

Two types of class B numeral modifiers: A reply to Nouwen 2010*

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Abstract Nouwen (2010) proposes that numeral modifiers like *at most* and *up to* belong to a natural class of expressions that share the same semantic interpretation. We identify several semantic contrasts between *at most* and *up to* that are inconsistent with this proposal. Moreover, we show that Nouwen's semantics cannot be right for *at most*. In attempting to apply his semantics to *up to*, we propose an amendment intended to accommodate certain observations not captured in Nouwen's original account.

Keywords: modified numerals, degree scales, ignorance implications, monotonicity, weak numerals

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1 Introduction

So-called numeral modifiers combine with numerals into expressions like *less than ten*, *at most five*, etc. The focus of [Nouwen 2010](#) is an interesting contrast between two types of numeral modifiers, illustrated in (1) and (2).

- (1) A hexagon has fewer than 11 sides. A
- (2) a. #A hexagon has at most 10 sides. B
 b. #A hexagon has up to 10 sides. B

([Nouwen 2010](#): 3:3)

[Nouwen](#) attributes the acceptability contrast between the sentences in (1) and (2) to a semantic contrast between the numeral modifiers they contain. Specifically, he takes the numeral modifiers in (2) to introduce an implication of speaker ignorance as to the number of sides a hexagon has. [Nouwen](#) attributes the oddness of the particular examples in (2) to the implausibility of this implication, which (1) lacks.¹

[Nouwen](#) refers to numeral modifiers that introduce ignorance implications as *class B* modifiers and those that do not as *class A* modifiers. Furthermore, [Nouwen](#) distinguishes *upper-bound* numeral modifiers, such as those in (1) and (2), from *lower-bound* numeral modifiers, such as *more than*, *at least* and *minimally*.

In this reply we focus on [Nouwen's](#) analysis of upper-bound class B numeral modifiers. [Nouwen](#) argues that all upper-bound class B numeral modifiers share the same interpretation. We present data that challenge this proposal, identifying two types of semantic contrasts between *at most* and *up to*: one involving NPI licensing and the corresponding inferences and another involving what we refer to as the *bottom-of-the-scale effect*. We explore how [Nouwen's](#) analysis applies to these two upper-bound class B modifiers. We argue that it fails for *at most*, but we explore how a conservative amendment of his analysis, extended with what we call the *range requirement*, applies to *up to*. One consequence, if our amendment is on the right track, is that the ignorance implications for *at most* and *up to* have different sources.

In section 2, we review [Nouwen's](#) analysis of upper-bound class B numeral modifiers. Section 3 identifies a contrast between *at most* and *up to* with regard to NPI licensing and the corresponding inferences. Section 4 describes

¹ We take [Nouwen's](#) use of “#” in (2) to indicate mere implausibility, as opposed to downright unacceptability. In later sections, we apply “#” differently to mark semantic/pragmatic ill-formedness in our own examples.

another contrast with regard to the bottom-of-the-scale effect, and we provide a characterization of it within the framework employed by [Nouwen](#). In section 5 we propose and explore an amended semantics for *up to* which incorporates the range requirement. We show that, apart from capturing the bottom-of-the-scale effect, it has the welcome consequence of deriving ignorance implications. In section 6 we identify and discuss a point of tension between our account and the existence of weak interpretations of numerals. Section 7 concludes.

2 [Nouwen's](#) semantics for class B numeral modifiers

2.1 Bare numerals

Following [Hackl 2000](#), [Nouwen \(2010\)](#) proposes that bare numerals, such as *three* in (3), serve as the argument of a silent counting quantifier, *many*, as shown in the *logical form* or *LF* in (4).

- (3) Three students attended.
 (4) [[three many] students] attended

[Nouwen](#) assigns to *many* the denotation in (5), where X is a singular or plural individual and $|X|$ is the number of singular parts of X .²

- (5) $\llbracket \text{many} \rrbracket = \lambda d_d. \lambda f_{et}. \lambda g_{et}. \exists! X [|X| = d \ \& \ f(X) = g(X) = T]$

Under this semantics for *many*, the LF in (4) will be assigned the truth conditions in (6), which states that exactly three students attended.

