamic version of the Ramsey test appeals to a notion of acceptance that is sensitive to the possibilities treated as irrelevant in discourse and reasoning. To see how this suggestion can be elaborated using our model of the common ground — given some minor additional twists — recall that each of its members is just weak enough to accommodate all the possibilities that we have agreed to take seriously in discourse. As such, coming to accept *p* as a live possibility is to impose a *lower bound* on the set of possible worlds satisfying everything that is common ground. But it also may make sense for discourse participants to impose an *upper bound* on the set of possible worlds satisfying everything that is common ground: certain possibilities, we may say, are *faint* possibilities in the sense that they are compatible with the common ground yet the discourse participants have agreed to treat them as irrelevant for current purposes of the conversation.[13] What is needed, then, is a context model that tracks not only the sets of possible worlds *weak* enough to accommodate all the live possibilities in discourse — which is just what a set of sets of possible worlds would do — but also which of those are *strong* enough so that they do not accommodate any faint possibilities. To introduce a handy label, we will say that *p* is a *live necessity* just in case each of these distinguished sets exclusively consists of *p* worlds. To accept *p* for purposes of the Ramsey test is then to treat *p* as a live necessity: its negation need not be excluded from the common ground, as long as it fails to be relevant in virtue of being a merely faint possibility.

To see how all of this may be implemented in a formal model of the common ground, start with the observation that a set of sets of possible worlds can be transformed into a set of systems of spheres that is ordered by the subset relation, each coming with a center accommodating all the live possibilities in the common ground. Obviously, agreeing to treat a possibility as live is to trim each system of spheres so that its center (or minimal element) includes a *p*-world — it refines our conception of what matters in inquiry by imposing a lower bound on the region of logical space to be taken seriously in discourse and reasoning. And it then makes perfect sense to say that agreeing to treat *p* as live necessity refines our conception of what matters in inquiry as well: it imposes an upper bound on the region of logical space to be taken seriously in discourse and reasoning by eliminating each system of spheres whose minimal element fails to exclusively consist of *p*-worlds. The possibility of *p* being false may still be open, but it can only be faint possibility. As such, communication can be understood as an effort to coordinate on a region of logical space on which joint inquiry should focus, and each set of possible worlds

13 Faint possibilities bear a conceptual resemblance to what Kratzer's (1991b) labels "slight" possibilities, though they differ in their theoretical roles and technical implementations.

that plays the role of a center in the common ground (understood as a set of centered spheres) is a possible candidate for being just that region.

Accordingly, the asserted content of an indicative conditional $\ulcorner(\text{if } \phi)(\psi)\urcorner$ is modeled dynamically as a process that establishes ψ as a live necessity in the result of strengthening the common ground with the information carried by ϕ, which is just to say that it establishes the material conditional $\ulcorner\phi \supset \psi\urcorner$ as a live necessity. Putting everything said so far together, the dynamic proposal for the indicative conditional $\ulcorner(\text{if } \phi)(\psi)\urcorner$ amounts to the following picture: it establishes ϕ as a live possibility in virtue of its presupposed content, and then establishes $\ulcorner\phi \supset \psi\urcorner$ as a live necessity in virtue of its asserted content. The two components conspire to account for the phenomenon of Sobel sequences for indicatives. The crucial observation is that live necessities may be *defeated* through the process of transforming certain possibilities into live possibilities and in particular, $\ulcorner\phi \supset \psi\urcorner$ may hold throughout the centers in the common ground but fail to hold throughout the expansion of those minimal elements triggered by the need to accommodate a hitherto ignored possibility.

To see how this proposal handles the indicative Sobel sequence in (2), start with a common ground Σ in which A is a live possibility and $A \supset F$ is a live necessity. Then "If Alice comes to the party, it will be fun" is established in Σ, but it may very well be that we entertain the possibility that Alice comes to the party without entertaining the possibility that Alice *and* Bert come to the party, even though for all we know it is possible that both will come. But then "If Alice and Bert come to the party, it will not be fun" triggers an expansion of the live possibilities to include the possibility that $A \wedge B$, and it may very well be that $A \supset F$ fails to be a live necessity in the resulting common ground. So it is possible to establish $(A \wedge B) \supset \neg F$ as a live necessity, but once more: it may very well be that in entertaining the possibility that Alice and Bert come to the party, we have ignored the possibility that Charles comes too, even though for all we know it is possible that he will come. Then an utterance of "If Alice and Bert and Charles come to the party, it will be fun" once *again* triggers an expansion of the set of relevant possibilities so that they include the possibility of $A \wedge B \wedge C$. And of course, $(A \wedge B) \supset \neg F$ need not be a live necessity in *those* lights, and so we may now establish $(A \wedge B \wedge C) \supset F$ as a live necessity, resulting in a common ground in which the even thinner conditional is established. So far for the intuitive outline of the proposal. The next step is to make what I have said here a little bit more precise.

3.2 Key details

In this section I elaborate on the key formal aspects of the proposal with the goal of highlighting what exactly is doing the explanatory lifting in a dynamic strict approach to indicative Sobel sequences. A comprehensive presentation of the technical details is left for the Appendix, which also includes proofs of some key results.

Let me begin here by stating briefly the central outcome of the upcoming formal proposal. Earlier I said that indicative conditionals establish their corresponding material conditional as live necessities but that such necessities may be *defeated* as hitherto ignored possibilities come into view. This perspective translates into the claim that the logical consequence relation for conditionals fails to be monotonic in the following sense:

$$\text{MONOTONICITY:} \quad \text{If } \phi_1, \ldots, \phi_n \vDash \psi\text{, then } \phi_1, \ldots, \phi_n, \phi_{n+1} \vDash \psi$$

Monotonicity requires that whatever has been established in discourse and reasoning remains established in light of additional information. In contrast, from a dynamic outlook a member of a Sobel sequence has the potential, in virtue of its presupposed content, to *remove* a commitment established by its predecessor. In particular, we will obtain the following result:

$$(\text{if } A)(F) \vDash (\text{if } A)(F) \text{ but } (\text{if } A)(F), (\text{if } A \wedge B)(\neg F) \nvDash (\text{if } A)(F)$$

The result brings into relief the hypothesis that indicative conditionals are bounded from below by the material conditional and the observation that — on pain of Sobel sequences suffering from presupposition violations — the first two members of our Sobel sequence must not entail "$\neg(A \wedge B)$." A commitment to an indicative conditional comes with a commitment to the corresponding material conditional. And for sure, a commitment to "$A \supset F$" *and* "$(A \wedge B) \supset \neg F$" would bring in its wake a commitment to "$\neg(A \wedge B)$." But at *no* point in a consistent Sobel sequence does such a joint commitment arise since each of its members *defeats* the commitment to its predecessor. The purpose of the next sections is to explain how such a nonmonotonic perspective arises naturally if we treat indicative conditionals as strict over a dynamically evolving domain of quantification.

3.2.1 Contexts and possibilities

I start by saying a bit more about the model of context I will employ and how it keeps track of the conceptual distinctions underlying the proposal. The basic picture, recall, is that discourse is an effort to coordinate on a region of logical space on which discourse should focus, that is, on a region of logical space weak enough to accommodate all live possibilities but strong enough to exclude all faint possibilities. Thinking of a context as a set of systems of spheres provides us with enough structure to make this picture more precise: each minimal element of a sphere is a possible region of logical space on which joint inquiry should focus. Discourse moves that aim at sharpening this space translate into operations on the spheres representing the relevant context.

A system of spheres is nothing but a (for our purposes nonempty) set of sets of possible worlds totally ordered by the subset relation. The ordering represents how relevant the region of logical space is for inquiry, with the innermost sphere being the most relevant and the outermost region being the least relevant region. I will thus speak of the elements of our context models as *relevance orderings* and of sets of such orderings as *context states*.[14] Modeling a context this way allows us to distinguish between plain and live possibilities as well as between plain and live necessities in a supervaluationist fashion. Propositions are, as usual, understood as sets of possible worlds and are thus of the same kind as are the elements of a relevance ordering.

i. A proposition p is a *possibility* in context Σ just in case p is compatible with the outermost sphere of *some* relevance ordering in Σ.

ii. A proposition p is a *necessity* in context Σ just in case p is entailed by the outermost sphere of *every* relevance ordering in Σ.

iii. A proposition p is a *live* possibility in context Σ just in case p is compatible with the innermost sphere of *every* relevance ordering in Σ.

iv. A proposition p is a *live* necessity in context Σ just in case p is entailed by the innermost sphere of *every* relevance ordering in Σ.

This is nothing but a more precise statement of the previous discussion in Section 3.1. Given some context c we will continue to speak of s_c as the context set of c, and we will now say that Σ_c is the context state of c. Intuitively, the union of the outermost spheres of the elements of Σ_c constitute s_c: p counts as a possibility in discourse just in case the context set contains at least one p-world, that is, just in case some relevance ordering in Σ_c leaves the possibility of p being true open. Conversely, p is a necessity just in case its negation is incompatible with the context set, that is, just in case no relevance ordering in Σ_c leaves the possibility of p being true open.

Context states are powerful enough to distinguish between possibilities and necessities in discourse, just as context sets do, but their additional structure allows us to define the notions of a live possibility and of a live necessity as well: if context keeps track of which regions of logical space we should focus on, then live

14 Appealing to a relevance ordering is *not* to rely on the same apparatus that drives the classical variably strict semantics since nothing I have said here requires that those orderings be generated by an underlying similarity relation. All that is needed is that context comes with a (ultimately dynamically evolving) fallback relation governing the accommodation procedure of hitherto ignored possibilities. See Gillies 2007 for relevant discussion.

possibilities must be compatible with each possible candidate for this region — that is, with every center of every relevance ordering in the context model. In other words, for p to be a live possibility, the discourse participants' conception of which possibilities to take seriously in discourse must entail the existence of a p-world (but not of some *particular* p-world). For p to be a live necessity, in turn, the discourse participants' conception of which possibilities to take seriously in discourse must entail p — every center of every relevance ordering in the context model must exclusively consist of p-worlds.[15] Faint possibilities are just possibilities whose negation are live necessities.

Let me illustrate these distinctions using a very simple model. Suppose that our context state Σ consists of the relevance orderings $\pi_1 = \{s_1, s_2, s_3\}$ and $\pi_2 = \{s_2, s_3\}$, where $s_1 \subset s_2 \subset s_3$. Then s_1 and s_2 are possible centers of inquiry but not s_3. We can keep track of spheres that are elements of the context but fail to be possible centers of inquiry by drawing their borders using dotted lines, like this:

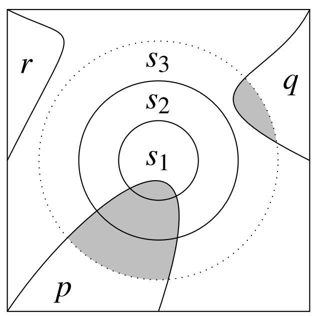

Figure 1: A possible context state Σ

Clearly, p is a live possibility in Σ since the possible centers of inquiry s_1 and s_2 include p-worlds. In contrast, q is only a faint possibility: while not excluded by every sphere in the context and hence a possibility, it is incompatible with every candidate for the center of inquiry. Conversely, then, $\neg q$ is a live necessity. Finally, r is incompatible with any relevance ordering in Σ — observe here that the context set of Σ coincides with s_3 — and so $\neg r$ is a necessity in Σ.

