

Quantification over alternative intensions*

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Abstract In footnote 13 on p. 85f. of his dissertation, Mats Rooth (1985) addresses certain peculiarities of his treatment of *only* as a quantifier over propositions. The current note elaborates on that footnote to conclude that the lack of adequacy of this approach to quantification is more severe than previously thought. Section 1 presents a gap in the alternative[s] semantics treatment of *only*. In Section 2 an attempt is made to close it by way of meaning postulates to eliminate ‘degenerate’ models (Rooth’s term) in which extensions do not vary enough across Logical Space. In view of the lack of feasibility and systematicity of that approach, Section 3 explores a more principled, yet ultimately futile, strategy for determining ‘realistic’ models (Rooth’s term) that reflect the extensional variation offered by Model Space as a whole. Section 4 points out the limitations any such repair encounters when it comes to sentences with non-contingent at-issue contents. Section 5 briefly discusses a variant of the interpretation of *only* as a quantifier over propositional alternatives and how it fares with respect to the problems addressed in the previous sections.

Keywords: alternative semantics, possible worlds semantics, model-theoretic semantics, meaning postulates

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Introduction

One of the principal assets to alternative semantics, originating with [Rooth 1985](#), is its straightforward account of the truth-conditional effects of certain focus-sensitive elements like *only*, which does without any (possibly problematic) movement operations.¹ At the same time it has long been known that the somewhat underspecified analyses of alternative semantics do not always get the truth conditions *quite* right. The present note scrutinises the lacunae in the alternative treatment of *only*. More specifically, it addresses possible enhancements to alternative semantics by eliminating certain unintended models so as to predict more specific and adequate truth conditions for sentences with *only*. Three objections will be raised against such attempts:

- In Section 2, it will be argued that the standard technique of employing meaning postulates to make up for the lack of specificity in the truth conditions does not appear to be feasible.
- In Section 3, a more principled approach to adequate truth conditions in terms of a model-theoretic reflection technique, will be explored but ultimately dismissed since it turns out to be not viable either.
- In Section 4, a specific counter-example will be presented to show that the inadequacy of the truth conditions is sometimes beyond repair.

For expository reasons, these arguments will only be directed at the simple, uniform account of *only* as a propositional quantifier developed in [Rooth 1985](#): ch. II, where in particular, quantification over alternatives to individual referents is simulated by quantification over alternative propositions. Section 5 will turn to an example of a more flexible (and more popular) approach to alternative semantics and show that the latter leads to the same problems, albeit only in a sub-class of examples where *only* focus-associates with a proper part of its argument and quantifies over VP-denotations instead of propositions; as it turns out, the inadequacies of propositional quantification carry over to quantifiers over properties. The final section then briefly discusses how these results may affect other analyses and theories beyond alternative treatments of *only* as a quantifier. Before all this, Section 1 will illustrate the core problem by going through a few pertinent examples.

¹ See, e.g., [Rooth 1985](#): 33ff.; [Stechow 1991](#): 813f.; [Kratzer 1991](#): 827f. for arguments adduced against movement accounts of (associated) focus, and [Wagner 2006](#), [2012](#) and [Erlewine & Kotek 2016](#) for evidence for a hybrid account of movement and alternative semantics.

1 Quantification over propositions vs. quantification over individuals

According to one version of alternative semantics, focus-sensitive operators like *only* quantify over propositions rather than individuals.² As a case in point, a sentence like:

(1) Only Mary is asleep.

comes out as expressing:³

(2) $(\forall p)[[\forall p \wedge (\exists x) p = \wedge \mathbf{S}(x)] \rightarrow p = \wedge \mathbf{S}(\mathbf{m})]$

rather than the more straightforward predicate logic formalisation:

(3) $(\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}]$

Models abound in which (2) comes out true where it should not. Thus, e.g., if John and Mary, while being distinct, happen to be asleep in exactly the same worlds (of a model) and are also the only ones who are asleep in the actual world (of that model), then (2) is true in the actual world (of that model) because the proposition denoted by ' $\wedge \mathbf{S}(\mathbf{m})$ ' — that Mary is asleep — is the same proposition as the one expressed by ' $\wedge \mathbf{S}(\mathbf{j})$ ' — that John is asleep — and it is the only proposition of the form ' $\wedge \mathbf{S}(x)$ ' that is true in the actual world (of the model).⁴ Intuitively, however, (1) should not count as true in that world (of the model) because the extension of \mathbf{S} — the constant expressing the property

² Cf. [Rooth 1985](#): ch. III, where a cross-categorial approach to focus-sensitive particles is developed. Other approaches to alternative quantification — like the one in [Rooth 1985](#): ch. II, which gets the truth conditions in constellations like (1) right — are plagued by the same problems when it comes to slightly more involved constellations but will not be addressed before Section 5. Certain aspects of both approaches, such as context dependence, domain restrictions, compositionality, and multiple or wide foci, will be neglected in the following, since they are irrelevant for the comparison with ordinary quantification; only the conventional implicature triggered by *only* will make a short appearance in Section 4.

³ Apart from the self-explanatory (and eliminable) terms for n -tuples of individuals, the intensional type logic formulae used here are largely in line with Montague's notation ([Montague 1970](#)). In particular, ' \wedge ' abstracts over the (implicit) world parameter, and ' \forall ' indicates application to the world of evaluation; thus if φ is a truth-valuable formula, ' $\wedge \varphi$ ' stands for the proposition expressed by φ ; and if π stands for a proposition, the formula ' $\forall \pi$ ' says that π is true (of the world at stake).

⁴ It is understood that the individual constants corresponding to proper names (like \mathbf{m} and \mathbf{j} , which are supposed to translate *Mary* and *John*, respectively) are rigid designators in the sense of [Kripke 1972](#); this assumption is standardly being taken care of by meaning postulates like ' $(\exists x) \square \mathbf{m} = x$ '.

of being asleep — contains two distinct individuals. And obviously (3) does bring out this truth condition correctly.

More generally, as Rooth (1985) (p. 85f fn. 13) observed, given the standard modeling of propositions as regions in Logical Space, this analysis requires a certain abundance of possible worlds: (2) only comes out as equivalent to (3) if (4) holds, i.e., if the proposition denoted by ‘ $\wedge\mathbf{S}(\mathbf{m})$ ’ differs from the proposition denoted by ‘ $\wedge\mathbf{S}(x)$ ’ for any x other than Mary:

$$(4) \quad (\forall x)[x \neq \mathbf{m} \rightarrow \wedge\mathbf{S}(x) \neq \wedge\mathbf{S}(\mathbf{m})]$$

Generalising (4) from (1) by abstracting from Mary, the equivalence of (2) and (3) turns out to impose a certain degree of granularity on the propositions expressed by simple predications:

$$(5) \quad (\forall x)(\forall y)[x \neq y \rightarrow \wedge\mathbf{S}(x) \neq \wedge\mathbf{S}(y)]$$