- (6) $\exists! X [|X| = 3 \ \& \ \llbracket \text{students} \rrbracket (X) = \llbracket \text{attended} \rrbracket (X) = T]$

2.2 Upper-bound class B numeral modifiers and covert modality

Following [Hackl 2000](#), [Nouwen \(2010\)](#) assumes that a numeral modifier combines with a numeral to form a degree phrase denoting a generalized quantifier over degrees. For interpretability, this degree phrase must raise

² Actually, [Nouwen](#) introduces two numeral counting quantifiers: a strong *many* (which he calls *many*₂), with the denotation in (5), and a weak *many* (*many*₁), as in [Hackl 2000](#). Since the weak *many* is not central to [Nouwen's](#) account of numeral modifiers, nor to our discussion of it, we disregard it until section 6.

covertly to derive a degree predicate in its scope. Under these assumptions, (8) is an interpretable LF for the example in (7).

- (7) At most three students attended.
 (8) \llbracket at most \rrbracket three $\lambda d \llbracket$ [[d many] students] attended \rrbracket

The lexical entry that [Nouwen](#) proposes for upper-bound class B numeral modifiers like *at most* and *up to* is shown in (9), according to which these modifiers equate the degree denoted by the numeral with the maximum element of the set given by the degree predicate.

- (9) \llbracket at most/up to $\rrbracket = \lambda d_d. \lambda f_{dt}. \max(f) = d$

Under this semantics for upper-bound class B numeral modifiers, the LF in (8) is assigned the truth conditions in (10).

- (10) $\max(\{d: \exists!X[|X| = d \ \& \ \llbracket$ students $\rrbracket(X) = \llbracket$ attended $\rrbracket(X) = T \}) = 3$

The set of degrees to which *max* applies in (10) is the singleton set containing the cardinality of the set of students who attended, i.e., the exact number of attending students.³ Of course, the maximum element of a singleton set is its single element. Accordingly, (10), like (6), states that exactly three students attended.

To be sure, this is not an attested interpretation of (7), nor does [Nouwen](#) claim that it is. [Nouwen](#) credits the actual truth conditions of (7) to a different LF, namely the one in (11). This LF differs from (8) only in that “ \diamond ”, a silent speaker-oriented epistemic possibility operator, appears in the immediate scope of the degree phrase.

- (11) \llbracket at most \rrbracket three $\lambda d[\diamond \llbracket$ [[d many] students] attended $\rrbracket]$

The LF in (11) receives the truth conditions in (12), which states that the maximal number of students who could have attended, according to the speaker’s evidence, is three. [Nouwen](#) suggests that this is the intended interpretation of (7).

- (12) $\max(\{d: \diamond \exists!X[|X| = d \ \& \ \llbracket$ students $\rrbracket(X) = \llbracket$ attended $\rrbracket(X) = T \}) = 3$

³ We are putting aside, for now, the problematic case where the set of students who attended is empty. We return to this issue in section 4, where it will be important.

Two questions remain. First, what excludes the unattested truth conditions in (10), hence what excludes the LF in (8)? Nouwen proposes that the exclusion of this LF is a *blocking* effect of the sort familiar from the morpho-phonological literature (e.g., Kiparsky 1973, 1983). Specifically, Nouwen takes the LF in (8) to be blocked by the truth conditionally equivalent but less *marked* LF in (4).⁴

Second, how does Nouwen’s account derive the ignorance implications which he takes to be a defining feature of class B numeral modifiers? Surprisingly, this is not addressed. Note that, (12), as it stands, does not entail speaker ignorance regarding the number of students who attended. We return to this issue in section 5.

3 NPI licensing and downward entailment

3.1 *At most*

It has been observed in the literature that phrases headed by *at most* license negative polarity items (NPIs), such as *ever* in (13) and *give a damn* in (14).

(13) At most three people had ever been in this cave. (Krifka 2007)

(14) At most three students give a damn about Pavarotti.
(Chierchia & McConnell-Ginet 2000: 522)

This observation puts Nouwen’s (2010) analysis of upper-bound class B modifiers in conflict with Ladusaw’s (1979) classic account of NPI licensing, which equates the class of NPI licensors with the class of downward entailing expressions. Under Nouwen’s account, *at most three* is clearly not downward entailing; in fact, it is non-monotone. Obviously, for any two sets of degrees A and B such that A is a subset of B, nothing guarantees that *max*(A) equals *max*(B).

Moreover, it would not be possible for Nouwen to resolve this conflict by simply rejecting Ladusaw’s analysis of NPI licensing. Putting aside NPIs, intuitions on inferences alone suggest that *at most three* should be analyzed as downward entailing. The validity of relevant inferences, like the one from (15a) to (15b), seems to be uncontroversial.⁵

⁴ Even though Nouwen doesn’t present a general theory of markedness, it indeed seems plausible to us to assume that (8) is more marked than (4).