What I have done so far is to outline how we can model the distinction between plain and live possibilities and necessities in discourse. The distinction as well as the way it can be captured by our models of context is of independent interest but here I

15 Observe here that the notion of a live necessity is weaker than the one of a plain necessity. This is unsurprising since possibility and necessity are duals and the notion of a live possibility is stronger than the one of a plain possibility.

will focus on explaining how conversational moves can modify these contexts and, in particular, establish certain propositions as live possibilities and necessities.

3.2.2 Modals

Dynamic semantic analyses construe the meaning of a sentence in terms of its context change potential and here I will start with a familiar dynamic proposal about epistemic modals. Veltman (1996) understands contexts as plain sets of possible worlds and lets propositional formulas eliminate from those contexts all possible worlds at which they are false, just as in Stalnaker's pragmatic model. So if ϕ is any propositional formula and s is some context understood as a set of possible worlds, then $s \uparrow \phi$, the result of *updating* s with ϕ, is just the result of intersecting s with the proposition expressed by ϕ. Epistemic modals are different since they *test* whether the input state possesses certain properties. Specifically, an epistemic possibility claim of the form $\ulcorner \Diamond\phi \urcorner$ checks whether the input context s is compatible with the prejacent ϕ. An epistemic necessity claim of the form $\ulcorner \Box\phi \urcorner$ checks whether the input context entails its prejacent ϕ in the sense that adding ϕ to s does not induce any change at all. A passed test simply returns the original input context, while a failed test results in the empty set of possible worlds:

$$s \uparrow \Diamond\phi = \begin{cases} s & \text{if } s \uparrow \phi \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases} \qquad s \uparrow \Box\phi = \begin{cases} s & \text{if } s \uparrow \phi = s \\ \emptyset & \text{otherwise} \end{cases}$$

This treatment of epistemic *might* and *must* is at the heart of proposals defended by, among others, Stalnaker (1970), Beaver (2001), von Fintel & Gillies (2008), and Willer (2013a), and they are — from an abstract point of view — very similar to Yalcin's (2007) domain semantics for epistemic modals. An equivalent (and notationally more convenient) way of stating the above update rules is the following:

$$s \uparrow \Diamond\phi = \{w \in s\colon\ s \uparrow \phi \neq \emptyset\}$$
$$s \uparrow \Box\phi = \{w \in s\colon\ s \uparrow \phi = s\}$$

The story I am about to tell here works with a notion of a context more complex than Veltman's and so we need to add a few — fairly straightforward — twists to the simple update model for *might* and *must*. Let us go through the details.

Epistemic modals, I have said, impose lower or upper bounds on the set of possible worlds satisfying everything that is common ground. Since the center of a relevance ordering is a candidate for being this set, it then makes sense to think of epistemic *might* as trimming each relevance ordering so that its center can be consistently updated with the prejacent, thus establishing its prejacent as relevant for inquiry. *Must* trims each relevance ordering so that its center entails the prejacent.

To make this idea more precise I let the update function operator "$\uparrow$" be sensitive to a relevance ordering π and modify Veltman's test for *might* and *must*. The state s that is here modified should be interpreted as a sphere in a relevance ordering:

$$s \uparrow_\pi \Diamond\phi = \{w \in s \colon\ \exists s' \in \pi.\, s' \subseteq s \text{ and } s' \uparrow_\pi \phi \neq \emptyset\}$$
$$s \uparrow_\pi \Box\phi = \{w \in s \colon\ \exists s' \in \pi.\, s' \subseteq s \text{ and } s' \uparrow_\pi \phi = s'\}$$

Let me highlight the two particular aspects of these update rules that will matter for our discussion. Assume that ϕ is a plain propositional formula and that we are updating a sphere s of a relevance ordering π. Then s passes the test imposed by $\ulcorner\Diamond\phi\urcorner$ if and only if some subset s' of s that constitutes a sphere in π includes a ϕ-world. Clearly, this is so just in case s itself includes a ϕ-world, and so there is no real difference between this treatment of epistemic *might* and Veltman's original proposal. The update rule for $\ulcorner\Box\phi\urcorner$ is a bit more interesting since it asks whether π contains some subset s' of s as a sphere that exclusively consists of ϕ-worlds, and this will be so if and only if the center of the ordering π exclusively consists of ϕ-worlds. Specifically, a formula such as "$\Box(p \supset q)$" tests whether every p-world in the minimal domain of π is also a q-world: such formulas are thus interpreted as strict material conditionals over a minimal domain of some system of spheres. As a consequence, a sphere s may pass the test imposed "$\Box(p \supset q)$" even if it includes worlds at which the material conditional is false: what matters is that the material conditional is entailed by the *center* of the sphere of which s is a member. Propositional formulas continue to update a sphere in the classical fashion.

The above update rules apply to spheres in a relevance ordering — it remains to elaborate on how orderings as a whole as well as context states change in response to utterances in discourse. A relevance ordering is updated by updating each of its members (in light of that ordering) and then collecting all the non-empty results:

$$\pi \uparrow \phi = \{s \colon\ s \neq \emptyset \text{ and } \exists s' \in \pi \colon\ s' \uparrow_\pi \phi = s\}$$

Basically the same procedure is at play when it comes to updating a context state: simply collect the results of updating each of its relevance orderings. The only additional condition we will impose here is that for an update with ϕ to be successful, its negation must not already be a live necessity in the context. Let us say then that Σ *admits* ϕ just in case some relevance ordering π of Σ comes with a minimal sphere s such that $s \uparrow_\pi \phi \neq \emptyset$. Then:

$$\Sigma[\phi] = \{\pi \colon\ \Sigma \text{ admits } \phi \text{ and } \exists \pi' \in \Sigma \colon\ \pi' \uparrow \phi = \pi\}$$

In sum then, updating a context state with ϕ involves the following steps: update the spheres of every relevance ordering and gather all the nonempty results. The output state is just the collection of these relevance ordering, provided that an update with ϕ is admissible (otherwise the update returns the empty set of relevance orderings).

It is easy to explain the basic mechanism using some illustrations. As in the previous example I appeal to a simple context state Σ consisting of two relevance orderings $\pi_1 = \{s_1, s_2, s_3\}$ and $\pi_2 = \{s_2, s_3\}$, but here I set up logical space slightly differently (Figure 2). Figure 3 shows the result of updating Σ with an atomic sentence, which is nothing but the result of adding the classical proposition expressed by that sentence (here the proposition p) to each sphere in the context:

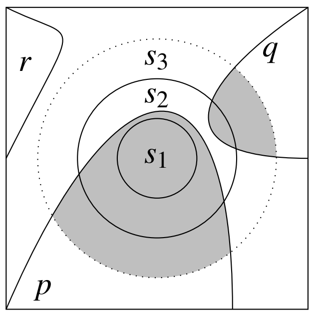

Figure 2: Σ

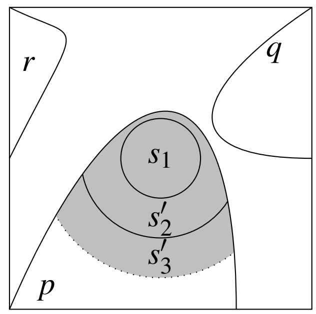

Figure 3: $\Sigma[p]$

The following illustrations exhibit the result of updating Σ with "$\Diamond q$" and "$\Box p$," respectively. The former eliminates every sphere not including a q-world — here s_1 — thus establishing q as a live possibility. The latter eliminates all relevance orderings not centered on a sphere exclusively consisting of p-words — here $\{s_2, s_3\}$ — thus establishing p as a live necessity. Note that s_2 is no longer a possible center of inquiry in $\Sigma[\Box p]$:

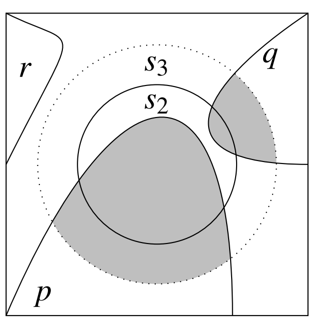

Figure 4: $\Sigma[\Diamond q]$

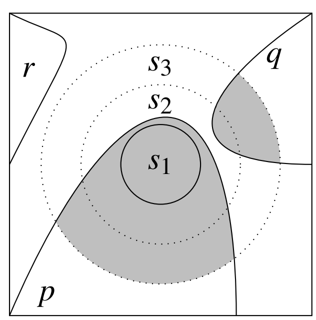

Figure 5: $\Sigma[\Box p]$

I have now outlined how to make formal sense of the dynamic dimension of epistemic possibility and necessity claims in discourse. The next item on the agenda is to put these ideas together to make the dynamic analysis of conditionals more precise.

3.2.3 Indicative conditionals

I need to add one minor wrinkle to the picture developed so far. Indicative conditionals, I have said, *presuppose* that their antecedent might be the case. To capture this we require a formal conception of a presupposition operator ∂. Following standard protocol presuppositions are definedness conditions on updating:

$$s \uparrow_\pi \partial\phi = s \text{ iff } s \uparrow_\pi \phi = s$$

An update with $\ulcorner \partial\phi \urcorner$ idles in case an update with ϕ idles, and is undefined otherwise (see Beaver 2001 and Heim 1982, among others).

It is then easy to capture the idea that indicative conditionals are strict material conditionals presupposing that their antecedent is a live possibility. To update an element of a relevance ordering with an indicative conditional, process its presupposition that the antecedent might be the case and then its assertion that the corresponding material conditional is a live necessity in those lights:

$$s \uparrow_\pi (\text{if } \phi)(\psi) = (s \uparrow_\pi \partial\Diamond\phi) \uparrow_{\pi'} \Box(\phi \supset \psi)$$

where $\pi' = \{s \uparrow_\pi \partial\Diamond\phi : \ s \in \pi\}$. It follows immediately that updating a context state with a conditional $\ulcorner(\text{if } \phi)(\psi)\urcorner$ amounts to an update with $\ulcorner\partial\Diamond\phi\urcorner$ followed by an update with $\ulcorner\Box(\phi \supset \psi)\urcorner$, that is, $\Sigma[(\text{if } \phi)(\psi)] = \Sigma[\partial\Diamond\phi][\Box(\phi \supset \psi)]$.