The possible worlds setting as such does not guarantee the truth of (5). In particular, (5) does not hold in models in which the number n of individuals by far exceeds the number m of worlds in that $n > 2^m$: for some x and y the propositions denoted by ‘ $\wedge\mathbf{S}(x)$ ’ and ‘ $\wedge\mathbf{S}(y)$ ’ would have to coincide, since there are only 2^m propositions to begin with. Similarly, if $n > 2$ and the intension of \mathbf{S} happens to be rigid, the propositions denoted by ‘ $\wedge\mathbf{S}(x)$ ’ and ‘ $\wedge\mathbf{S}(y)$ ’ coincide for at least two x and y in the (constant) extension of \mathbf{S} . In order to guarantee (4), then, such ‘degenerate’ models (Rooth 1985) would have to be discarded by semantic theory. This could be achieved by way of meaning postulates (or similar constraints) to the effect that the lexical predicates must satisfy (5) in lieu of \mathbf{S} .⁵ However, the above reasoning extends way beyond the realm of lexical intransitives. Thus, e.g., in order for the interpretations (b) of the sentences (a) under (6)–(9) to come out right, the corresponding assumptions (c) turn out to be crucial:⁶

- (6) a. John only met MARY.
 b. $(\forall p)[[\vee p \wedge (\exists x) p = \wedge\mathbf{M}(\mathbf{j}, x)] \rightarrow p = \wedge\mathbf{M}(\mathbf{j}, \mathbf{m})]$
 c. $(\forall x_1)(\forall x_2)(\forall y_1)(\forall y_2)[(x_1, x_2) \neq (y_1, y_2) \rightarrow \wedge\mathbf{M}(x_1, x_2) \neq \wedge\mathbf{M}(y_1, y_2)]$

- (7) a. John only introduces MARY to Sue.

⁵ See Beaver & Clark 2008: 84 for an explicit suggestion along these lines.

⁶ To see this, as in the case of (1), variants of the (a)-sentences with different names must be considered.

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- b. $(\forall p)[[\forall p \wedge (\exists x) p = \wedge \mathbf{I}(\mathbf{j}, x, \mathbf{s})] \wedge \rightarrow p = \wedge \mathbf{I}(\mathbf{j}, \mathbf{m}, \mathbf{s})]$
c. $(\forall x_1)(\forall x_2)(\forall x_3)(\forall y_1)(\forall y_2)(\forall y_3)[(x_1, x_2, x_3) \neq (y_1, y_2, y_3) \rightarrow \wedge \mathbf{I}(x_1, x_2, x_3) \neq \wedge \mathbf{I}(y_1, y_2, y_3)]$
- (8) a. Only Mary is both drunk and asleep.
b. $(\forall p)[[\forall p \wedge (\exists x) p = \wedge [\mathbf{D}(x) \wedge \mathbf{S}(x)]] \rightarrow p = \wedge [\mathbf{D}(\mathbf{m}) \wedge \mathbf{S}(\mathbf{m})]]$
c. $(\forall x)(\forall y)[x \neq y \rightarrow \wedge [\mathbf{D}(x) \wedge \mathbf{S}(x)] \neq \wedge [\mathbf{D}(y) \wedge \mathbf{S}(y)]]$
- (9) a. John only knows that MARY knows that Harry introduces Bill to Sue.
b. $(\forall p)[[\forall p \wedge (\exists x) p = \wedge \mathbf{K}(\mathbf{j}, \wedge \mathbf{K}(x, \wedge \mathbf{I}(\mathbf{h}, \mathbf{b}, \mathbf{s})))] \rightarrow p = \wedge \mathbf{K}(\mathbf{j}, \wedge \mathbf{K}(\mathbf{m}, \wedge \mathbf{I}(\mathbf{h}, \mathbf{b}, \mathbf{s})))]$
c. $(\forall x_1) \dots (\forall x_5)(\forall y_1) \dots (\forall y_5)[(x_1, \dots, x_5) \neq (y_1, \dots, y_5) \rightarrow \wedge \mathbf{K}(x_1, \wedge \mathbf{K}(x_2, \wedge \mathbf{I}(x_3, x_4, x_5))) \neq \wedge \mathbf{K}(y_1, \wedge \mathbf{K}(y_2, \wedge \mathbf{I}(y_3, y_4, y_5)))]$

(6) and (7) illustrate that an adequate generalisation of (5) would have to cover lexical verbs of higher valencies, like the binary and ternary predicates **M**[*meet*] and **I**[*introduce*]. (8) and (9) indicate that the role of the predicate **S** in (5) can be played by non-lexical (n -ary) predicates like:

- (10) $\lambda x. [\mathbf{D}(x) \wedge \mathbf{S}(x)]$
(11) $\lambda x_5. \lambda x_4. \lambda x_3. \lambda x_2. \lambda x_1. \mathbf{K}(x_1, \wedge \mathbf{K}(x_2, \wedge \mathbf{I}(x_3, x_4, x_5)))$

the general pattern being:⁷

- (12) $(\forall \vec{x})(\forall \vec{y})[\vec{x} \neq \vec{y} \rightarrow [\wedge R\{\vec{x}\}] \neq [\wedge R\{\vec{y}\}]]$

where ' \vec{x} ' and ' \vec{y} ' range over n -tuples of individuals (for fixed but arbitrary n).

2 Extensional Variation by Meaning Postulates

Closer inspection of the counter-examples to (5) reveals that there is something wrong with them for independent reasons: such 'degenerate' models according to which Logical Space is severely limited, ought to be done away with anyway. What distinguishes such models from more 'realistic' (Rooth 1985 p. 85f fn. 13) contenders, is their austerity: rather than varying freely across Logical Space, they restrict the range of possible extensions of **S**[*leap*] in that

⁷ The curly brackets, which indicate application of the extension of a property, are again part of the intensional logic of Montague 1970, where they reflect a certain asymmetry between non-logical constants (like **S**) and variables (like **R**).

it does not cover every set (of individuals). This defect could be remedied in a natural way by demanding a maximal degree of extensional variation:

$$(13) \quad (\forall X)\diamond[S = X]$$

where ‘ X ’ ranges over arbitrary sets of individuals. It is readily seen that (13) implies (5) (though not *vice versa*).⁸ In the case of **M** and **I**, too, it may seem natural to derive the pertinent requirements (6c) and (7c) from some more general principle of variation, analogous to (13). Arguably, however, in these cases the variation is conceptually bounded. In particular, it would seem that the extension of **M** needs to be irreflexive; for it does not make (literal) sense for anyone to meet him- or herself. Similarly, one may want to exclude from the possible extensions of **I** any triples the third components of which coincide with either of the other two; for no one can introduce anyone (or anything) to him- or herself — on either reading of that clause.⁹ Further, possibly more clear-cut cases along these lines may be found in comparatives and degree verbs like *exceed*. Given these considerations, it appears dubious that (13) should be postulated for lexical predicates across the board. If anything, the range of possible extensions needs to be determined predicate by predicate. On top of (13) we may thus have:

$$(14) \quad (\forall R)[[\neg(\exists x)R(x, x)] \rightarrow \diamond[\mathbf{M} = R]]$$

$$(15) \quad (\forall S)[[\neg(\exists x)(\exists y)[S(x, y, x) \vee S(x, y, y)]] \rightarrow \diamond[\mathbf{I} = S]]$$

where ‘ R ’ and ‘ S ’ respectively range over binary and ternary relations, and so on, for any pertinent lexical predicate. However, this cannot be the whole story. For conjoined possibility does not imply compossibility, and so while both **D**[*runk*] and **S** may have an unlimited range, (5) and its **D**-variant do

⁸ If (13) holds, then for any individual x there is a world at which the extension of **S** is the singleton $\{x\}$, thus excluding any individual y distinct from x and satisfying (5). On the other hand if, e.g., the extension of **S** could be any singleton but nothing else, (5) would hold, but (13) would not. (13) strengthens, and is in the spirit of the notion of, *lexical freedom* as defined in Keenan 1987 (methinks), which implies that for any X , there is some lexical predicate **P** satisfying ‘ $\diamond[\mathbf{P} = X]$ ’.