⁵ Actually, in view of the ignorance implications with class B modifiers described by Nouwen (see section 1 above), the intuitive validity of this inference is not necessarily expected. To

- (15) a. At most three students smoke.
 b. At most three students smoke cigars.
 (Chierchia & McConnell-Ginet 2000: 522)

And yet, by virtue of the non-monotone semantics Nouwen assigns to *at most three*, the satisfaction of the truth conditions of (16a), shown in (17a), does not guarantee the satisfaction of the truth conditions of (16b), shown in (17b).

- (16) a. $[[\text{at most}] \text{ three}] \lambda d[\diamond [[[d \text{ many}] \text{ students}] \text{ smoke}]]$
 b. $[[\text{at most}] \text{ three}] \lambda d[\diamond [[[d \text{ many}] \text{ students}] \text{ [smoke cigars]}]]$
- (17) a. $\max\{d: \diamond \exists! X [|X| = d \ \& \ \llbracket \text{students} \rrbracket(X) = \llbracket \text{smoke} \rrbracket(X) = T]\} = 3$
 b. $\max\{d: \diamond \exists! X [|X| = d \ \& \ \llbracket \text{students} \rrbracket(X) = \llbracket \text{smoke cigars} \rrbracket(X) = T]\} = 3$

For example, in a scenario where the speaker is sure that exactly one, two or three students smoke, while also being sure that exactly one or two (but not three) students smoke cigars, (17a) is true, while (17b) is false. Thus, it is clear that Nouwen cannot account for the attested inference from (15a) to (15b).

3.2 *Up to*

Recall now that one of Nouwen's (2010) main points is that upper-bound class B modifiers, including *up to* and *at most*, share the same semantic interpretation. It turns out that *up to* is markedly different from *at most* when it comes to NPI licensing. (18) and (19) are unacceptable, unlike their counterparts with *at most* in (13) and (14).

- (18) **Up to three people had ever been in this cave.*
 (19) **Up to three students give a damn about Pavarotti.*

Furthermore, in accordance with Ladusaw's (1979) analysis of NPI licensing, *up to* fails to support the corresponding inferences. That is, our intuitions indicate that (20b) cannot be inferred from (20a).

be sure, it is possible to be uncertain about the exact number of students who smoke while being certain about the exact number of students who smoke cigars. Apparently, ignorance implications with *at most* are not obligatory. Looking ahead, note that this is consistent with our working hypothesis in section 5 that ignorance implications with *up to* are part of the asserted, truth-conditional content.

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- (20) a. Up to three students smoke.
b. Up to three students smoke cigars.

Specifically, the same scenario described above readily comes to mind according to our intuitions: In a scenario where the speaker is sure that exactly one, two or three students smoke, while also being sure that exactly one or two (but not three) students smoke cigars, (20a) is true and appropriate, while (20b) is not.⁶

These observations cast doubt on [Nouwen's](#) proposal that all class B modifiers share the same semantic interpretation.⁷ However, while we have excluded the lexical entry in (9) for *at most*, it might be just what is needed for *up to*. In particular, not only does [Nouwen's](#) analysis predict the unacceptability of (18) and (19) and the invalidity of the inference from (20a) to (20b), it also fits with our intuitions, noted above, as to why this inference is invalid.⁸

⁶ It is clear that it would be inappropriate for a speaker to utter (20b) in the scenario provided. However, it is not so clear that (20b) is judged to be false, even though [Nouwen's](#) analysis predicts that it is false. Under [Nouwen's](#) analysis, there is a missing link between falsity and the judgment of inappropriateness. This issue could be attributed to the fact that the sentences in question are statements about the speaker's epistemic state. The question, which we leave to future research, is how predicted truth values in such cases translate into judgments on appropriateness. Thanks to David Beaver for raising this issue.

Note also that, as predicted by the non-monotone semantics for *up to*, we do not judge the reverse inference from (20b) to (20a) to be valid, either. (20a) conveys information that (20b) does not, viz. that (the speaker is certain that) no more than three students smoke.

⁷ Citing [Gajewski's \(2008\)](#) analysis of *only* and *no one but*, a reviewer suggests that the NPI licensing contrast presented above could be consistent with semantic equivalence of *at most* and *up to* if it could be argued that the two differ in internal syntactic structure. The reviewer's point is well taken. However, we are not sure how to spell out this suggestion, since it appears to us that a structural contrast in the spirit of [Gajewski 2008](#) would necessitate a semantic contrast as well. Moreover, the following section presents an additional contrast between *up to* and *at most* which we interpret as further indication that the two are not semantically equivalent.

⁸ As illustrated by the contrast in pragmatic well-formedness between (i) and (ii) below, there are dramatic differences in use between *up to* and *at most*.