Let me once again use some illustrations to highlight the key ideas at work and why they matter. Consider a context state $\Sigma = \{\pi_1, \pi_2, \pi_3\}$, where $\pi_1 = \{s_1, s_2\}$, $\pi_2 = \{s_1, s_3\}$, $\pi_3 = \{s_2\}$, and logical space is set up as follows:

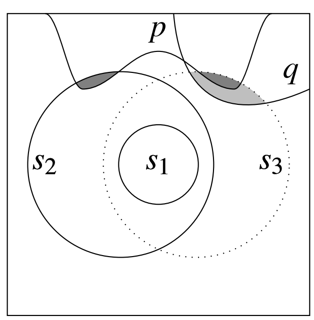

Figure 6: Σ

Two observations about this context state: first, p fails to be a live possibility since s_1 is a possible center of inquiry; second, q is merely a faint possibility since s_3 fails to be a possible center of inquiry and so $\neg q$ is a live necessity. All of this matters once we consider the result of updating Σ with the conditional "(if p)(q)." Its presupposed content raises p to a live possibility by eliminating s_1 as a possible center of inquiry and thus transforming s_3 into a possible center of inquiry (Figure 7). It then establishes the material conditional $p \supset q$ as a live necessity, thus eliminating s_2 as a possible center of inquiry (Figure 8).

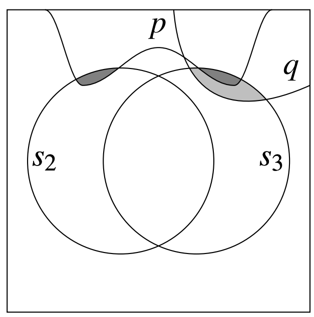

Figure 7: $\Sigma[\partial\Diamond p]$

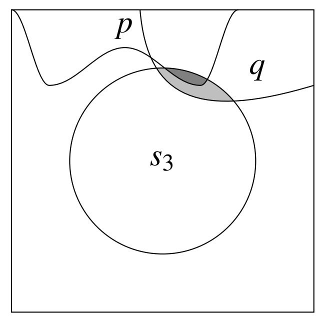

Figure 8: $\Sigma[(\text{if } p)(q)]$

This is a very simple example, clearly, but it highlights the potential of conditionals to defeat a live necessity (in this case $\neg q$) in virtue of their presupposed content and to establish a live necessity (in this case $p \supset q$) in virtue of their asserted content.

The example under consideration also illustrates the key mechanism behind the dynamic treatment of Sobel sequences. For let "$\top$" stand for any tautology: then the example shows what happens if we update Σ with a sequence of conditionals starting with "(if $\top$)($\neg q$)" and followed by "(if $\top \wedge p$)(q)." $\Sigma[(\text{if } \top)(\neg q)] = \Sigma$ since the presupposition of the possibility of the trivial proposition is satisfied and $\top \supset \neg q$ is already a live necessity in Σ. The result of updating Σ with the second conditional, which is just illustrated by Figure 8, is *consistent* in the sense that if fails to be the empty set, and it is noteworthy in two respects: first, it *fails* to be committed to the of negation of $\top \wedge p$ — that is, the negation of the antecedent of the second member of the Sobel sequence — since it treats p and thus $\top \wedge p$ as a live possibility. Second, it *no longer* supports the first member of the sequence since the presupposition of the second conditional has effectively defeated $\neg q$ and thus $\top \supset \neg q$ as a live necessity. More complex Sobel sequences can be processed in more complex context states.

The fact that conditionals establish certain possibilities as live necessities that may be defeated as hitherto ignored possibilities come into view is the key driver behind the nonmonotonicity of a dynamic logic for conditionals. Let me explain.

3.2.4 Entailment

The final step is to state the notion of logical consequence more precisely. Here I will follow a familiar dynamic path and say that a sequence $\phi_1, \ldots, \phi_n$ entails ψ just in case any context that has been updated with that sequence results in a context committed to ψ. What matters here is how exactly the notion of a commitment is to be understood, and I already suggested earlier that commitment is a matter of which possibilities are deemed most relevant in inquiry. Specifically, we will say that a context state Σ is committed to ϕ just in case every center of each relevance ordering in Σ supports ϕ:

Σ is *committed* to ϕ, $\Sigma \Vdash \phi$, iff for every $\pi \in \Sigma$ and every center s of π: $s \uparrow_\pi \phi = s$

The notion of logical consequence and consistency are then as follows:

i. $\phi_1, \ldots, \phi_n$ *entails* ψ, $\phi_1, \ldots, \phi_n \vDash \psi$, iff for all Σ: $\Sigma[\phi_1] \ldots [\phi_n] \Vdash \psi$

ii. $\phi_1, \ldots, \phi_n$ is *consistent* iff for some Σ: $\Sigma[\phi_1] \ldots [\phi_n] \neq \emptyset$

An argument is valid just in case its conclusion is accepted by every context state once updated with its premises. A sequence is consistent just in case we can find some context state that can be updated with that sequence without resulting in the absurd state. These definitions could undergo some optional tweaking (see Section 4.2 for discussion) but for now I choose the most straightforward path.

3.3 Output

The proposal developed here, while nonclassical, preserves all the semantic and pragmatic constraints we imposed earlier on indicative conditionals. Start with a simple and intuitive assumption about pragmatics: an utterance of ϕ in a context c is a proposal to update Σ_c with ϕ. In other words, the result of uttering ϕ in c is such that $\Sigma_{c+\phi} = \Sigma_c[\phi]$. Since propositional formulas eliminate from every sphere in the context all possible worlds at which the proposition expressed is false, and since the context set is nothing but the union of all the spheres in a context, assertions of propositional formulas just do what Stalnaker (1978) claims they do.

The fact that indicative conditionals presuppose that their antecedent might be the case immediately predicts that $\Sigma_c[(\text{if } \phi)(\psi)] \neq \emptyset$ only if $\Sigma_{c+\phi} \neq \emptyset$ and thus only if $s_{c+\phi} \neq \emptyset$. It follows immediately that the story told here preserves the textbook explanation of why (5a) is marked. Notice, furthermore, that the *might*-content carried by a conditional projects just in the way we expect in case it is presupposed: for instance, we predict that an update with a conditional as well as with any complex formula containing it are undefined in case the input context state does not treat its antecedent as a possibility.

Finally, an indicative conditional is bounded from below by the material conditional: updating a context with an indicative conditional of the form $\ulcorner(\text{if } \phi)(\psi)\urcorner$ establishes the corresponding material conditional as a live necessity. But that is just what it takes for a material conditional to be accepted in a context and so, given our definition of logical consequence, $(\text{if } \phi)(\psi) \vDash \phi \supset \psi$. Importantly, since $(\text{if } \phi)(\psi) \vDash \Diamond\phi$ as well — conditionals establish their antecedent as a live possibility — indicative conditionals detach in the sense that $(\text{if } \phi)(\psi), \phi \vDash \psi$.

All of that, and we can *still* account for the felicity of indicative Sobel sequences. To show why this is so, it is sufficient to show that they are consistent, since updating with a presupposition that cannot be accommodated is guaranteed to result in the absurd state. I will do so for our initial example (2) using a detailed example in the Appendix, but it is easy enough to see why the story told here will do the trick. The first observation is that conditionals resist strengthening of the antecedent in the sense that $(\text{if } A)(F), (\text{if } A \wedge B)(\neg F)$ is consistent. For consider $\Sigma' = \Sigma[(\text{if } A)(F)]$: then $\Sigma' \Vdash \Diamond A$ and $\Sigma' \Vdash A \supset F$. But $\Diamond A \nvDash \Diamond(A \wedge B)$ and so it may very well be that $\Sigma' \nVdash \Diamond(A \wedge B)$. In that case, accommodating the presupposition carried by $(\text{if } A \wedge B)(F)$ in Σ' will induce a non-trivial change and, in particular, may defeat $A \supset F$ as a live necessity, leaving room for consistent update with $\Box((A \wedge B) \supset \neg F)$.

What caused the trouble for a variably strict semantics was that (2a) and (2b) are consistent but entail that Alex and Bert will not both come to the party. The second observation, then, is that the dynamic framework avoids this result: $(\text{if } A)(F), (\text{if } A \wedge B)(\neg F) \nvDash \neg(A \wedge B)$. True enough, $(\text{if } A)(F) \vDash A \supset F$ and $(\text{if } A \wedge B)(\neg F) \vDash (A \wedge B) \supset \neg F$, and also $A \supset F, (A \wedge B) \supset \neg F \vDash \neg(A \wedge B)$. But crucially $(\text{if } A)(F), (\text{if } A \wedge B)(\neg F) \nvDash A \supset F$, and for the familiar reason that live necessities may be defeated once open but hitherto ignored possibilities come into view. So just as it is possible to update $\Sigma[(\text{if } A)(F)]$ with $(\text{if } A \wedge B)(\neg F)$ since $\Diamond A \nvDash \Diamond(A \wedge B)$, it is possible to update $\Sigma[(\text{if } A)(F)][(\text{if } A \wedge B)(\neg F)]$ with $(\text{if } A \wedge B \wedge C)(F)$ since $\Diamond(A \wedge B) \nvDash \Diamond(A \wedge B \wedge C)$.

The crucial feature that brings into relief the validity of modus ponens with the felicity of indicative Sobel sequences in this framework is thus the nonmonotonicity of the dynamic logical consequence relation, specifically the fact that $(\text{if } A)(F) \vDash (\text{if } A)(F)$ but $(\text{if } A)(F), (\text{if } A \wedge B)(\neg F) \nvDash (\text{if } A)(F)$. On the view developed here, then, indicative Sobel sequences are consistent because each member of the sequence *defeats* its predecessor. The predecessor establishes its corresponding material conditional as a live necessity, but live necessities may be defeated as hitherto ignored possibilities come into view — which is just what the subsequent conditional does in virtue of its presupposed content.

The key mechanisms at work here are also highlighted by the (alleged) counterexample to modus ponens from Lycan (1993, 2001) Consider the first two members of our indicative Sobel sequence again — (2a) that the party will be fun if Alice comes and (2b) that the party will not be fun if Bert comes too — and suppose that:

(8) Alice and Bert come to the party.

Lycan's puzzle is that if modus ponens were valid, then (8) and (2b) would entail that the party will not be fun, but assuming conjunction elimination, (8) and (2a) would *also* entail that the party will be fun, and so we end up with a contradiction. Where did we go wrong?

A textbook variably strict analysis that preserves modus ponens must insist that (8) is incompatible with (2a) and (2b), as we have seen. Lycan maintains that this is a counter-intuitive result but earlier we observed that it comes with a deeper problem: it conflicts with the uncontroversial datum that the sequence consisting of (2a) and (2b) may be continued with (2c) without fuss. It does not follow, however, that one has to reject modus ponens since one might question Lycan's assumption of a monotonic logical consequence relation. Specifically, on the dynamic story told here (2b) *defeats* (2a) and so once we have updated with (2b), we may no longer appeal to (2a) in drawing inferences. So the result of strengthening the common ground with (2a), (2b), and (8) supports that the party will not be fun, but *not* that the party will be fun. A nonmonotonic perspective on Sobel sequences allows us to resist Lycan's case against modus ponens.