⁹ To the extent that time travel into the past is a coherent concept, it might seem that persons could meet, be introduced to, or introduce others to, themselves. Arguably, however, such scenarios involve multiple copies or representations of one person — and thus more than one individual — rather than one person with more than one body. Similar things could be said about encountering and acquainting representations of persons in mirrors or on TV. I am indebted to Peter Smith for valuable discussion and sharing his native intuitions relating to the meaning and use of *to meet*.

not exclude models according to which they are contradictories, say, and thus the extension of (10) is necessarily empty. To ensure (8c), the variation postulates would thus have to cover the compound predicate in (10):

$$(16) \quad (\forall X) \diamond [\lambda x. [\mathbf{D}(x) \wedge \mathbf{S}(x)] = X]$$

Similarly, for (9c), one would need:

$$(17) \quad (\forall T) \diamond [[\lambda x_5. \lambda x_4. \lambda x_3. \lambda x_2. \lambda x_1. \mathbf{K}(x_1, \wedge \mathbf{K}(x_2, \wedge \mathbf{I}(x_3, x_4, x_5)))] = T]$$

with ‘ T ’ ranging over five-place relations among individuals.

Apart from idiosyncratic limitations due to lexical meanings as in (14)–(16), further restrictions on extensional variation ensue from interdependencies of lexical meanings. Thus, e.g., while unlimited variation seems to be desirable for the intersection of logically independent predicates like \mathbf{D} and \mathbf{S} , constraints would have to be imposed on the possible extensions when it comes to Boolean combinations of, say, \mathbf{S} and $\mathbf{A}[\textit{wake}]$, \mathbf{K} and $\mathbf{B}[\textit{believe}]$, etc. In fact, the range of extensional variation of complex predicates seems to be largely delimited by independent meaning postulates that the variation principles would have to somehow take into account — and it is not clear how this could be done in a systematic way.

3 Extensional variation by reflection principles

Formulating extensional variation postulates for ever more complex predicates appears neither feasible nor particularly insightful. Thus a more principled way to proceed ought to be found; and, indeed, the examples of the previous section suggest a direction the search may take. For the kind of extensional variation that would secure the equivalence of (2) and (3) in a natural way, cannot only be found across the worlds of Logical Space but also across different models: even if the extension of a predicate does not vary sufficiently *within* a given (‘degenerate’) model, its extensions in *other* models (‘degenerate’ or not) may still display a maximal range of variation. Since Model Space appears to have the richness we are after, we may want to look for models that reflect this richness within their own (‘local’) Logical Spaces. The current section is about this quest for models whose worlds represent all of model space. We will start by taking a closer look at the variation of Model Space on the extensions of the above examples and observe that this variation as such does not guarantee the richness needed for analysing quantification

over individuals in terms of quantification over properties: not all models are rich enough. We will then define what it means for a model's Logical Space to be as rich as Model Space; such models will be said to *reflect* Model Space. The section ends with an argument that such reflective models are unlikely to exist.

Let us first see how the richness of Model Space affects the predicate **S**. If D and W are arbitrary domains of individuals and possible worlds, the following holds:

- (18) For any $X \subseteq D$, there is a \mathcal{K}_0 -model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:

$$F_{\mathfrak{M}}(\mathbf{S})(w) = X$$

where \mathcal{K}_0 is the class of all models of intensional type logic. While (13) grants the predicate **S** a maximal amount of extensional variation across Logical Space, (18) records that the same kind of variation can be observed across Model Space. Yet the latter variation does not depend on (13): \mathcal{K}_0 also contains models that do not satisfy (13) at every world — which means that they do not satisfy (13) at any world.¹⁰

In a similar vein, the extensions of **M** and **I** vary wildly across Model Space — even in the absence of meaning postulates (14) and (15):

- (19) For any $R \subseteq D^2$ there is a \mathcal{K}_0 -model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:

$$F_{\mathfrak{M}}(\mathbf{M})(w) = R.$$

For any $S \subseteq D^3$ there is a \mathcal{K}_0 -model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:

$$F_{\mathfrak{M}}(\mathbf{I})(w) = S.$$

As a consequence, the variation noted in (19) abides if Model Space is restricted by irreflexivity postulates for **M** and **I**:

- (20) a. $\neg(\exists x)\mathbf{M}(x, x)$
 b. $\neg(\exists x)(\exists y)[\mathbf{I}(x, y, x) \vee \mathbf{I}(x, y, y)]$

In other words, if \mathcal{K}_1 is the class of models satisfying (20), then (21) holds:

¹⁰ This is so because (13) does not make implicit reference to a world of evaluation, i.e., the formula is *modally closed* in the sense of Gallin 1975: 14.

- (21) a. For any irreflexive $R \subseteq D^2$ there is a \mathcal{K}_1 -model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:

$$F_{\mathfrak{M}}(\mathbf{M})(w) = R.$$
b. For any irreflexive₃ $S \subseteq D^3$ there is a \mathcal{K}_1 -model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:

$$F_{\mathfrak{M}}(\mathbf{I})(w) = S.$$

where *irreflexivity*₃ is the property attributed to (the extension of) **I** in (20b). Moreover, it is readily shown (not here, though) that the unlimited variation does not stop at compound predicates like (10) and (11), whose extensions vary freely across Model Space, independently of any trans-world variation principles like (16) and (17).

It is important to realise that cross-model variation cannot make up for a lack of cross-world variation.¹¹ Yet it is the latter that leads to a lack of intensions in general and propositions in particular, which again may have dramatic consequences such as the non-equivalence of (2) and (3). Since propositions, and intensions in general, are constructed by abstracting from worlds rather than models, the varying extensions of the latter are inaccessible to them. However, one may still hope that some, ‘realistic’ models would capture enough cross-model variation within their Logical Spaces. More concretely, given a class \mathcal{K} of models, a model \mathfrak{M}^* may be said to *reflect* \mathcal{K} in that for any $\mathfrak{M} \in \mathcal{K}$ and any \mathfrak{M} -world w there is an \mathfrak{M}^* -world w^* in which the same (type-logical) sentences are true:

$$(22) \quad \{\varphi \mid \mathfrak{M} \models_w \varphi\} = \{\varphi \mid \mathfrak{M}^* \models_{w^*} \varphi\}^{12}$$

In particular, a model that reflects a class of models across which the extension of a (definable) predicate varies *ad libitum*, makes any combination of attributions of that predicate true of at least one of its worlds. Hence, e.g., any set Σ of sentences of the form ‘**S**(**n**)’, where ‘**n**’ is an individual constant, will have to be true at some world w_{Σ} . And if pertinent postulates guarantee the distinctness of sufficiently many constants, it would seem that the equivalence of (2) and (3) ought to emerge.