- (i) Save up to 50% on airtime rates!
(ii) #Save at most 50% on airtime rates!

While we leave the detailed analysis of such contrasts as a topic for future work, we speculate that they relate to the monotonicity contrast described in this section.

4 Bottom-of-the-scale effect

4.1 Another contrast

The pattern in (21) and (22) illustrates another systematic contrast between *at most* and *up to*, showing that *up to* is sensitive to the choice of the numeral being modified in a way that *at most* is not.

- (21) a. At most ten people died in the crash.
 b. Up to ten people died in the crash. (Nouwen 2008: 580)
- (22) a. At most one person died in the crash.
 b. #Up to one person died in the crash.

While *at most* combines freely with either *one* or a higher numeral like *ten*, the use of *up to* in (22b) is unacceptable. A natural speculation as to the relevant difference between *one* and *ten* is that *one* in some sense denotes the *bottom of the scale*. The acceptability contrast in (22) suggests, then, that *up to* cannot combine with a bottom-of-the-scale numeral. We refer to this restriction on *up to* as the *bottom-of-the-scale effect*.

The observation that this effect is only attested with one of the two modifiers under consideration further questions Nouwen's (2010) proposal that class B modifiers share the same semantic interpretation. In the previous section, we concluded that Nouwen's semantics does not apply correctly to *at most*, but is still an option for *up to*. Therefore, for the remainder of this paper we confine our attention to the latter.

It turns out that, as it stands, Nouwen's analysis of class B modifiers fails to predict the bottom-of-the-scale effect with *up to*. The analysis assigns sentence (22b) the LF in (23), which receives the truth conditions in (24).

(23) $[[\text{up to}] \text{one}] \lambda d[\diamond [[d \text{ many}] \text{people died in the crash}]]$

(24) $\max(\{d: \diamond \exists! X[|X| = d \ \& \ [[\text{people}]](X) = [[\text{died in the crash}]](X) = T\}) = 1$

The statement in (24) amounts to saying that, according to the speaker's evidence, no more than one person could have died in the crash. Since this would seem to be a useful meaning for a sentence to convey, no explanation for the unacceptability of (22b) emerges.

To accommodate the bottom-of-the-scale effect, we now present and explore a conservative revision of Nouwen's account.

4.2 The bottom of the scale

Naturally, to explain the bottom-of-the-scale effect, we will first have to elucidate the notion of bottom of the scale. We begin by taking a closer look at the set of degrees in (25), the set to which *max* applies in (24) above.

$$(25) \quad \{d: \diamond \exists! X [|X| = d \ \& \ \llbracket \text{person} \rrbracket (X) = \llbracket \text{died in the crash} \rrbracket (X) = T] \}$$

The smallest possible element of (25) depends in part on what exactly the variable *X* is taken to range over. [Nouwen's \(2010\)](#) assumptions in this regard are unclear to us. But the issue is addressed in [Hackl 2000](#), which established the framework for [Nouwen's](#) analysis of numeral modifiers. [Hackl](#) is explicit in not assuming the existence of a zero individual, i.e., an individual corresponding to the empty set in set theory.⁹ The absence of such a possible value for *X* guarantees that (25) cannot have 0 as an element.

In fact, we can be certain that (25) does not contain any degree smaller than 1: While it is perhaps possible for half a person to be photographed or to be covered with sand, it is impossible for half a person to die in a crash. The smallest possible degree contained in the set (25) is accordingly 1, i.e., the degree that we have characterized as the bottom of the scale.

The bottom of the scale, then, is the smallest possible element of the set of degrees to which the degree phrase headed by *up to* applies under (our construal of) [Nouwen's](#) analysis. In more theory-neutral terms, a bottom-of-the-scale numeral denotes the lowest non-zero degree that can satisfy the degree property provided by the sentence that contains it. So 1 is the bottom of the scale in (22b) because a fatal crash necessarily implies the death of at least one person.

This general characterization of the bottom of the scale suggests that it need not always be given by *one*, hence that the bottom-of-the-scale effect can be reproduced with other numerals. This is indeed so. Suppose, for example, that eggs can be bought in half-dozen cartons only. Under this assumption, our judgments on the examples (26b) and (27b) are parallel to those we gave for (21b) and (22b) above. In particular, we detect the same unacceptability in (27b) as in (22b).¹⁰

⁹ [Hackl \(2000: 169\)](#) writes: "Note that the definition does not allow us to count zero individuals, i.e., there is no individual that is assigned degree zero in a complete join semi-lattice. I will have to leave it for future research to extend the definition so that expressions like 'more than zero students', 'fewer than one students', etc. can be covered."