4 Loose ends

In this section I address a few remaining issues that are of immediate relevance to the dynamic story told so far. The first is Moss's (2012) challenge from reverse Sobel sequences. The second concerns Stalnaker's (2011) recent puzzle about the direct argument. Finally, I briefly comment on the prospects of expanding the framework to counterfactuals.

4.1 Reverse Sobel sequences

Heim observes that reverse counterfactual Sobel sequences are in general marked. This is just as true for their indicative cousins:

(9) #(a) If Alice and Bert come to the party, it will not be fun. (b) But if Alice comes, it will be fun.

As von Fintel (2001) and Gillies (2007) detail, Heim's observation is readily explained by a dynamic strict analysis of conditionals, and the framework developed here is no exception. For suppose that Σ treats $A \wedge B$ as a live possibility and $(A \wedge B) \supset \neg F$ as a live necessity. Then accommodating the presupposition carried by (9b) idles and updating with its asserted content results in the absurd state. A variably strict semantics for conditionals, in contrast, does not *by itself* account for the difference between plain Sobel sequences and their ugly reverse cousins, for whenever our original Sobel sequence is classically consistent, so is (9).

Moss (2012) maintains that a variably strict semantics may appeal to pragmatic considerations in order to explain why reverse Sobel sequences, while perfectly consistent, are in general marked, but she also holds that there are circumstances in which they are not marked in the first place. Her discussion focusses on counterfactuals but the scenarios considered also come with some force in the indicative case. Here is a representative one: imagine a speaker who intends to convey that Mary will turn down a marriage proposal from John, but cannot explicitly say so (for instance because the speaker promised not to do it). In such a scenario (10) could be acceptable:

(10) (a) If John proposes to Mary and Mary says *yes*, he will very happy. (b) But if John proposes, he will be very unhappy.

What makes cases like these interesting is that reverse Sobel sequences now seem to pose a problem for a dynamic strict analysis of conditionals rather than support it since (10) is dynamically inconsistent by design.[16]

However, it is crucial for the case under discussion that the antecedent of (10a) pertains to a possibility that can be ignored in discourse and that the speaker intends to indirectly communicate to the audience that this is so. But this opens up the option of accounting for the acceptability of (10) and related examples by appealing to some general principles about rational communication. For speakers are in general cooperative and thus do not propose to add information to the common ground that is incompatible with what has been established. In these lights, it makes good sense to say that a speaker may communicate ψ by asserting ϕ in case updating the discourse

16 Similar problems on first sight arise with sequences such as:

> Alice and Bert might come to the party. If Alice comes, it will be fun. But if Alice and Bert come, it won't be fun.

But cases like these are a bit more complicated since the first member of the sequence is ambiguous between (a) $\Diamond A \wedge \Diamond B$ and (b) $\Diamond(A \wedge B)$. The framework developed here fails to predict the consistency of the sequence only if the second interpretation is in play, and it is not clear that this is a bad result. To wit, enforcing the second reading makes the sequence much less acceptable:

> ?? Alice and Bert might come to the party together. If Alice comes, it will be fun. But if Alice and Bert come, it won't be fun.

It thus makes sense to focus on examples such as (10), which pose a more clear-cut problem to the dynamic analysis presented here.

with ϕ results in the absurd state but updating that context with ψ and then with ϕ does not. More precisely:

Dynamic Quality Suppose that $\Sigma_c[\phi] = \emptyset$ but $\Sigma_c[\psi][\phi] \neq \emptyset$ and for all χ such that $\Sigma_c[\chi][\phi] \neq \emptyset$, $\Sigma_c[\chi] \Vdash \psi$. Then an assertion of ϕ in context c by default pragmatically implies a proposal to update $\Sigma_c[\psi]$ with ϕ.

The crucial observation now is that whenever $\Sigma_c \Vdash (\text{if } \phi \wedge \psi)(\neg\chi)$, then an utterance of $\ulcorner(\text{if } \phi)(\chi)\urcorner$ in context c amounts to an assertion of $\ulcorner\Box(\phi \supset \chi)\urcorner$ in Σ_c. And while asserting $\ulcorner\Box(\phi \supset \chi)\urcorner$ in Σ_c inevitably results in the absurd state, making this assertion in Σ_c *as strengthened with* $\ulcorner\neg(\phi \wedge \psi)\urcorner$ may be perfectly consistent.

The general observation is that while a dynamic strict analysis is committed to conditional antecedents bringing certain possibilities into view, these possibilities may be consistently eliminated as discourse proceeds. If what I have said before is right, we may then appeal to more general communicative principles to explain why utterances of conditionals can be used to convey that certain previously highlighted possibilities may be eliminated. For instance, an assertion of (10b) after uttering (10a) may communicate that Mary will not say *yes* to John's proposal since updating with this information allows for a consistent update with the asserted content of (10b) while simply doing so with the asserted content of (10b) does not. Suppose that at w_1 Mary accepts John's proposal and he is very happy while at w_2 Mary rejects John's proposal and he is very unhappy, and consider a really simple context state $\Sigma_c = \{\pi\}$ with $\pi = \{\{w_1, w_2\}\}$. This context rejects the claim that if John proposes to Mary, he will be very unhappy, for according to Σ_c she might say *yes*, and then John will be very happy. The context becomes compatible with that claim, however, once we strengthen it with the information that Mary will not accept John's proposal — in fact the resulting (and perfectly consistent) context state $\Sigma_{c'} = \{\pi'\}$ with $\pi' = \{\{w_2\}\}$ is committed to (10b).

The difference between felicitous and infelicitous instances of reverse Sobel sequences then is that in the former, but not in the latter case, the speaker is in a position to assert, and may be interpreted as intending to communicate, that the possibility raised by the first member of the sequence can safely be ignored. Developing a story that predicts when *exactly* this is the case goes beyond the scope of this paper: this, I maintain, is a task for pragmatics, and I refer the reader to Moss's (2012) discussion for inspiration as to how such a story should go. The point is that such a pragmatic story may be smoothly integrated into the dynamic semantic proposal developed here once we realize that it leaves room for possibilities brought into view to be eliminated to bring speech acts in line with general principles governing cooperative communication.

The outlined story about reverse Sobel sequences, I should add, is compatible with there being additional pragmatic constraints on when possibilities brought into view by conditional antecedents may subsequently be eliminated. As observed by an anonymous reviewer, the following sequence sounds odd:

(11) #(a) If John is coming over for Thanksgiving, we should make a vegan dish.
(b) No one is coming over for Thanksgiving.

A plausible explanation of what is going on here is that (11a) plays no obvious communicative purpose: the speaker asserts a conditional just to state explicitly that its definedness condition is not met. The situation is different in (10). For sure, here the speaker also communicates that the definedness condition of the first conditional (10a) is not met. But *that* very communicative act — the one of communicating that Mary will not accept John's proposal — crucially relies on the utterance of the first conditional: it establishes a context in which an utterance of the second conditional (10b) triggers the dynamic quality implicature.

The upshot of this discussion is that the possibility of felicitous reverse Sobel sequences can be accommodated in the dynamic framework developed here. Granted, since the story I have told appeals to pragmatic considerations in order to account for the complexity of the data surrounding reverse Sobel sequences, it is hard to make a compelling case to the conclusion that these data favor a dynamic strict analysis of conditionals over its static variably strict alternative. My modest goal here has been to show that, given plausible pragmatic assumptions, a dynamic strict outlook on Sobel sequences remains a viable alternative even if reverse Sobel sequences are not infelicitous across the board.

4.2 The direct argument

Stalnaker (1975) observes that there is a compelling inferential connection between disjunctions and conditionals:

Direct Argument $\quad \phi \vee \psi \vDash (\text{if } \neg\phi)(\psi)$

A variably-strict analysis of conditionals does not deliver the validity of the direct argument, though Stalnaker explains how it can still count as a "reasonable" inference. Many dynamic analyses of conditionals (see for instance Gillies 2004) semantically license the direct argument, but the story told here does not deliver this result in its current form: simply observe that nothing about $\ulcorner \phi \vee \psi \urcorner$ being a necessity in the common ground guarantees that $\ulcorner \neg\phi \urcorner$ is a live possibility in the common ground. So while $\ulcorner \phi \vee \psi \urcorner$ certainly entails the asserted content of a conditional of the form

$\ulcorner(\text{if } \neg\phi)(\psi)\urcorner$, this is not true of the conditional's presupposed content and so accommodation may very well require nontrivial changes to the common ground even after the disjunction has been established.

The result is to be expected given the fact that presuppositions can often be informative and thus induce non-trivial changes in the common ground. It also allows us to make sense of certain problematic cases for the direct argument such as Stalnaker's (2011) murder mystery, in which one investigator, Alice, firmly believes that the gardener is innocent and that the chauffeur has a good alibi, and thus concludes that the butler did it (who in fact is the murderer). Her partner Bert also thinks that the chauffeur is innocent but has misleading evidence that he takes to exonerate the butler, thus concluding that the gardener did it. He now presents the following line of reasoning to Alice:

(12) We disagree about who did it, but we agree that it was either the butler or the gardener. So you should agree that if the butler did not do it, then the gardener did it.

The issue here, according to Stalnaker, is that Alice may agree with the disjunction and *still* be justified in resisting Bert's conclusion: were she to learn, much to her surprise, that the butler was innocent after all, she would conclude that the chauffeur's alibi must not be as good as it looks and that he is the culprit.

Nothing about this case is surprising in light of the framework proposed here, for even if Bert's disjunctive premise is granted, his conclusion rests on the presupposition that the butler's being innocent is a live possibility in the common ground. In the scenario under consideration, accommodating the presupposition induces a non-trivial change in the common ground and, what is more, this change is one that Alice is unlikely to accept — no wonder that she resists Bert's line of reasoning. Notice, furthermore, that this diagnosis crucially relies on the idea that conditionals carry *might*-presuppositions rather than simply requiring that their antecedent be compatible with the common ground. If the latter were the case, Bert's conditional would carry a presupposition that is already satisfied and thus needs no accommodation, leaving it a mystery what is going wrong in his argument.[17]

Still, there remains the fact that the direct argument is not without intuitive appeal, and it is easy to see why: the premise entails its conclusion under the assumption that the latter's presuppositions are met. Say that $\pi_\phi = \{s \in \pi\colon\ s \uparrow_\pi \phi \text{ is defined}\}$

17 In discussing this case I follow Stalnaker in assuming that Alice agrees with Bert's disjunctive premise. But one may also challenge this assumption: disjunctions in general imply that both disjuncts might be true, and so Alice should not agree with Bert's premise in the first place. In any case the explanation of Alice's reaction would have to rest on the distinction between p being a live possibility in the common ground and it being compatible with the common ground.

and that $\Sigma \circ \phi = \{\pi\colon\ \exists \pi' \in \Sigma.\, \pi'_\phi = \pi\}$. Taking some inspiration from von Fintel 1999, we can then define the notion of a conditional entailment:

$$\phi_1, \ldots, \phi_n \textit{ conditionally entails } \psi \text{ iff for all } \Sigma\colon\ \Sigma[\phi_1]\ldots[\phi_n] \circ \psi \Vdash \psi$$

Entailment thus defined is conditional in the sense that the premises license the conclusion only if the presuppositions of the latter (if any) are accommodated. It is straightforward to verify that the direct argument is valid in this sense:

$$\phi \vee \psi \text{ conditionally entails } (\text{if } \neg\phi)(\psi).$$

The fact that the direct argument is in general compelling is thus compatible with the observation that certain contexts fail to license the inference of $\ulcorner(\text{if } \neg\phi)(\psi)\urcorner$ from $\ulcorner\phi \vee \psi\urcorner$. There is not much to choose between plain entailment and conditional entailment since in many cases — including those that were of interest in the previous discussion — they make the same predictions. One notable observation is that there is an intuitive connection between conditional entailment and inconsistency:

$$\phi_1, \ldots, \phi_n \text{ conditionally entails } \psi \text{ just in case } \phi_1, \ldots, \phi_n, \neg\psi \text{ is inconsistent.}$$

The main point to notice is that the basic apparatus allows for some potentially fruitful extensions and modifications. All of this should give us some confidence that the dynamic story told here is one that deserves to be taken seriously.