It is therefore natural to look for Rooth’s ‘realistic’ models among the ones that reflect the class \mathcal{K}^* of models satisfying all pertinent meaning postulates, including irreflexivity assumptions like (20) but excluding the

¹¹ See also Zimmermann 2011: 798f. for this point.

¹² ‘ \models_w ’ relates (type-logical) models \mathfrak{M} to the *sentences* (closed, truth-valuable formulae) that are true at world w (in \mathfrak{M} ’s Logical Space).

variation principles (13)–(17); after all, the intended effect of the latter shows in the variation across \mathcal{K}^* and should thus be reflected in any \mathfrak{M}^* satisfying (22). Alas, this search strategy is futile. To see this, one may consider a random contingency:

(23) Mary is asleep.

Clearly, \mathcal{K}_0 contains models \mathfrak{M}_0 and \mathfrak{M}_1 according to which (23) expresses a contradiction and a possibility, respectively:

(24) a. $\mathfrak{M}_0 \not\models_w \mathbf{S}(\mathbf{m})$, for any \mathfrak{M}_0 -world w ;
b. $\mathfrak{M}_1 \models_{w'} \mathbf{S}(\mathbf{m})$, for some \mathfrak{M}_1 -world w' .

With the help of the unrestricted possibility operator ‘ \diamond ’, (24) may be expressed as a statement about arbitrary \mathfrak{M}_0 - and \mathfrak{M}_1 -worlds w and w' :¹³

(25) a. $\mathfrak{M}_0 \models_w \neg \diamond \mathbf{S}(\mathbf{m})$
b. $\mathfrak{M}_1 \models_{w'} \diamond \mathbf{S}(\mathbf{m})$

In the (assumed) absence of any lexical postulates restricting the extensional range of the constants ‘ \mathbf{S} ’ and ‘ \mathbf{m} ’, \mathcal{K}^* , too, ought to contain models \mathfrak{M}_0 and \mathfrak{M}_1 satisfying (24) and (25). Hence a ‘realistic’ model \mathfrak{M}^* that reflects \mathcal{K}^* would have to contain worlds w_0 and w_1 that reflect arbitrary \mathfrak{M}_0 - and \mathfrak{M}_1 -worlds w and w' :

(26) a. $\mathfrak{M}^* \models_{w_0} \neg \diamond \mathbf{S}(\mathbf{m})$
b. $\mathfrak{M}^* \not\models_{w_1} \diamond \mathbf{S}(\mathbf{m})$

But the formulae in (26) are modally closed and so the truth of neither depends on the particular choice of worlds w_0 and w_1 ; hence (27) would have to hold for all w^* in \mathfrak{M}^* ’s Logical Space:

(27) $\mathfrak{M}^* \models_{w^*} [[\neg \diamond \mathbf{S}(\mathbf{m})] \wedge \diamond \mathbf{S}(\mathbf{m})]$

... which cannot be.¹⁴ So the strategy of narrowing down the class of all models to the more realistic ones in one fell swoop, leads to a contradiction.

¹³ Again, the two formulae under scrutiny are modally closed. Without loss of generality, one may actually assume that \mathfrak{M}_0 and \mathfrak{M}_1 share their Logical Space and hence that $w = w'$; this is so because Model Space is closed under arbitrary isomorphisms.

¹⁴ The situation is vaguely reminiscent of modal logic: in the canonical model of the logic of universal accessibility (S5), accessibility is an equivalence relation but not universal; cf. Hughes & Cresswell 1996: 118.

One may try and escape this conclusion by excluding modal formulae like the ones in (25) from the reflection requirement (22). Again, it is not obvious how this could be done in a sensible way. In particular, the above reasoning does not depend on the fact that the formulae under scrutiny are modally closed; a conjunction of either with some contingent sentence would have done just as well. Moreover, while it is crucial that the formulae involve intensionality, i.e., abstraction from particular worlds, restricting (22) to purely extensional φ would make the reflection property unattractively weak. Thus, e.g., in order for (9a) and its focus variants to come out right, a ‘realistic’ model ought to reflect the extensional range of the property in (11) and thus make arbitrary combinations of attribution of it true, as long as they obey the relevant postulates (like the veridicality of **K**). However, these attributions are not extensional in that they involve the propositional attitude **K**[knowledge].

4 Limits of propositional quantification

Since it does not seem to be possible to formulate general principles of extensional variation that would us allow to derive them, the relevant instances of (12) — here repeated as (28) — would have to be assumed as meaning postulates in their own right:

$$(28) \quad (\forall \vec{x})(\forall \vec{y})[\vec{x} \neq \vec{y} \rightarrow [{}^{\wedge}R\{\vec{x}\}] \neq [{}^{\wedge}R\{\vec{y}\}]] \quad [= (12)]$$

The problem is to define these instances. To begin with, (28) cannot be assumed for arbitrary (n -place) relations R ; for its universal closure (29) turns out to be inconsistent:

$$(29) \quad (\forall R)(\forall \vec{x})(\forall \vec{y})[\vec{x} \neq \vec{y} \rightarrow [{}^{\wedge}R\{\vec{x}\}] \neq [{}^{\wedge}R\{\vec{y}\}]]$$

Among others, certain trivial properties such as self-identity, or being identical with some fixed n -tuple, contradict (29). Hence (29) ought to be suitably relativised by some (2nd order) property \wp , if only for consistence:

$$(30) \quad (\forall R)[\wp(R) \rightarrow (\forall \vec{x})(\forall \vec{y})[\vec{x} \neq \vec{y} \rightarrow [{}^{\wedge}R\{\vec{x}\}] \neq [{}^{\wedge}R\{\vec{y}\}]]]$$

It is not clear how \wp should be specified so as to imply all relevant instances of (28), including (5) and (6c)-(9c). In particular, filtering out those R that contradict (28) will not do: (28) is satisfied by precisely the relations that never conflate the propositions that two distinct n -tuples stand in them.

Hence in (30), \wp cannot be the property \wp^* of being consistent with (28): given any relation R , either $\wp^*(R)$ holds or its negation does; and R is inconsistent with (28) precisely in the latter case. So the properties that are consistent with (28) are the ones that satisfy (28), which means that equating \wp with \wp^* would turn (30) into the tautology that any R that satisfies (28), satisfies (28). The problem, then, is to find a property \wp shared by relations like **S**, **L**, **I**, (10), and (11) so that (28) would make sure that they also have \wp^* .

One may thus ask what the relations mentioned have in common. For one thing, they are all definable in terms of the lexical predicates featuring in indirect interpretation.¹⁵ However, \wp must not be identified with definability in **S**, **L**, **I**, **K**, etc.; for not all definable relations R can be assumed to satisfy (28): as already indicated, at least some of them, like self-identity (which is definable in these constants, even without them), should be excluded for logical reasons.¹⁶

In order to define a suitable restriction \wp , one may try to characterise those relations that are definable from the logical representations of sentences like (1) and (6a)–(9a). In principle, this could be done in terms of an indirect interpretation algorithm. Yet again, this method of defining \wp would overshoot unless at least tautologies and contradictions are exempted from it. In fact, Mats Rooth (1985 p. 85 fn. 13) already observed that the strategy of mimicking individual by propositional quantification reaches its limits when it comes to rigid intensions:¹⁷

(31) Only three is an odd number.