¹⁰ Replacing *a dozen* and *half a dozen* with *twelve* and *six*, respectively, does not significantly alter our judgments.

- (26) (Assumption: Eggs can be bought in half-dozen cartons only.)
 a. He bought at most a dozen eggs.
 b. He bought up to a dozen eggs.
- (27) (Assumption: Eggs can be bought in half-dozen cartons only.)
 a. He bought at most half a dozen eggs.
 b. #He bought up to half a dozen eggs.

Under Nouwen's analysis, (27b) has the LF in (28), which receives the truth conditions in (29); (30) isolates the set of degrees to which *max* applies in (29).

- (28) [[up to] half a dozen] $\lambda d[\diamond [\text{he bought [d many] eggs}]]$
 (29) $\max(\{d: \diamond \exists!X[|X| = d \ \& \ \llbracket \text{eggs} \rrbracket(X) = \llbracket \text{he bought} \rrbracket(X) = T]\}) = 6$
 (30) $\{d: \diamond \exists!X[|X| = d \ \& \ \llbracket \text{eggs} \rrbracket(X) = \llbracket \text{he bought} \rrbracket(X) = T]\}$

By assumption, any purchase of eggs has to be a purchase of at least six. Accordingly, in the absence of a zero individual, 6 is the smallest possible element of the set of degrees in (30). Thus, *half a dozen* in (27b) denotes the bottom of the scale, and so we are not surprised that (27b) shares the oddness of (22b).

It is also possible, of course, for some scale not to have a bottom at all. In such a case, the bottom-of-the-scale effect is not expected to arise with *one* or with any degree-denoting phrase, for that matter. For example, since the amount of cake consumed in a cake-eating event can be arbitrarily small, and in particular less than one entire cake, *one* in (31) need not denote the bottom of the scale. Hence, we correctly expect this example to be acceptable.¹¹

- (31) She ate up to one whole cake.

¹¹ Example (31) sounds better with *whole* (or *entire*) than without, and we speculate that this is because *whole* signals the possibility of eating less than one cake, i.e., it signals that the relevant scale reaches below 1. Note that this speculation is supported by the oddness of the sentence *One entire person died in the crash*, whose content is inconsistent with a scale reaching below 1.

Note also that, while it seems clear that the contrast between (31) and (22b) is to be explained along the lines suggested here, the framework that Nouwen adopts from Hackl 2000 is not actually set up to support formalization of this account. The reason is that Hackl does not countenance quantification over proper parts of a singular individual such as a cake. For relevant discussion, including analyses of *entire* and *whole*, see Morzycki 2002 and Moltmann 2005.

5 The range requirement

Providing a general characterization of the bottom of the scale is of course just one step towards understanding the bottom-of-the-scale effect. In this section, we take the next step by exploring a possible derivation of this effect. We amend [Nouwen's \(2010\)](#) semantics for *up to* by adding an entailment that we refer to as the *range requirement*. We then show that this addition has other welcome consequences. In particular, we show that it derives the ignorance implications described in section 1.¹²

5.1 Deriving the bottom-of-the-scale effect

Having identified 1 and 6 as the smallest possible elements of the sets in (25) and (30), respectively, let us revisit the truth conditions in (24) and (29). According to these truth conditions, the smallest possible degree in a set of degrees, the so-called bottom of the scale, is also the maximal element of the set. Note that this is tantamount to saying that the bottom of the scale is the only element of the set in question. This means that (24) and (29), in our scenario, can be rewritten as (32) and (33), respectively.

$$(32) \quad \{d: \diamond \exists!X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\} = \{1\}$$

$$(33) \quad \{d: \diamond \exists!X[|X| = d \ \& \ \llbracket \text{eggs} \rrbracket(X) = \llbracket \text{he bought} \rrbracket(X) = T]\} = \{6\}$$

More generally, when *up to* modifies a bottom-of-the-scale numeral, the resulting truth conditions entail that the degree denoted by that numeral is the only element of the set of degrees that the modified numeral composes with.