4.3 Counterfactuals

The dynamic story about indicative conditionals may be expanded so that it addresses counterfactuals. The key idea is simple: we already said that context provides a set of relevance orderings that matter for the interpretation of epistemic modals; we now add that it also provides a set of relevance orderings that matter for the interpretation of *might have* and *would have*. Counterfactuals play with these relevance orderings in the same way that indicative conditionals played with those that mattered for epistemic *might* and *must*. Let me briefly outline one way of implementing this idea.

It is a familiar slogan that counterfactuals may pertain to possibilities that are incompatible with the common ground (see Stalnaker 1975) and that this is so in virtue of their carrying a past tense morphology that receives a modal instead of its usual temporal interpretation (see Iatridou 2000, Isard 1974, Lyons 1977, and Starr 2014a).[18] The obvious move to make then is to embellish each set of possible

18 The second bit of the slogan is not uncontroversial: Arregui (2009) and Ippolito (2006), for instance, maintain that the relevant past tense morphology in counterfactuals receives a temporal interpre-

worlds with a counterfactual domain of quantification that may evolve dynamically as discourse proceeds and then model counterfactuals in terms of their update effects on such domains. Start with the already familiar idea that each possible world w comes with a metaphysical similarity relation determining a system of spheres $\pi_w = \{S_1(w), \ldots, S_n(w)\}$ that is strongly centered on w so that $S_1(w) = \{w\}$. On that basis, we may now define a set of functions $F = \{f_1, \ldots, f_n\}$ such that for all $s \subseteq W$ and $1 \leq m \leq n$, $f_m(s) = \{\bigcup S_m(w) :\ w \in s\}$. We will allow elements of F apply to relevance orderings as well and say that $f_m(\pi) = \{f_m(s) :\ s \in \pi\}$ for all $\pi \in \Pi$ and $1 \leq m \leq n$. Embellishing the elements of a context state (relevance orderings) with a *counterfactual* domain of quantification determined by some f then leads to a more complex conception of a context state.

We add a special operator "$\triangleleft$" that scopes over elements of a modal propositional language embellished with the conditional operator "(if $\cdot$)($\cdot$)" and the presupposition operator "∂." The basic idea is that a formula of the form $\ulcorner \triangleleft\phi \urcorner$ has the same semantic update effect that ϕ has, with the exception that it affects the counterfactual domain of quantification associated with a relevance ordering. Formulas of the form $\ulcorner \triangleleft\phi \urcorner$ thus update sets of possible worlds that may lie outside the common ground.

The proposal now is to analyze counterfactual conditionals as plain conditionals with an additional meaning component that corresponds to the presence of past tense morphology that receives a modal instead of its usual temporal interpretation. Specifically, the proposal is that such conditionals are of the form $\ulcorner \triangleleft(\text{if } \phi)(\psi) \urcorner$. The first consequence is that updating with a counterfactual involves updating with $\ulcorner \triangleleft\partial\Diamond\phi \urcorner$ and thus enforces ϕ as a live possibility in each counterfactual relevance ordering. Counterfactuals do not presuppose that their antecedent is compatible with what is common ground but have the potential to bring into view possibilities that matter for counterfactual discourse.

The second consequence is that updating with a counterfactual involves updating with $\ulcorner \triangleleft\Box(\phi \supset \psi) \urcorner$, which is just to test whether the counterfactual relevance ordering supports $\ulcorner \phi \supset \psi \urcorner$. It should be obvious that the proposal made in this section will account for the felicity of counterfactual Sobel sequences by appealing to the same mechanism that stood behind the explanation of the felicity of indicative Sobel sequences: just as indicatives, counterfactuals are strict material conditionals but also carry certain presuppositions that may trigger an expansion of the domain of quantification. Elaborating this idea in detail requires only minimal extensions of the

tation after all; see also also Edgington 1995 and references therein for corresponding views in the philosophical literature. See Schlenker 2004 for a different path toward a uniform analysis of indicatives and counterfactuals that takes the latter as the basic phenomenon and the former as the special case. The issues raised by these alternative views are too complex to be efficiently discussed here. I am simply interested in pursuing one concrete path toward a theory that treats indicatives and counterfactuals as strict over distinct though related relevance orderings.

framework that has been developed in the previous section, and the resulting analysis of counterfactuals is very much in the spirit of the proposal laid out by von Fintel (2001) and Gillies (2007) (though again see Willer 2013b for remarks on differences at crucial levels of detail).

5 Conclusion

Folklore has it that Sobel sequences favor a variably strict analysis of conditionals over its plainly strict alternative, but a closer look at their indicative cousins shows that this is simply not so. Given minimal semantic and pragmatic assumptions, indicative Sobel sequences turn out to be an embarrassment for classical implementations of the variably strict and of the plainly strict analysis alike. They thus raise a very general puzzle about the semantics and pragmatics of conditionals, and it is anything but trivial that the solution to the puzzle is bound to have a variably strict semantics at its core. In fact, if what I have said before is right, there is at least one attractive solution to the puzzle that treats conditionals as strict material conditionals over a dynamically evolving domain of quantification.

I have not argued that the positive proposal developed here is without alternatives, and I am not going to start now. The key hypothesis of this paper is that conditionals establish live necessities that may be defeated as hitherto ignored possibilities become live: Sobel sequences exploit the dynamic interaction between live necessities and live possibilities in that each member of the sequence brings the possibility of its antecedent into view and thus defeats a commitment to its immediate predecessor. To fix ideas I have opted for an implementation of this idea in a dynamic theory of meaning, which allows us to translate the defeasibility of discourse commitments into a nonmonotonic entailment relation. This solves the problem at the level of semantics: indicatives are bounded from below by the material conditional, and *still* two members of a consistent Sobel sequence fail to entail that the antecedent of the second conditional is false. Others may prefer to go for a strict and static semantic analysis of indicatives and let all the dynamic effects I have described play out at the level of pragmatics. On this view, Sobel sequences are semantically inconsistent, and $\ulcorner(\text{if } \phi)(\chi)\urcorner$ together with $\ulcorner(\text{if } \phi \wedge \psi)(\neg\chi)\urcorner$ semantically entails $\ulcorner\neg(\phi \wedge \psi)\urcorner$; but extra-semantic accommodation can salvage the felicity of Sobel sequences by bringing hitherto ignored $\ulcorner\phi \wedge \psi\urcorner$-worlds into view. I did not choose this path since it is unclear to me how a possibility — hitherto ignored or not — could be accommodated if it is ruled out by what is literally said. But it is a path worth exploring, and consistent with the key message of the story told here.

Everything I have said is also compatible with there being alternative responses to the problem that have a variably strict analysis at its core: the perhaps most obvious path is to reject the hypothesis that indicative conditionals are bounded from below by the material conditional. To fix ideas, consider a framework that is inspired by Kratzer's analysis of modals and conditionals (see Kratzer 1979, 1981, 1991a,b, 2012; again I am streamlining the discussion by making the Limit Assumption). Context fixes for each possible world a set of possible worlds (the *modal base*) and an ordering on that set. Modals are just quantifiers over those worlds that are minimal in the modal base given the relevant ordering, the difference between modal flavors — epistemic, deontic, alethic, and so on — amounting to different choices for the modal base and ordering. For epistemic modals, the modal base $f(w)$ is a set of possible worlds compatible with some contextually provided body of information, while the ordering $\leq_w$ is "stereotypical" in that it reflects the normal course of events in w. Finally, *if*-clauses function as restrictors of modals; in plain indicative conditionals, the modal is implicit and an epistemic necessity operator. The truth-conditions for such conditionals may then be stated as follows:

i. $\min(B, \leq_w) = \{w' \in B : \text{ there is no } w'' \in B \text{ such that } w'' <_w w'\}$

ii. $[\![(\text{if } \phi)(\psi)]\!]^{c,w} = 1 \text{ iff } \min(f(w) \cap [\![\phi]\!]^c, \leq_w) \subseteq [\![\psi]\!]^c$

This is just another spin on the variably strict semantics. But at some possible world w things may not go as they normally go, and so there is no reason to think that a c-provided stereotypical ordering relation is weakly centered by design. Whenever we have weak centering failures, the fact that $[\![(\text{if } \phi)(\psi)]\!]^{c,w} = 1$ and that w is a ϕ-world leaves the status of the consequent at w wide open: w does not have to be a ψ-world itself (since it may be outranked by some $\ulcorner \phi \wedge \psi \urcorner$-worlds from the modal base) and it does not have to be a $\ulcorner \Box \psi \urcorner$-world either (since the $\leq_w$-minimal worlds in $f(w)$ may all be $\ulcorner \neg \phi \wedge \neg \psi \urcorner$-worlds).

The implications of this proposal are complex, too complex to be efficiently discussed here. But they are also complex enough to emphasize once more that Sobel sequences — in their indicative incarnation anyway — do not so much provide a data point that favors a variably strict semantics but rather create complex issues that such a theory needs to address. For starters, rejecting weak centering for epistemic modals is to say that epistemic necessity claims fail to semantically entail their prejacent — a result that, as von Fintel & Gillies (2010) have shown, is anything but unproblematic. Moreover, every story about indicative conditionals worth its salt needs to explain why the inference rule of modus ponens seems so compelling, and the most straightforward explanation would be that it is, in fact, a semantic validity.[19] I leave it an open question how these and related concerns are best to be

19 The claim that modus ponens is an intuitive rule of inference is not just made from the armchair: across experimental studies of deductive reasoning, participants reliably endorse modus ponens. See Cariani & Rips Forthcoming and references therein for discussion. McGee 1985 is the *locus classicus*

addressed, but submit that the resulting story will not be obviously superior to the one told here in terms of explanatory power and theoretical simplicity.