(31) is clearly false, and so is a straightforward predicate logic formalisation of it:

(32) $(\forall x)[\mathbf{O}(x) \rightarrow x = \mathbf{3}]$

However, (31) comes out true on an analysis of *only* as a quantifier over propositions:

(33) $(\forall p)[[\forall p \wedge (\exists x)p = \wedge \mathbf{O}(x)] \rightarrow p = \wedge \mathbf{O}(\mathbf{3})]$

¹⁵ By *indirect interpretation* I mean a framework along the lines of Montague 1970 where, instead of directly giving their semantic values (as in, say, Heim & Kratzer 1998), an algorithm is specified that takes (the LFs of) natural language expressions to their type-logical translations.

¹⁶ Actually, self-identity does satisfy (28) in totally ‘degenerate’ models with just one individual.

¹⁷ For expository reasons, Rooth (1985) uses a slightly more involved example to prove the point: *Nine is only the square of THREE*.

The reason for this failure lies in a peculiarity of the predicate $O[*dd number*]$, whose extension is taken to not vary across Logical Space.¹⁸ Hence, in particular, any proposition of the form ‘ $\wedge O(x)$ ’ will coincide with either the empty set or all of Logical Space. So the latter is the only true proposition of that form, thus verifying (33). Unlike in the case of (1) and (2), though, there is no escape from the conclusion that (33) is an inadequate formalisation of the truth conditions of (31), which, again, seem aptly captured by (32). One may therefore doubt that the failure to adequately capture the truth-conditions of (31) should be adduced to the notorious lack of fine-grainedness of possible world semantics, as Rooth (p. 85f fn. 13) seems to suggest: after all, the adequate formalisation (32), too, can be expressed in that very framework. Moreover, the inadequacies due to rigid intensions may lead to contingencies:¹⁹

(34) Only Mary is one of John and Mary and exactly as tall as either one.

Clearly, under the (hardly objectionable) assumption that *John* and *Mary* are (rigid) names of different persons and the predicate *be one of John and Mary* rigidly denotes the pair of them, (34) ought to come out as a contradiction. And again it does if the sentence is analysed in terms of quantification over individuals:

(35) $(\forall x)[[x = \mathbf{j} \vee x = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow h(x) = h(y)]] \rightarrow x = \mathbf{m}$

To see what (35) comes down to, one should first notice that its matrix (= the scope of the outermost universal quantifier) is trivially satisfied by any individual distinct from John: Mary satisfies it in view of the triviality of the consequent (of the matrix); and any other individual satisfies it because it trivially fails to satisfy the left conjunct of the antecedent. As a consequence, the universal quantification in (35) boils down to:

¹⁸ As in the case of proper names (cf. fn. 4 above), this would have to be guaranteed by a specific meaning postulate, e.g., ‘ $(\exists X) \square O = X$ ’.

¹⁹ Slightly less artificial examples may be obtained if the domain of quantification is kept fixed: *Only this object is exactly as heavy as everything else*. — The rest of this section deviates from the Early Access version, which contained a faulty predicate logic analysis of (34) instead of (35). Apart from getting the brackets right, the correction required some re-arrangement of the material. I apologise for any consternation caused by my previous error and thank the editors for allowing me to fix it for the final version.

$$[[[\mathbf{j} = \mathbf{j} \vee \mathbf{j} = \mathbf{m}] \wedge (\forall \mathcal{Y})[[\mathcal{Y} = \mathbf{j} \vee \mathcal{Y} = \mathbf{m}] \rightarrow h(\mathbf{j}) = h(\mathcal{Y})]] \rightarrow \mathbf{j} = \mathbf{m}]$$

which contains a redundant conjunct and reduces to:

$$[(\forall \mathcal{Y})[[\mathcal{Y} = \mathbf{j} \vee \mathcal{Y} = \mathbf{m}] \rightarrow h(\mathbf{j}) = h(\mathcal{Y})] \rightarrow \mathbf{j} = \mathbf{m}]$$

which in turn is equivalent, by the distinctness assumption, to the negation of its antecedent and thus (by familiar quantifier laws) to:

$$(\exists \mathcal{Y})[[\mathcal{Y} = \mathbf{j} \vee \mathcal{Y} = \mathbf{m}] \wedge h(\mathbf{j}) \neq h(\mathcal{Y})]$$

Eliminating the existential quantifier yields:

$$[h(\mathbf{j}) \neq h(\mathbf{j}) \vee h(\mathbf{j}) \neq h(\mathbf{m})]$$

which boils down to:

$$(36) \quad h(\mathbf{j}) \neq h(\mathbf{m})$$

This is not yet the desired contradiction, which only arises once the conventional implicature triggered by *only* is taken into consideration. Before we get to it, though, let us take a look at the ordinary content that alternative semantics ascribes to (34):

$$(37) \quad (\forall p)[[\vee p \wedge (\exists x)p = \wedge [[x = \mathbf{j} \vee x = \mathbf{m}] \wedge (\forall \mathcal{Y})[[\mathcal{Y} = \mathbf{j} \vee \mathcal{Y} = \mathbf{m}] \rightarrow \mathbf{h}(x) = \mathbf{h}(\mathcal{Y})]]] \\ \rightarrow p = \wedge [[\mathbf{m} = \mathbf{j} \vee \mathbf{m} = \mathbf{m}] \wedge (\forall \mathcal{Y})[[\mathcal{Y} = \mathbf{j} \vee \mathcal{Y} = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{m}) = \mathbf{h}(\mathcal{Y})]]]$$

Closer inspection reveals that (37) is logically valid. To see this, one may observe that (37) can be equivalently rewritten as the tautology (38), where $p_{\mathbf{j} \approx \mathbf{m}}$ abbreviates ' $\wedge [\mathbf{h}(\mathbf{j}) = \mathbf{h}(\mathbf{m})]$ ':

$$(38) \quad (\forall p)[[\vee p \wedge p = p_{\mathbf{j} \approx \mathbf{m}}] \rightarrow p = p_{\mathbf{j} \approx \mathbf{m}}]$$

The details of the reformulation are kindly left to the reader. As in the case of (31), then, the alternative treatment misanalyses a contradiction, (34), as valid. However, there is a difference that comes to the fore once *full [conventional] content* is taken into consideration, i.e., the combination of ordinary (truth-conditional, assertoric, at-issue) content and conventional implicature. In the above formalisations the latter has been suppressed in the interest of

readability.²⁰ In the case of sentences containing *only*, it can be identified with the (ordinary) content of the sentence without *only*. As a case in point, the predicate logic and alternative formalisations of the full conventional content of (1) come out as (40) and (39), respectively, where the second line gives a logically equivalent (' \equiv ') reformulation of the first:

$$\begin{aligned}
 (39) \quad & [(\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}] \wedge \mathbf{S}(\mathbf{m})] \\
 & \equiv (\forall x)[\mathbf{S}(x) \leftrightarrow x = \mathbf{m}] \\
 (40) \quad & [(\forall p)[[\forall p \wedge (\exists x)p = \wedge \mathbf{S}(x)] \rightarrow p = \wedge \mathbf{S}(\mathbf{m})] \wedge \mathbf{S}(\mathbf{m})] \\
 & \equiv [(\forall p)[[\forall p \wedge (\exists x)p = \wedge \mathbf{S}(x)] \leftrightarrow p = \wedge \mathbf{S}(\mathbf{m})]]
 \end{aligned}$$

By the same token, the full content of (31) ought to be (41), as predicted by standard predicate logic formalisation, but comes out as (42) in alternative semantics:

$$\begin{aligned}
 (41) \quad & (\forall x)[\mathbf{O}(x) \leftrightarrow x = \mathbf{3}] \\
 (42) \quad & (\forall p)[[\forall p \wedge (\exists x) p = \wedge \mathbf{O}(x)] \leftrightarrow p = \wedge \mathbf{O}(\mathbf{3})]
 \end{aligned}$$

Since the ordinary content (32) of (31) is already contradictory, so is the stronger full content (41). On the other hand, (42) only adds the uninformative conjunct that 3 is odd to the trivial ordinary content (33) of (31), and is thus also trivially true. So on both analyses of (31), ordinary content and full content coincide. This may be taken to confirm the suspicion that (31) is an exceptional or even neurotic case, somewhat beyond the proper area of application of possible worlds semantics.