¹² While preparing the final version of this reply, we were reminded of a closely related proposal by [Penka \(2010\)](#). [Penka](#) applies a general semantics for superlatives to *at least* and *at most* in an attempt to compositionally derive their truth conditional contributions. By decomposing *at least* and *at most*, [Penka](#) effectively adds a range requirement (as a presupposition) to their semantics. Moreover, [Penka](#) intends for the range requirement to have the same effects regarding blocking and the derivation of ignorance implications that we discuss in sections 5.2 and 5.3 below, and analogous to our discussion in sections 6.3 and 6.4, [Penka](#) identifies the potential problems for this approach coming from weak interpretations of numerals (and other measure phrases). Naturally, however, being concerned with superlative modifiers, rather than *up to*, [Penka](#) does not establish a link between the range requirement and our bottom-of-the-scale effect. In fact, under [Penka's](#) proposal, we wonder how one might accommodate the observation, reported in section 4, that this effect is absent with *at most*. It also appears that [Penka's](#) account inherits from [Nouwen's](#) the problems we have identified in section 3 regarding NPI licensing and the corresponding inferences.

This fact suggests a possible way to account for the bottom-of-the-scale effect. Suppose that *Nouwen's* lexical entry for *up to* in (9) above is strengthened so as to yield the implication that the relevant set of degrees contains more than one element. Let us assume for concreteness that this implication is part of the asserted content. We then arrive at the revised lexical entry in (34), where $range(f)$, the range requirement, expresses that (the set characterized by) f has more than one element.^{13,14}

$$(34) \quad \llbracket \text{up to} \rrbracket = \lambda d_d. \lambda f_{dt}. \max(f) = d \ \& \ range(f)$$

Under our revised entry for *up to*, the truth conditions of (23), repeated below, are the conjunction of the equality in (32) and the range requirement in (35); likewise, the truth conditions of (28), also repeated below, are the conjunction of the equality in (33) and the range requirement in (36).

$$(23) \quad \llbracket \text{up to} \rrbracket \text{ one} \ \lambda d[\diamond \llbracket [d \text{ many}] \text{ people died in the crash} \rrbracket]$$

$$(35) \quad range(\{d: \diamond \exists !X[|X| = d \ \& \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\})$$

$$(28) \quad \llbracket \text{up to} \rrbracket \text{ half a dozen} \ \lambda d[\diamond \llbracket \text{he bought } [d \text{ many}] \text{ eggs} \rrbracket]$$

$$(36) \quad range(\{d: \diamond \exists !X[|X| = d \ \& \llbracket \text{eggs} \rrbracket(X) = \llbracket \text{he bought} \rrbracket(X) = T]\})$$

The LFs in (23) and (28) are now contradictory: In their respective truth conditions, one conjunct implies that the relevant set of degrees is a singleton set, while the other implies that it is not. Under the assumption that contradictory truth conditions can be a source of unacceptability, a view recently defended in *Gajewski 2003*, the range requirement therefore correctly excludes (22b) and (27b) and more generally derives the bottom-of-the-scale effect.

More accurately, so far we have only shown that the particular LFs in (23) and (28) are contradictory. A complete explanation of the unacceptability of the corresponding examples will have to exclude other potential LFs for these examples as well. In particular, we will have to exclude LFs that, unlike (23) and (28), feature no possibility modal in the scope of the degree phrase.

¹³ It should be clear that under this revised entry, *up to* is still not downward entailing (in fact, it is still non-monotone), so the relevant observations in section 3 are accounted for as before.

¹⁴ In *Nouwen 2008*, *up to* is a universal quantifier over degrees in an interval. Under this view, the range requirement could be taken to follow from the general requirement that the restrictor of a universal quantifier have more than one element (see, for example, *Schwarzschild 2002* for discussion).

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These are indeed excluded as well, as we will show in the next section. Just like in [Nouwen's](#) account, under the amendment we have proposed, LFs that feature no possibility modal in the scope of *up to* will not give rise to attested interpretations.

5.2 The range requirement and blocking

Introducing the range requirement forces us to abandon [Nouwen's \(2010\)](#) rationale for the obligatory presence of a silent possibility operator. We can illustrate this with [\(21b\)](#), repeated below.

[\(21b\)](#) Up to ten people died in the crash.

Recall that [Nouwen's](#) account rests on the assumption that an LF can be blocked by the availability of a semantically equivalent LF that counts as less marked. For [Nouwen](#), the LF for [\(21b\)](#) given in [\(37\)](#) is true just in case exactly ten people died in the crash.

[\(37\)](#) $[[\text{up to}] \text{ten}] \lambda d[[d \text{ many}] \text{people died in the crash}]$

[Nouwen's](#) account ensures that the bare numeral example in [\(38a\)](#), or rather its LF in [\(38b\)](#), is likewise true just in case exactly ten people died in the crash and hence is equivalent to [\(37\)](#). Moreover, [Nouwen](#) assumes that [\(37\)](#) is more marked than [\(38b\)](#), and so the latter blocks the former.