In fact, assuming that the data discussed here would allow one to make a straightforward case for any distinct semantic theory — may it be strict or variably strict, dynamic or static — runs the risk of replacing one mistaken lesson from Sobel sequences with another. The correct and perhaps even more interesting moral to draw is that any comprehensive story about conditionals must eventually negotiate between a small set of intuitive semantic and pragmatic principles that, in a classical setting anyway, stand in tension with the felicity of Sobel sequence in their indicative incarnation. Telling such a story with a nonclassical dynamic spin and a rich conception of the kind of possibilities and necessities that context provides — which is what I have done here — is an attractive option that deserves to be taken seriously.

A Appendix: Apparatus

A.1 Basic framework

I begin by defining the basic framework discussed in Section 3.2.

Definition 1 (Language) $\mathscr{L}$ is the smallest set that contains a set of sentential atoms $\mathscr{A}$ and is closed under negation ($\neg$), conjunction ($\wedge$), the modals *might* ($\Diamond$) and *must* ($\Box$), and the natural language conditional $(\text{if}\cdot)(\cdot)$. $\mathscr{L}^{+} = \mathscr{L} \cup \{\partial\phi\colon\ \phi \in \mathscr{L}\}$. $\mathscr{L}_0$ is defined as the non-modal fragment of $\mathscr{L}$. Disjunction ($\vee$) and the material conditional ($\supset$) are defined in the usual way.

Definition 2 (Possible Worlds, Relevance Orderings) w is a *possible world* iff $w\colon\ \mathscr{A} \mapsto \{0,1\}$. W is the set of such w's, $\mathscr{P}(W)$ is the powerset of W. Given some total preorder $\preceq$ on some $U \subseteq W$, $s \subseteq W$ is a *sphere* iff $\exists w \in U$ such that $s = \{v\colon\ v \preceq w\}$, and $S(\preceq, U)$ is the set of these spheres. π is a *relevance ordering* iff $\pi = S(\preceq, U)$ for some choice of $\preceq$ and nonempty U, and Π is just the set of all such systems of spheres. s_π is the *center* (or *minimal element*) of π.

Definition 3 (Context States) Σ is a *context state* iff $\Sigma \subseteq \Pi$ and I is the set of such Σ's. The set of *centers* of Σ is defined as $\mathscr{C}(\Sigma) = \{s_\pi\colon\ \pi \in \Sigma\}$. The *initial* context state Σ_0 is identical with Π, the *absurd* context state $\Sigma_\emptyset$ is identical with $\emptyset$.

of opposition to the semantic validity of modus ponens; see Gillies 2004 for a response to McGee's case.

Definition 4 (Propositions) The function $[\![.]\!]$ assigns to each $\phi \in \mathscr{L}_0$ a proposition, understood as a subset of W, as follows:

i. $[\![p]\!] = \{w \in W\colon\ w(p) = 1\}$

ii. $[\![\neg\phi]\!] = W \setminus [\![\phi]\!]$

iii. $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cap [\![\psi]\!]$

Definition 5 (Possibilities, Necessities) Consider any $\Sigma \in I$ and $[\![\phi]\!] \subseteq W$:

i. $[\![\phi]\!]$ is a *possibility* in Σ iff $\exists s \in \bigcup\Sigma\ \exists w \in s\colon\ w \in [\![\phi]\!]$

ii. $[\![\phi]\!]$ is a *live* possibility in Σ iff $\forall s \in \mathscr{C}(\Sigma)\ \exists w \in s\colon\ w \in [\![\phi]\!]$

iii. $[\![\phi]\!]$ is a *necessity* in Σ iff $\forall s \in \bigcup\Sigma\ \forall w \in s\colon\ w \in [\![\phi]\!]$

iv. $[\![\phi]\!]$ is a *live necessity* in Σ iff $\forall s \in \mathscr{C}(\Sigma)\ \forall w \in s\colon\ w \in [\![\phi]\!]$

Definition 6 (Updates on Elements of Context States) Consider any $\Sigma \in I$ and $p \in \mathscr{A}$, $\phi, \psi \in \mathscr{L}^+$. The update operation $+\colon\ \Pi \mapsto (\mathscr{L}^+ \mapsto \mathscr{P}(W)^{\mathscr{P}(W)})$ is recursively defined as follows:

(1) $s \uparrow_\pi p = \{w \in s\colon\ w(p) = 1\}$

(2) $s \uparrow_\pi \neg\phi = s \setminus (s \uparrow_\pi \phi)$

(3) $s \uparrow_\pi (\phi \wedge \psi) = (s \uparrow_\pi \phi) \uparrow_{\pi'} \psi$, where $\pi' = \{s \uparrow_\pi \phi\colon\ s \in \pi\}$

(4) $s \uparrow_\pi \Diamond\phi = \{w \in s\colon\ \exists s' \in \pi . s' \subseteq s\ \&\ s' \uparrow_\pi \phi \neq \emptyset\}$

(5) $s \uparrow_\pi \Box\phi = \{w \in s\colon\ \exists s' \in \pi . s' \subseteq s\ \&\ s' \uparrow_\pi \phi = s'\}$

(6) $s \uparrow_\pi \partial\phi = s$ iff $s \uparrow_\pi \phi = s$

(7) $s \uparrow_\pi (\text{if } \phi)(\psi) = s \uparrow_\pi (\partial\Diamond\phi \wedge \Box(\phi \supset \psi))$

Definition 7 (Acceptance, Admission) Consider any $\pi \in \Pi$, $\Sigma \in I$, and $\phi \in \mathscr{L}^+$:

i. π *supports* ϕ, $\pi \Vdash \phi$, iff $s_\pi \uparrow_\pi \phi = s_\pi$

ii. Σ *accepts* ϕ, $\Sigma \Vdash \phi$, iff for all $\pi \in \Sigma\colon\ \pi \Vdash \phi$

iii. Σ *admits* ϕ, $\Sigma \rhd \phi$, iff $\Sigma \nVdash \neg\phi$

Definition 8 (Updates on Context States) Consider arbitrary $\Sigma \in I$ and $\phi \in \mathscr{L}^+$, and say that $\pi + \phi = \{s \uparrow_\pi \phi \colon s \in \pi\} \setminus \{\emptyset\}$. The update operation $[\phi] \colon I \mapsto I$ is defined as follows:

$$\Sigma[\phi] = \{\pi \colon \Sigma \rhd \phi \ \& \ \exists \pi' \in \Sigma. \ \pi' + \phi = \pi\}$$

Definition 9 (Entailment, Consistency) Consider arbitrary $\phi_1, \dots, \phi_n, \psi \in \mathscr{L}^+$:

i. $\phi_1, \dots, \phi_n$ *entails* ψ, $\phi_1, \dots, \phi_n \vDash \psi$, iff for all $\Sigma \in I \colon \Sigma[\phi_1] \dots [\phi_n] \Vdash \psi$

ii. $\phi_1, \dots, \phi_n$ is *consistent* iff for some $\Sigma \in I \colon \Sigma[\phi_1] \dots [\phi_n] \neq \Sigma_\emptyset$

A.2 Presupposition and conditional entailment

Definition 10 offers a more precise statement of what it takes for one formula to presuppose another (inspired by Beaver 2001) while Definition 11 articulates the notion of conditional entailment from Section 4.2 (inspired by von Fintel 1999).

Definition 10 (Presupposition) ϕ *presupposes* ψ, $\phi \gg \psi$, iff for all $s \subseteq W$ and $\pi \in \Pi$: if $s \uparrow_\pi \phi$ is defined, then $s \uparrow_\pi \psi = s$.

Definition 11 (Conditional Entailment) Consider arbitrary $\pi \in \Pi$, $\Sigma \in I$ and $\phi_1, \dots, \phi_n, \psi \in \mathscr{L}^+$:

i. $\pi_\phi = \{s \in \pi \colon s \uparrow_\pi \phi \text{ is defined}\}$

ii. $\Sigma \circ \phi = \{\pi \colon \exists \pi' \in \Sigma. \pi'_\phi = \pi\}$

We say that $\phi_1, \dots, \phi_n$ *conditionally entails* ψ iff for all $\Sigma \colon \Sigma[\phi_1] \dots [\phi_n] \circ \psi \Vdash \psi$

A.3 Counterfactuals

The following definitions expand the analysis of indicatives to counterfactuals.

Definition 12 (Expanded Language) $\mathscr{L}^+_\lhd$ is just like $\mathscr{L}^+$ except that it also contains a special operator $\lhd$ that scopes over elements of $\mathscr{L}$ so that $\mathscr{L}^+_\lhd = \mathscr{L}^+ \cup \{\lhd\phi \colon \phi \in \mathscr{L}^+\}$.

Definition 13 (Metaphysical Similarity) Associate with each $w \in W$ a metaphysical similarity relation determining a system of spheres $\pi_w = \{S_1(w), \ldots, S_n(w)\}$ that is strongly centered on w so that $S_1(w) = \{w\}$. Define on that basis a set of functions $F = \{f_1, \ldots, f_n\}$ such that for all $s \subseteq W$ and $1 \leq m \leq n$, $f_m(s) = \{\bigcup S_m(w)\colon\ w \in s\}$. We will allow elements of F apply to relevance orderings as well and say that $f_m(\pi) = \{f_m(s)\colon\ s \in \pi\}$ for all $\pi \in \Pi$ and $1 \leq m \leq n$.

Remark. Whenever $S_m(w) = W$ we may introduce another sphere $S_{m+1}(w)$ around w so that $S_m(w) = S_{m+1}(w)$. This guarantees that we may associate the same number of (not necessarily distinct) spheres with every $w \in W$ and so that F is well-defined.

Definition 14 (Complex Context States) $\Gamma \subseteq \Pi \times F$ is a *complex* context state and G is just the set of all complex context states. Given some $\rho \in \Gamma$, π_ρ and f_ρ represent the relevance ordering and counterfactual selection function of ρ, respectively, and s_ρ is the center of π_ρ. The *initial* complex context state Γ_0 is identical with $\Pi \times F$. The *absurd* complex context state $\Gamma_\emptyset$ is identical with $\emptyset$.

Definition 15 (Updates on Elements of Complex Context States) Consider any $\Gamma \in G$ and $\phi \in \mathscr{L}^+_\triangleleft$. The operation $\Uparrow\colon\ \Pi \times F \mapsto (\mathscr{L}^+_\triangleleft \mapsto (\mathscr{P}(W)^{\mathscr{P}(W)}))$ is defined as follows:

i. If $\phi \in \mathscr{L}^+$, then $s \Uparrow_\rho \phi = s \uparrow_{\pi_\rho} \phi$

ii. $s \Uparrow_\rho \triangleleft\phi = \{w \in s\colon\ w \in f_\rho(s) \uparrow_{f_\rho(\pi_\rho)} \phi\}$

Remark. If $\phi \in \mathscr{L}^+$ we require that the output state is the result of updating the input state with ϕ in light of π_ρ, which guarantees that everything we said about our basic target language remains in place. An update with $\ulcorner\triangleleft\phi\urcorner$ is shifty in the sense that $w \in s$ survives an update with $\ulcorner\triangleleft\phi\urcorner$ just in case it survives an update of the image of s under f_ρ in light of the image of π_ρ under f_ρ (that is, the counterfactual relevance ordering associated with π_ρ).