However, things stand differently with (34). Given that, according to the alternative analysis, its ordinary content comes out as trivial, its full content coincides with its conventional implicature, i.e., the proposition that Mary is [one of John and Mary and] as tall as [either Mary or] John:

$$\begin{aligned}
 (43) \quad & [(\forall p)[[\forall p \wedge p = p_{j \approx m}] \rightarrow p = p_{j \approx m}] \wedge \\
 & \quad [[\mathbf{m} = \mathbf{j} \vee \mathbf{m} = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{m}) = \mathbf{h}(y)]]] \\
 & \equiv [(\forall p)[[\forall p \wedge p = p_{j \approx m}] \rightarrow p = p_{j \approx m}] \wedge \mathbf{h}(\mathbf{m}) = \mathbf{h}(\mathbf{j})] \\
 & \equiv \mathbf{h}(\mathbf{m}) = \mathbf{h}(\mathbf{j})
 \end{aligned}$$

²⁰ The analysis in Rooth 1985: 120ff. takes both assertions and implicatures into account. While I follow Rooth 1985: 40& *passim* in classifying the non-at-issue part as a conventional implicature (rather than, say, a presupposition), I am actually agnostic about this question; I do not think anything in my argumentation hinges on this.

On the predicate logic analysis, on the other hand, the conventional implicature of (34) is (44), which does, obviously, contradict its ordinary content (36), as expected and desired:

$$(44) \quad [[\mathbf{m} = \mathbf{j} \vee \mathbf{m} = \mathbf{m}] \wedge (\forall \mathcal{Y})[[\mathcal{Y} = \mathbf{j} \vee \mathcal{Y} = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{m}) = \mathbf{h}(\mathbf{j})]] \\ \equiv \mathbf{h}(\mathbf{m}) = \mathbf{h}(\mathbf{j})$$

(34) differs from run-of-the-mill examples for lacking fine-grainedness: though some apparent contingencies are known to come out as contradictions in possible worlds semantics²¹, contradictions that express contingent propositions are unheard of (to my knowledge, anyway). The fact that certain contradictory sentences are misanalysed as expressing contingent propositions should not be blamed on the notorious lack of fine-grainedness of the framework alone: after all, the predicate logic account of *only* does get (34) right and is within the reach of possible worlds semantics. Hence the example may and should be taken as indication against the analysis of *only* as a propositional quantifier in possible worlds semantics.

5 Quantification over alternative properties

The upshot of the above reasoning is that quantification over individuals cannot be fully captured by quantification over propositions. But then the latter seems to be a peculiarity of the cross-categorial treatment of Rooth 1985: ch. III anyway: later analyses in the tradition of alternative semantics follow Rooth 1985: ch. II and Rooth 1992 in treating *only* as quantifying over alternative intensions of the constituent it modifies. It may thus seem that the above considerations do not bear on these approaches where (alternative) propositions give way to (alternative) intensions in general. Indeed, (1) then does come out as directly quantifying over alternatives to proper name denotations — and thus as a quantifier over individuals, just as the predicate logic formalisation (3) would have it.²² However, if *only* modifies a larger constituent that (properly) contains the focussed name it associates with, the alternative intensions of this constituent — which need not be propositions (and typically are not) — are plagued by the very same potential lack of vari-

²¹ Cf. Kaplan 1995 and Kripke 2011 for interesting cases.

²² These denotations may be referents or intensions, depending on the details of the framework. In view of the rigidity of names (cf. fn. 4 above), this difference has no bearing on the truth-conditions.

ation deplored in connection with propositions above, and thus susceptible to the same kind of reasoning.

To begin with, the argument of the previous section may be adapted to certain cases in which *only* modifies a proposition-denoting clause:

- (45) Bill knows only that MARY is one of John and Mary and exactly as tall as either one.

It appears that (45) is contradictory in that its full content implies (46), which can only be true if Bill believes that both Mary and John satisfy (47) (in lieu of x), thus contradicting (45):

- (46) Bill knows that MARY is one of John and Mary and exactly as tall as either one.
 (47) Bill knows only that x is one of John and Mary and exactly as tall as either one.

These examples are admittedly marginal and hardly the stuff that kills a theory. However, when quantification over individuals is mimicked by quantification over intensions other than propositions, the worries addressed in Sections 1-3 return in a different guise. As a case in point, instead of analysing (48a) in terms of alternatives to its conventional implicature, as in (6b) above (and repeated below), alternatives to the predicate intensions, as in (48c), are supposed to achieve the intended effect:²³

- (48) a. John only met MARY. [= (6a)]
 b. $(\forall p)[[\forall y p \wedge (\exists y) p = \wedge \mathbf{M}(\mathbf{j}, \mathbf{y})] \rightarrow p = \wedge \mathbf{M}(\mathbf{j}, \mathbf{m})]$ [\approx (6b)]
 c. $(\forall P)[[P\{\mathbf{j}\} \wedge (\exists y) P = \hat{x} \mathbf{M}(x, \mathbf{y})] \rightarrow P = \hat{x} \mathbf{M}(x, \mathbf{m})]$

where ' P ' ranges over properties, i.e., possible intensions of unary predicates. One may wonder whether (48c) fares better than (48b) with respect to the problems discussed above. To begin with, it should be noted that the two formulae are not equivalent. For although (48c) implies (48b), this implication does not reverse.²⁴

²³ ' $\hat{x} \varphi$ ' abbreviates ' $\wedge \lambda x. \varphi$ ', thus denoting the intension of a λ -abstracted predicate.

²⁴ To show this, one may consider a (totally 'degenerate') model with one world w and four individuals h, j, m , and s that assigns to \mathbf{M} the characteristic function of the set $\{(h, j), (h, s), (j, h), (j, m), (m, h)\}$ as its extension (at w). In such a model, (48b) comes out true (at w), but (48c) does not. A formal proof can be found on p. 13f. of the archived first draft of this paper: <http://semanticsarchive.net/Archive/mRhOGNjN/propquant.pdf>.