[\(38\)](#) a. Ten people died in the crash.
b. $[[\text{ten many}] \text{people died in the crash}]$

It should be clear that under the revised lexical entry in [\(34\)](#), the LF in [\(37\)](#) is no longer equivalent to [\(38b\)](#). The addition of the range requirement renders [\(37\)](#) strictly stronger than [\(38b\)](#). Therefore, assuming that semantic equivalence is a precondition on blocking, an alternative explanation must be sought for the unavailability of [\(37\)](#).

Fortunately, such an alternative explanation is readily available. The LF in [\(37\)](#) is, in fact, not merely stronger than [\(38b\)](#), but actually has contradictory truth conditions. To see why, consider the set given by the degree predicate in [\(37\)](#), shown in [\(39\)](#).

[\(39\)](#) $\{d: \exists!X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\}$

This set can never have more than one element: If no one died in the crash, it will be empty, and it will contain only the exact number of fatalities if someone did; so the range requirement in (40) is necessarily false.

$$(40) \quad \text{range}(\{d: \exists!X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\})$$

Therefore, under the assumption (already employed above) that contradictory truth conditions can keep an LF from being attested, the effect of blocking in *Nouwen's* account is replicated.

Moreover, the addition of a possibility operator can be shown to result in an LF whose truth conditions correspond to an observable interpretation. The reason is that a possibility operator renders an otherwise contradictory range requirement satisfiable. This will become clear in the next subsection, which examines the effect of the range requirement on attested interpretations of acceptable *up to* sentences.¹⁵

5.3 The range requirement and ignorance implications

In this section we have so far demonstrated the utility of the range requirement in preventing overgeneration by excluding problematic LFs containing bottom-of-the-scale numerals or lacking possibility modals. The range requirement was seen to exclude the LFs in question by rendering them contradictory. But surely the range requirement will have an effect on the observable interpretations derived for acceptable *up to* sentences, and this effect remains to be identified.

Let us therefore return to sentence (21b), repeated below. The sentence has the intended LF in (41). The truth conditions we assign to this LF are given by the conjunction of (42a) and (42b).

(21b) Up to ten people died in the crash.

$$(41) \quad \llbracket [\text{up to}] \text{ ten} \rrbracket \lambda d[\diamond \llbracket [d \text{ many}] \text{ people died in the crash} \rrbracket]$$

$$(42) \quad \begin{array}{l} \text{a.} \quad \max(\{d: \diamond \exists!X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) \ \& \\ \llbracket \text{died in the crash} \rrbracket(X) = T]\}) = 10 \\ \text{b.} \quad \text{range}(\{d: \diamond \exists!X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) \ \& \\ \llbracket \text{died in the crash} \rrbracket(X) = T]\}) \end{array}$$

¹⁵ This account of the modal interpretation of class B modifiers is very similar to *Menéndez-Benito's* (2010) analysis of so-called *universal free choice items*.

Two types of class B numeral modifiers

The conjunct in (42a), according to which 10 is the maximal number of fatalities in the crash consistent with the speaker's evidence, is of course an entailment of the LF in (41) under Nouwen's (2010) original account as well. We accept Nouwen's assumption that this is indeed an attested entailment of sentence (21b).

Turning to (42b), let us attend to the set serving as the argument of *range*. This set is isolated in (25), repeated below.

(25) $\{d: \diamond \exists!X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\}$

This set contains all non-zero degrees that could be the exact number of fatalities given the speaker's evidence. To be sure, since it is possible for this set to contain more than one element, the range requirement in (42b) can be true. This supports our conclusion at the end of the previous subsection that the addition of a possibility operator can render an otherwise contradictory LF contingent.

Moreover, saying that the set in (25) has more than one element amounts to saying that the set of worlds consistent with the speaker's evidence varies in terms of the exact number of people who died in the crash. Thus, (42b) amounts to the statement that the speaker considers it possible that there were fatalities in the crash, but does not have sufficient evidence to identify the exact number. In other words, (42b) entails speaker ignorance regarding the exact number of fatalities.

Sentence (21b) is indeed judged to imply such speaker ignorance. Of course, this is an instance of Nouwen's observation, reviewed in section 1, that all so-called class B modifiers introduce such implications. Recall that Nouwen attributes the oddness of example (2b), repeated below, to the implausibility of speaker ignorance regarding the number of sides a hexagon has.

(2b) #A hexagon has up to 10 sides.

Sentence (21b) illustrates the same effect, though perhaps in a less dramatic way, given that there is nothing implausible about the implied speaker ignorance regarding fatalities in the crash.