Definition 16 (Support, Acceptance, Admission (Revised)) Consider arbitrary $\rho \in \Pi \times F$, $\Gamma \in G$, and $\phi \in \mathscr{L}^+_\triangleleft$:

i. ρ *supports* ϕ, $\rho \Vdash \phi$, iff $s_\rho \Uparrow_\rho \phi = s_\rho$

ii. Γ *accepts* ϕ, $\Gamma \Vdash \phi$, iff for all $\rho \in \Gamma\colon\ \rho \Vdash \phi$

iii. Γ *admits* ϕ, $\Gamma \rhd \phi$, iff $\Gamma \nVdash \neg\phi$

Definition 17 (Updates on Context States) Consider arbitrary $\Gamma \in G$ and $\phi \in \mathscr{L}^{+}_{\lhd}$, and say that $\pi_\rho \Uparrow \phi = \{s \Uparrow_\rho \phi : \ s \in \pi_\rho\} \setminus \{\emptyset\}$. The update operation $+ : \ \mathscr{L}^{+}_{\lhd} \mapsto (G \mapsto G)$ is defined as follows:

$$\Gamma + \phi = \{\rho : \ \Gamma \rhd \phi \ \& \ \exists \rho' \in \Gamma. \ f_{\rho'} = f_\rho \ \& \ \pi_{\rho'} \Uparrow \phi = \pi_\rho\}$$

Remark. There is no need to rewrite the whole semantics but Definition 16 states the revised notions of support, acceptance, and admission while Definition 17 states the update rules for complex context states. The definitions of the notions of (conditional) entailment and consistency are minor rewrites of what has been said before.

B Appendix: Observations

B.1 Basic framework

This subsection articulates some key observations about the basic framework.

Fact 1 If $\Sigma \Vdash \phi$, then $\Sigma[\Diamond\neg\phi] = \Sigma_\emptyset$.

Proof. If $s \uparrow_\pi \phi = s$, then $s \uparrow_\pi \Diamond\neg\phi = \emptyset$ and hence $s \uparrow_\pi \neg\Diamond\neg\phi = s$. So if $\pi \Vdash \phi$, then $\pi \Vdash \neg\Diamond\neg\phi$. But if $\Sigma \Vdash \phi$, then $\pi \Vdash \phi$ for all $\pi \in \Sigma$. It follows that whenever Σ accepts ϕ, then Σ does not admit $\ulcorner\Diamond\neg\phi\urcorner$. Accordingly, whenever $\Sigma \Vdash \phi$, $\Sigma[\Diamond\neg\phi] = \Sigma_\emptyset$. □

Fact 2 $\Box\phi \vDash \phi$

Proof. Consider arbitrary $\Sigma \in I$: for all $\pi \in \Sigma[\Box\phi]$, $\pi \Vdash \phi$. But this is exactly what it takes for $\Sigma[\Box\phi]$ to accept ϕ and so $\Sigma[\Box\phi] \Vdash \phi$. □

Remark. The fact that Σ accepts ϕ is sufficient for it not to admit an update with $\ulcorner\Diamond\phi\urcorner$ (Fact 1). The epistemic necessity operator is strong in the sense that it entails its prejacent (Fact 2).

Fact 3 $\Box\phi \vDash \neg\Diamond\neg\phi$ and $\Diamond\phi \vDash \neg\Box\neg\phi$

Proof. We already saw that if $\pi \Vdash \phi$, then $\pi \Vdash \neg\Diamond\neg\phi$. Since an update with $\ulcorner\Box\phi\urcorner$ establishes ϕ as a live necessity, we have for arbitrary Σ that $\Sigma[\Box\phi] \Vdash \neg\Diamond\neg\phi$. Similar considerations establish that $\Sigma[\Diamond\phi] \Vdash \neg\Box\neg\phi$ for all Σ. □

Fact 4 $\Diamond\phi \vDash \Box\Diamond\phi$

Proof. Consider arbitrary $\Sigma \in I$ and let $\Sigma' = \Sigma[\Diamond\phi]$. Clearly, $\Sigma' \Vdash \Diamond\phi$ and so for all $\pi \in \Sigma'$, $s_\pi \uparrow_\pi \Diamond\phi = s_\pi$. But then of course for all $\pi \in \Sigma'$, $s_\pi \uparrow_\pi \Box\Diamond\phi = s_\pi$ as well. Hence $\Sigma' \Vdash \Box\Diamond\phi$, as required. □

Remark. The current framework validates the characteristic axiom of S5. Moss (2015) takes issue with this prediction in the epistemic modal domain but a proper discussion of how it can be avoided — and of whether it should be avoided — is better left to another day.

Fact 5 Consider arbitrary $\phi \in \mathscr{L}_0 : \bigcup\Sigma_{c+\phi} = \bigcup\Sigma_c \cap [\![\phi]\!]$

Proof. Simply observe that if $\phi \in \mathscr{L}_0$, then $s \uparrow_\pi \phi = s \cap [\![\phi]\!]$ for all $\pi \in \Pi$ and $s \subseteq W$. □

Fact 6 $(\text{if } \phi)(\psi) \gg \Diamond\phi$

Proof. By definition $s \uparrow_\pi (\text{if } \phi)(\psi) = s \uparrow_\pi (\partial\Diamond\phi \wedge \Box(\phi \supset \psi))$. But $s \uparrow_\pi \partial\Diamond\phi$ is defined only if $s \uparrow_\pi \Diamond\phi = s$, which together with the definition of presupposition establishes the point. □

Fact 7 $(\text{if } \phi)(\psi) \vDash \Diamond\phi \wedge (\phi \supset \psi)$

Proof. The fact follows immediately from the update rule for $(\text{if } \cdot)(\cdot)$ together with the definition of logical consequence. □

Remark. The previous observations establish that the framework developed preserves the principles about assertion, presupposition, and modus ponens that figured prominently in the previous discussion. Since $\bigcup\bigcup\Sigma_c = s_c$ it follows that whenever ϕ is an element of $\mathscr{L}_0$ and thus expresses a proposition, then an assertion of ϕ in some context c is designed to eliminate from the context set of c all possible worlds at which that proposition is false. Stalnaker's model of assertion thus holds as long as we focus on the propositional fragment of $\mathscr{L}^+$ (Fact 5). $\Sigma_c[(\text{if } \phi)(\psi)] \neq \Sigma_\emptyset$ only if $\Sigma_{c+\phi} \neq \Sigma_\emptyset$ and thus only if $s_{c+\phi} \neq \emptyset$ (Fact 6). Finally, an indicative conditional establishes its antecedent as a live possibility in virtue of its presupposed content and the corresponding material conditional as a live necessity in virtue of its asserted content (Fact 7). Conditionals are thus bounded from below by the material conditional, as required, and support modus ponens: for all Σ, $\Sigma[(\text{if } \phi)(\psi)][\phi] \Vdash \psi$.

Fact 8 Take any $\phi, \psi \in \mathscr{L}_0$: $\Box\phi, (\text{if } \neg\phi)(\psi)$ is consistent.

Proof. Observe that if Σ treats $[\![\phi]\!]$ as a live necessity, then $s_\pi \uparrow_\pi (\text{if } \neg\phi)(\psi)$ is undefined for all $\pi \in \Sigma$, and hence $\Sigma \nVdash \neg(\text{if } \neg\phi)(\psi)$. It follows that Σ admits an update with $\ulcorner(\text{if } \neg\phi)(\psi)\urcorner$ and — as long as $[\![\neg\phi]\!]$ is a possibility in Σ — may in fact support $\ulcorner\neg\phi \supset \psi\urcorner$ once the presupposition of $\ulcorner\Diamond\neg\phi\urcorner$ has been accommodated. □

Remark. Note that if $[\![\neg\phi]\!]$ is not even a possibility in Σ, an update with $\ulcorner(\text{if } \neg\phi)(\psi)\urcorner$ is guaranteed to result in the absurd state, and so Fact 8 is compatible with our earlier claim that sequences such as (5a) are marked. The question then is whether we want to allow for possibility presuppositions to be accommodated if the proposition in question is a faint possibility. This is a complex question that ultimately depends on what one wants to say about the acceptability of sequences like the following (thanks to an anonymous reviewer for drawing my attention to the second example):

(a) Mary must be in New York. If she is not in New York, she is in Chicago.

(b) If it rains, we won't go golfing. If we go golfing in the rain, Mary will watch.

These sequences are consistent given the current admission criterion, and — while far from perfect — better (to my ears anyway) than (5a). But we do not need to take a firm stand here since it is easy to strengthen the current admission criterion so that Fact 8 no longer holds and the sequences under consideration are inconsistent:

$$\Sigma \textit{ admits } \phi, \Sigma \rhd \phi, \text{ iff } \Sigma \nVdash \neg\phi \text{ and } s_\pi \uparrow_\pi \phi \text{ is defined for some } \pi \in \Sigma$$

The important point is that the question of which admission criterion we endorse is orthogonal to our story about Sobel sequences: these will be consistent regardless of how we choose since an update with $(\text{if } A)(F)$ and $(\text{if } A \wedge B)(\neg F)$ fails to establish $\neg(A \wedge B)$ as a live necessity, as we will see now. (I continue to assume the weaker and simpler admission criterion to facilitate the discussion.)

Fact 9 $(\text{if } A)(F), (\text{if } A \wedge B)(\neg F)$ is consistent.

Fact 10 $(\text{if } A)(F), (\text{if } A \wedge B)(\neg F) \nvDash \neg(A \wedge B)$

Fact 11 $(\text{if } A)(F) \vDash (\text{if } A)(F)$ but $(\text{if } A)(F), (\text{if } A \wedge B)(\neg F) \nvDash (\text{if } A)(F)$

Proof. All of these facts will be demonstrated by going through an example highlighting the key mechanisms at work in the semantic apparatus. Start by considering the following distribution of truth-values across possible worlds:

	A	B	C	F
w_1	t	f	f	t
w_2	t	t	f	f
w_3	t	t	t	f
w_4	t	t	t	t

If $X = \{w_1, w_2, w_3, w_4\}$, then $\mathscr{P}(X)$ is the powerset of X and $\Sigma \subseteq 2^{\mathscr{P}(X)}$ is a possible context state:

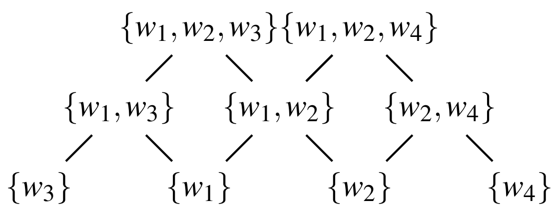

Figure 9: A possible context state Σ

Line segments going upward from one set to another represent the subset relation (the singleton sets of Σ are arranged in a way that enhances readability) and we shade sets of possible worlds that, while not eliminated, fail to be centers of Σ.