Despite an increase in granularity, the analysis (48c) still cannot escape the worries and objections previously raised against quantification over propositions. In fact, a closer look at counter-models to “(48b) \Rightarrow (48c)” mentioned in the preceding paragraph (and fn. 24) reveals that properties are almost as indiscriminate as propositions: though (48a) does come out false (as it should), the analogous (49a) comes out true (though it shouldn’t) if analysed as (49b):

- (49) a. Harry only met SUE.
 b. $(\forall P)[[P\{\mathbf{h}\} \wedge (\exists y) P = \hat{x} \mathbf{M}(x, y)] \rightarrow P = \hat{x} \mathbf{M}(x, \mathbf{s})]$

The reason for this again lies in a lack of fine-grainedness: ‘ $\hat{x} \mathbf{M}(x, \mathbf{j})$ ’ and ‘ $\hat{x} \mathbf{M}(x, \mathbf{s})$ ’ denote the same property, whose only possible extension is $\{h\}$, and this is the only property of the form ‘ $\hat{x} \mathbf{M}(x, y)$ ’ that h has (at w), just as (49b) requires. So (49b) is true even though, according to the only world of the model described, the referent of *Harry* stands in the relation denoted by \mathbf{M} to more than one individual. Needless to say, even for the above ‘degenerate’ model, a predicate logic formalisation taking *only* as quantifying over individuals would get the truth values of both (48a) and (49a) right.

The example illustrates that the more general approach involving predicate intensions inherits the deficiencies of propositional quantification. This may come as a surprise in view of the adequacy of the predicate logic formalisation, according to which *only* is a quantifier over individuals and thus ought to be less discriminative than a quantifier over alternative properties. However, the finesse of the latter is lost on the individuals to be quantified over as the sample analysis (48c) indicates and, more generally, a survey of the three quantifiers in question makes clear:

- (50) a. $\lambda y. \lambda Q. (\forall z) [Q\{z\} \rightarrow z = y]$
 b. $\lambda p. \lambda \mathcal{A}. (\forall q) [[\vee q \wedge \mathcal{A}(q)] \rightarrow q = p]$
 c. $\lambda x. \lambda P. \lambda \mathcal{S}. (\forall S) [[S\{x\} \wedge \mathcal{S}(S)] \rightarrow S = P]$

The quantifier in (50a) applies to (the referent of) the focussed element y_0 associated with *only* and (the intension of) the predicate Q_0 expressed by the rest of the sentence (minus *only*); the latter may be obtained by a standard λ -abstraction mechanism.²⁵ Taking (48a) as an example, y_0 would be the bearer of the name *Mary*, and Q_0 the property denoted by ‘ $\hat{z} \mathbf{M}(\mathbf{j}, z)$ ’. Thus (50a) is

²⁵ Cf. Heim & Kratzer 1998: 96.

a special case of a folklore analysis of *only* as a binary (non-conservative) *quantifier over individuals*.²⁶

The quantifier in (50b), on the other hand, applies to the proposition $p_0 = \{w | y_0 \in Q_0(w)\}$ expressed by the whole sentence (minus *only*) and the set \mathcal{A} of its alternatives (including p_0 itself):

$$\mathcal{A} = \{q | (\exists z) q = \{w | z \in Q_0(w)\}\}$$

In the above example (48a), p_0 would thus be the proposition denoted by ‘ $\wedge \mathbf{M}(\mathbf{j}, \mathbf{m})$ ’, and \mathcal{A} would be the set of propositions of the form ‘ $\wedge \mathbf{M}(\mathbf{j}, z)$ ’. Hence (50b) is Rooth’s *propositional quantifier* analysis Rooth 1985: 120, with some minor (and mostly notational) changes.²⁷

The quantifier in (50c) applies to (the referent of) the subject x_0 , (the intension of) the (clause) predicate P_0 , and the set \mathcal{S} of its alternatives:

$$\mathcal{S} = \{S | (\exists z)(\forall w)S(w) = R_0(w)(z)\}$$

where R_0 is constructed from the predicate (minus the focussed element) by a λ -abstraction device, in analogy to the construction of Q_0 from the whole sentence; in particular, for any world $w : P_0(w) = R_0(w)(y_0)$. In the case of (48a), R_0 would coincide with the relation expressed by \mathbf{M} . And the three arguments would be: the referent of the name *John* ($= x_0$), the property denoted by ‘ $\hat{x} \mathbf{M}(x, \mathbf{m})$ ’ ($= P_0$), and the set of properties of the form ‘ $\hat{x} \mathbf{M}(x, z)$ ’ ($= \mathcal{S}$). (50c) is adapted from the ‘domain selection’ analysis of Rooth 1985: 44, according to which *only* may quantify over alternative VP-denotations.²⁸

If analysed as the quantifier in (50a), *only* imposes the assumedly correct truth condition on the resulting sentence, viz., that the singleton $\{y_0\}$ cover the extension of Q_0 . On the other hand, a careful comparison of the above

26 Cf. Geach 1980 [1962]: 207ff. for an early source.

27 In particular, Rooth’s Russellian format has been Frege-Churched (in the sense of Kaplan 1975); the variable ranging over the set of alternatives has been λ -bound (and renamed from ‘ C ’ to ‘ \mathcal{A} ’); and the conventional implicature has been omitted, in line with the analyses in Section 1.

28 This ‘direct’ account of *only* as a quantifier over properties, which can be found in most alternative semantics approaches from Rooth 1992 onward, must not be confused with the type-shifted version derived from (50b) in the cross-categorial account of Rooth 1985: ch. III, which reads (on p. 121, again glossing over matters of notation and type regimentation):

$$(*) \lambda x. \lambda P. \lambda \mathcal{A}. (\forall p)[[\forall p \wedge \mathcal{A}(p)] \rightarrow p = \wedge P\{x\}]$$

Obviously, (*) boils down to the propositional quantifier analysis scrutinised in Sections 1–4 above.

quantifiers brings out the inadequacy of both (50b) and (50c).²⁹ On the one hand, the observations in Section 1 already indicate why the adequate truth conditions are not guaranteed to be expressible in terms of the two arguments of the quantifier in (50b). And despite its more complex type and finer granularity, the quantifier in (50c) does not attain the adequacy of the truth conditions imposed by (50a) either. Loosely speaking, the reason is that its extra argument is wasted on the subject instead of relating directly to the focussed element: the injective (one-one) function required for the equivalence between (50c) and (50a) does not concern the subject argument. Of course, this does not make the quantifier in (50c) as such inadequate, but only the way it is matched with its linguistic environment, following the alternative semantics approach. In fact, if applied to the focussed element y_0 , the property Q_0 of abstracting from it, and the set of all properties that nothing but y_0 has, the quantifier (50c) would impose the truth conditions of the full content; but then this procedure would not be in line with the alternative semantics architecture.

6 Conclusion

Now that we have seen that not all is well with the alternative treatment of *only* as a quantifier over propositions,³⁰ we may wonder whether similar deficiencies could not be found in other applications of alternative semantics, or in other theoretical frameworks that make use of similar assumptions and analyses. Given the wide-ranging literature on these topics, I will have to confine myself to some general remarks here.

The cause for all the trouble observed above is that the truth conditions of certain sentences involve quantification over individuals, which is mimicked by quantification over propositions. In terms of (two-sorted) types this means that operators on the domain (*et*) are ‘coded’ by corresponding operators on the domain (*(st)t*). The mismatch between these domains has been observed before and elsewhere, notably in the semantics of interrogatives, where it has also been held responsible for a number of inadequacies of

²⁹ Rigorous proofs can be found on p. 15ff. of the archived first draft of this paper: <http://semanticsarchive.net/Archive/mRhOGNjN/propquant.pdf>.