Notice that $\Sigma \Vdash \Diamond A$ but $\Sigma \nVdash \Box(A \supset F)$ since $\{w_2\}, \{w_3\} \in \mathscr{C}(\Sigma)$ and $\{w_2, w_3\} \subseteq [\![A \wedge \neg F]\!]$. Accordingly, updating Σ with (2a) — that the party will be fun if Alice comes — eliminates each relevance ordering whose center includes w_2 or w_3:

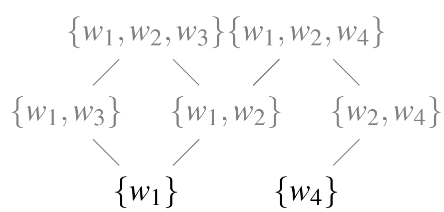

Figure 10: $\Sigma' = \Sigma[(\text{if } A)(F)]$

Keep in mind that a set of possible worlds s passes the test imposed by $\Box p$ in case there is a subset of s in π treating p as a necessity. Accordingly, the update of Σ with $\Box(A \supset F)$ — the asserted content of (2a) — will eliminate each relevance ordering in Σ not centered on $\{w_1\}$ or $\{w_4\}$. However, even if, for instance,

$\{w_1, w_3\} \notin \mathscr{C}(\Sigma')$, still $\{w_1, w_3\} \in \bigcup \Sigma'$ since $\{w_1, w_3\}$ is an element of the relevance ordering centered on $\{w_1\}$. Observe that $\Sigma' \nVdash \Diamond(A \wedge B)$ since $\mathscr{C}(\Sigma') = \{\{w_1\}, \{w_4\}\}$ and $w_1 \in [\![\neg(A \wedge B)]\!]$ but also $\Sigma \nVdash \neg\Diamond(A \wedge B)$ since $w_4 \in [\![A \wedge B]\!]$.

Updating Σ' with the second member of (2) — that the party will not be fun if Alice and Bert come — establishes Alice and Bert's coming as a live possibility in virtue of its presupposed context. Doing so requires eliminating $\{w_1\}$ from $\bigcup \Sigma'$:

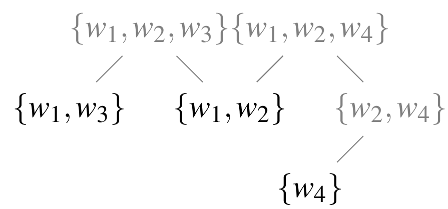

Figure 11: $\Sigma'[\partial\Diamond(A \wedge B)]$

Importantly, accommodating the presupposition that Alice and Bert might come defeats $A \supset F$ as a live necessity: while $\Sigma' \Vdash A \supset F$, clearly $\Sigma'[\partial\Diamond(A \wedge B)] \nVdash A \supset F$ since, for instance, $\{w_1, w_2\} \notin \mathscr{C}(\Sigma')$ but $\{w_1, w_2\} \in \mathscr{C}(\Sigma'[\partial\Diamond(A \wedge B)])$. It is thus possible to consistently update $\Sigma'[\partial\Diamond(A \wedge B)]$ with the asserted content of (2b):

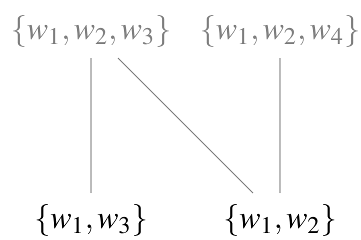

Figure 12: $\Sigma'' = \Sigma'[\partial\Diamond(A \wedge B)][\Box((A \wedge B) \supset \neg F)] = \Sigma'[(\text{if } A \wedge B)(\neg F)]$

Since Σ'' is a consistent context state, it follows that the first two members of the Sobel sequence in (2) are consistent, as stated by Fact 8. Furthermore, $\Sigma'' \nVdash \neg(A \wedge B)$ — in fact, no member of $\mathscr{C}(\Sigma'')$ treats $\neg(A \wedge B)$ as a necessity — which is sufficient to establish Fact 9. Finally, $\Sigma'' \nVdash (\text{if } A)(F)$ even though $\Sigma' \Vdash (\text{if } A)(F)$, which establishes Fact 10.

Before concluding this proof, it might be helpful for the reader to illustrate the result of processing the final member of the sequence in (2) — that the party will be fun if Alice and Bert and Charles come. Notice that $\Sigma'' \nVdash \Diamond(A \wedge B \wedge C)$ but also $\Sigma'' \nVdash \neg\Diamond(A \wedge B \wedge C)$. Presupposition accommodation eliminates $\{w_1, w_2\}$ from $\bigcup \Sigma''$:

$$\{w_1, w_2, w_3\} \quad \{w_1, w_2, w_4\}$$

$$\{w_1, w_3\}$$

Figure 13: $\Sigma''[\partial\Diamond(A \wedge B \wedge C)]$

Once again presupposition accommodation mechanism defeats a live necessity: $\Sigma''[\partial\Diamond(A \wedge B \wedge C)] \nVdash (\text{if } A \wedge B)(\neg F)$ — simply observe that $\{w_1, w_2, w_4\}$ has become a center of that context state — even though $\Sigma'' \Vdash (\text{if } A \wedge B)(\neg F)$, which had $\{w_1, w_3\}$ and $\{w_1, w_2\}$ as its centers. The result of updating with the asserted content of (2c) thus results in the non-absurd context state $\Sigma''' = \{\{w_1, w_2, w_4\}\}$. This demonstrates that the indicative Sobel sequence in (2) is consistent and thus — since presupposition failures result in the absurd context state — felicitous. More complex Sobel sequences can be accommodated by more complex context states. $\square$

Fact 12 For all $\phi, \psi, \chi \in \mathscr{L}_0$: $\Diamond(\phi \wedge \psi), \Box((\phi \wedge \psi) \supset \neg\chi), \neg(\phi \wedge \psi), \Box(\phi \supset \chi)$ is consistent.

Proof. Suppose that $w_1 \in [\![\phi \wedge \neg\psi \wedge \chi]\!]$ and that $w_2 \in [\![\phi \wedge \psi \wedge \neg\chi]\!]$ and consider

i. $\pi_1 = \{\{w_1\}, \{w_1, w_2\}\}$

ii. $\pi_2 = \{\{w_2\}, \{w_1, w_2\}\}$

iii. $\pi_3 = \{\{w_1, w_2\}\}$

Consider $\Sigma = \{\pi_1, \pi_2, \pi_3\}$ and say that $\Sigma' = \Sigma[\Diamond(\phi \wedge \psi)]$. Clearly $\Sigma' = \{\pi_2, \pi_3\}$ and so $\Sigma' \Vdash \Box((\phi \wedge \psi) \supset \neg\chi)$, hence $\Sigma'[\Box((\phi \wedge \psi) \supset \neg\chi)] = \Sigma'$. Observe that $\Sigma'[\Box(\phi \supset \chi)] = \Sigma_\emptyset$. However, $\Sigma'' = \Sigma'[\neg(\phi \wedge \psi)] = \{\{w_1\}\}$ and since $\Sigma'' \Vdash \Box(\phi \supset \chi)$, $\Sigma''[\Box(\phi \supset \chi)] = \Sigma''$, which is sufficient to establish the point. $\square$

Remark. Fact 12 together with the Dynamic Quality principle from Section 4.1 correctly predicts that the reverse Sobel sequence in (10) may be felicitous given suitable modification of the context and is pragmatically interpreted as a proposal to update the context with the information that Mary will not accept any marriage proposal from John.

Fact 13 $\phi \vee \psi$ conditionally entails $(\text{if } \neg\phi)(\psi)$.

Proof. Observe that for all $\Sigma \in I$, $\Sigma[\phi \vee \psi][\Diamond\neg\phi] \Vdash \Box(\neg\phi \supset \psi)$. Since $\Sigma[(\text{if } \neg\phi)(\psi)] = \Sigma[\partial\Diamond\neg\phi][\Box(\neg\phi \supset \psi)]$, for all $\Sigma \in I$, $\Sigma[\phi \vee \psi] \circ (\text{if } \neg\phi)(\psi) \Vdash (\text{if } \neg\phi)(\psi)$. □

Fact 14 $\phi_1, \ldots, \phi_n$ conditionally entails ψ iff $\phi_1, \ldots, \phi_n, \neg\psi$ is inconsistent.

Proof. The key observation here is that for all $\Sigma \in I$ and $\phi \in \mathscr{L}^+$, $\Sigma \circ \phi = \Sigma \circ \neg\phi$ and $\Sigma[\neg\phi] = \Sigma_\emptyset$ just in case $(\Sigma \circ \neg\phi)[\neg\phi] = \Sigma_\emptyset$. The fact that $\Sigma \circ \phi \Vdash \phi$ just in case $(\Sigma \circ \phi)[\neg\phi] = \Sigma_\emptyset$ is then sufficient to establish the fact. □

B.2 Expansion to counterfactuals

I conclude with some brief remarks on the framework as expanded to counterfactuals.

Fact 15 $\lhd(\text{if } \phi)(\psi) \vDash \lhd(\phi \supset \psi)$

Proof. Take any $\Gamma \in G$, let $\Gamma' = \Gamma + \lhd(\text{if } \phi)(\psi)$: then $\Gamma' \Vdash \lhd\Box(\phi \supset \psi)$ and so $\rho \Vdash \lhd\Box(\phi \supset \psi)$ for all $\rho \in \Gamma'$. Hence for all $\rho \in \Gamma'$, $\rho \Vdash \lhd(\phi \supset \psi)$. □

Remark. Since s_ρ is always a subset of the center of $f_\rho(\pi_\rho)$, $\lhd(\text{if } \phi)(\psi) \vDash \phi \supset \psi$ as long as ϕ and ψ are non-modal. Importantly, it may very well be that the context set is incompatible with ϕ and thus any $\rho \in \Gamma$ may trivially support $\ulcorner\phi \supset \psi\urcorner$, the reason being that the presupposition carried by a subjunctive conditionals does not operate on the set of sets of possible worlds representing what is common ground.

Fact 16 Take any $\phi, \psi, \chi \in \mathscr{L}_0$: $\lhd\Diamond(\phi \wedge \psi), \lhd\Box((\phi \wedge \psi) \supset \neg\chi), \lhd\neg(\phi \wedge \psi)$, $\lhd\Box(\phi \supset \chi)$ is consistent.

Proof. Choose relevance orderings π_1, π_2, and π_3 from the proof of Fact 12, observe that $f_1(\pi) = \pi$ for all $\pi \in \Pi$. Clearly, it follows from Fact 12 that the complex context state $\Gamma = \{\langle \pi_a, f_1\rangle, \langle \pi_b, f_1\rangle, \langle \pi_c, f_1\rangle\}$ can be consistently updated with the sequence under consideration. □

Remark. Due to Fact 16 everything said about reverse indicative Sobel sequences in Section 4.1 extends to reverse counterfactual Sobel sequences.

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