³⁰ I am returning to the uniform cross-categorial treatment of Rooth 1985: ch. II because I trust it that the observations in Section 5 have made it clear that the core problems are preserved in the more sophisticated versions employing quantification over alternative intensions.

proposition-based approaches.³¹ More specifically (and ignoring irrelevant intensional parameters), the alternative analysis essentially replaces the (apparently adequate) quantifier O_{only} of type $((et)(et))$, defined in (51), by an operator O^- of type $((st)t)((st)t)$, satisfying the equivalence in (52) (for all individuals x and properties P):

$$(51) \quad \lambda E.\lambda x. (\forall y)[E(y) \rightarrow y = x] \quad [\approx (50a)]$$

$$(52) \quad O^-(\wedge[P\{x\}], \lambda p. (\exists y) p = \wedge[P\{y\}]) \equiv O_{only}(x, \vee P)$$

The discussion in Sections 1-4 brought out that the equivalence (52) does not hold for all instances and that its exact scope is not easy to define. Since similar assumptions have been made concerning the treatment of other focus-sensitive elements within alternative semantics, these arguments are likely to carry over to them. As a case in point, according to a standard analysis of *even*, it contributes a (non-at-issue) proposition by operating on the alternative intensions of its scope. Again, the situation is essentially as in (51) and (52), with (53) being the operator O_{even} (of type $((s(et))(et))$) that expresses the intended reading, and (54) the equivalence condition that alternative semantics imposes on its surrogate (of type $((st)t)((st)t)$):³²

$$(53) \quad \lambda P.\lambda x. (\forall y)\mu(P\{x\}) \leq \mu(P\{y\})$$

$$(54) \quad O^-(\wedge[P\{x\}], \lambda p. (\exists y) p = \wedge[P\{y\}]) \equiv O_{even}(x, P)$$

It should not come as a surprise that (54) leads to the very same problems as the alternative reduction (52) of *only*; finding the relevant examples and adapting the above arguments is left to the reader. A more principled investigation into the domain switching strategy underlying alternative semantics will have to be left to future research on type-shifting.

So is the alternative semantics analysis of *only* doomed to assign inadequate truth conditions? Maybe not. Part of the trouble is the identification of propositions with sets of possible worlds. As Mats Rooth (*op. cit.*) already

³¹ See, e.g., Groenendijk & Stokhof 1984, p. 28off., Zimmermann 1985, p. 436f., Krifka 2001, or Aloni et al. 2007 for pertinent remarks and observations concerning leading frameworks of interrogative semantics. A reviewer also pointed out an interesting connection between question semantics and the discussion in Sections 1-3: ‘The danger would be that in degenerate models, *Mary* and *Bill* would incorrectly count as the same answer to the question *Who left?*’

³² μ is some contextually fixed probability measure; see, e.g., Wilkinson 1996: 194 for an analysis along these lines. The type difference between (51) and (52) is due to the intensionality of *even*, which *only* lacks.

pointed out, going ‘‘more intensional’’ — along the lines of property theory³³ — might be an option. However, rather than replacing the indeterminacy of intended models by an indeterminacy of fine-grained properties, one may stick to the possible worlds framework but incorporate (parts of) a competing semantic approach to focus: the theory of structured propositions and meanings,³⁴ which offers precisely the degree of granularity needed to capture the truth conditions of ordinary quantification over individuals. As a case in point, (1) could be accounted for adequately after replacing propositions in (2) by pairs of properties and individuals, as in (55):

- (1) Only Mary is asleep
 (2) $(\forall p)[[\forall p \wedge (\exists x) p = \wedge \mathbf{S}(x)] \rightarrow p = \wedge \mathbf{S}(\mathbf{m})]$
 (55) $(\forall \pi)[[\downarrow \pi \wedge (\exists x) \pi = (\mathbf{S}, x)] \rightarrow \pi = (\mathbf{S}, \mathbf{m})]$

where π ranges over structured meanings (of appropriate types) and \downarrow evaluates them in a way that ‘ $\downarrow (P, x)$ ’ comes down to ‘ $P\{x\}$ ’. It is not hard to see that (55) is equivalent to the intended analysis (3):³⁵

- (3) $(\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}]$

In any case it would seem that life in the alternative paradise that Rooth has created does not come for free.

³³ Bealer 1982. Rooth quotes Chierchia 1984 in this connection.

³⁴ I am indebted to Irene Heim (p.c., June 2014) for bringing up this suggestion. See Stechow 1991 and Krifka 2001 for expositions and comparison of the frameworks. In fact, the structured alternatives employed by Onea 2013: ch. 8 come close to this kind of hybrid architecture.

³⁵ This may be established by the following chain of *IL*-equivalences:

$$\begin{aligned}
 & (\forall \pi)[[\downarrow \pi \wedge (\exists x) \pi = (\mathbf{S}, x)] \rightarrow \pi = (\mathbf{S}, \mathbf{m})] \\
 \equiv & (\forall \pi)[(\exists Q)(\exists y)\pi = (Q, y) \rightarrow [[\downarrow \pi \wedge (\exists x) \pi = (\mathbf{S}, x)] \rightarrow \pi = (\mathbf{S}, \mathbf{m})]] \\
 & \hspace{15em} \text{def. of struct. prop.} \\
 \equiv & (\forall \pi)(\forall Q)(\forall y)[\pi = (Q, y) \rightarrow [[\downarrow \pi \wedge (\exists x) \pi = (\mathbf{S}, x)] \rightarrow \pi = (\mathbf{S}, \mathbf{m})]] \\
 & \hspace{15em} \text{quantifier law} \\
 \equiv & (\forall Q)(\forall y)[[\downarrow (Q, y) \wedge (\exists x) (Q, y) = (\mathbf{S}, x)] \rightarrow (Q, y) = (\mathbf{S}, \mathbf{m})] \\
 & \hspace{15em} \text{identity law} \\
 \equiv & (\forall Q)(\forall y)[[Q\{y\} \wedge (\exists x) (Q, y) = (\mathbf{S}, x)] \rightarrow (Q, y) = (\mathbf{S}, \mathbf{m})] \\
 & \hspace{15em} \text{def. of } \downarrow \\
 \equiv & (\forall Q)(\forall y)[[Q\{y\} \wedge (\exists x)[Q = \mathbf{S} \wedge x = y]] \rightarrow [Q = \mathbf{S} \wedge y = \mathbf{m}]] \\
 & \hspace{15em} \text{def. of pair} \\
 \equiv & (\forall Q)(\forall y)[Q = \mathbf{S} \rightarrow [Q\{y\} \rightarrow [Q = \mathbf{S} \wedge y = \mathbf{m}]]] \\
 & \hspace{15em} \text{predicate logic} \\
 \equiv & (\forall y)[\mathbf{S}(y) \rightarrow [\mathbf{S} = \mathbf{S} \wedge y = \mathbf{m}]] \\
 & \hspace{15em} \text{identity law + prop. logic} \\
 \equiv & (\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}] \\
 & \hspace{15em} \text{identity law + renaming}
 \end{aligned}$$

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Quantification over alternative intensions

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