We therefore conclude that in addition to accounting for the bottom-of-the-scale effect, the range requirement has the welcome effect of deriving attested ignorance implications contributed by *up to*, which under Nouwen's original proposal remained unaccounted for.

This is a pleasing result; however, in outlining [Nouwen’s](#) analysis of upper-bound class B modifiers in section 2, we have permitted ourselves a slight departure from his actual account. While inconsequential for [Nouwen’s](#) analysis of numeral modifiers, this departure raises questions for our modification of it. This is the topic of the next section.

6 Weak *many*

We reported that [Nouwen \(2010\)](#) posits a counting quantifier, *many*, that combines with numerals to form degree phrases. Actually, however, [Nouwen](#) assumes that there are two such counting quantifiers (see footnote 2). In addition to the *many* in (5), repeated below, [Nouwen](#) also posits a second *many*, given in (43), which we write as *many_w*.

$$(5) \quad \llbracket \text{many} \rrbracket = \lambda d_d. \lambda f_{et}. \lambda g_{et}. \exists! X [|X| = d \ \& \ f(X) = g(X) = T]$$

$$(43) \quad \llbracket \text{many}_w \rrbracket = \lambda d_d. \lambda f_{et}. \lambda g_{et}. \exists X [|X| = d \ \& \ f(X) = g(X) = T]$$

The subscript *w* is meant to signal that the *many_w* in (43) derives truth conditions that are weaker than those derived by the strong *many* in (5). Consider again example (3) and the LF in (4), repeated below. We have seen that (4) is true just in case exactly three students attended. In contrast, (43) ensures that (44) is true just as long as no fewer than three students attended.

(3) Three students attended.

(4) \llbracket three many \rrbracket students \rrbracket attended

(44) \llbracket three many_w \rrbracket students \rrbracket attended

Postulating *many_w* in (43) is motivated by the well-known observation that sentences with bare numerals indeed allow for such weak interpretations (see [Geurts 2006](#) and the references provided therein). In this section, we describe the consequences of introducing *many_w* for [Nouwen’s](#) original account as well as for our own modification of it. We show that it is innocuous for the former, but not for the latter. Specifically, we show that while our account of the bottom-of-the-scale effect is preserved, our derivation of ignorance implications is not. Note that we will not solve this problem. The purpose of this section is merely to identify the price to pay for the kind of modification to [Nouwen’s](#) account that we have proposed.

6.1 Nouwen's account revisited

Nouwen (2010) carefully demonstrates that when the two counting quantifiers combine with upper-bound class B modifiers, the semantic difference between the strong version in (5) and the weak version in (43) is systematically neutralized. It will be useful to review the reason for this neutralization by attending again to (21b), repeated below. Consider first the LF with *many* in (37), repeated below, and its counterpart with *many_w* in (45).

(21b) Up to ten people died in the crash.

(37) $[[\text{up to}] \text{ten}] \lambda d [[d \text{ many}] \text{people died in the crash}]$

(45) $[[\text{up to}] \text{ten}] \lambda d [[d \text{ many}_w] \text{people died in the crash}]$

Consider the sets denoted by the derived degree predicates in (37) and (45), shown in (39) (repeated from earlier) and (46).

(39) $\{d: \exists!X [|X| = d \ \& \ \llbracket \text{people} \rrbracket (X) = \llbracket \text{died in the crash} \rrbracket (X) = T]\}$

(46) $\{d: \exists X [|X| = d \ \& \ \llbracket \text{people} \rrbracket (X) = \llbracket \text{died in the crash} \rrbracket (X) = T]\}$

Suppose that there have been fatalities in the crash and let f be their exact number. We have seen that (39) is $\{f\}$. In contrast, (46) is the interval $[1, f]$, which apart from f includes all lower cardinalities down to the bottom of the scale, 1. This implies that the two sets in (39) and (46) necessarily have the same maximum. Accordingly, the two statements in (47) and (48) are equivalent, for any cardinality n , and therefore under Nouwen's semantics for *up to*, the LFs in (37) and (45) have identical truth conditions.

(47) $\max(\{d: \exists!X [|X| = d \ \& \ \llbracket \text{people} \rrbracket (X) = \llbracket \text{died in the crash} \rrbracket (X) = T]\}) = n$

(48) $\max(\{d: \exists X [|X| = d \ \& \ \llbracket \text{people} \rrbracket (X) = \llbracket \text{died in the crash} \rrbracket (X) = T]\}) = n$

The same is true for the LFs in (41) (repeated from earlier) and (49), where the silent possibility modal has been added to the scope of *up to*.

(41) $[[\text{up to}] \text{ten}] \lambda d [\diamond [[d \text{ many}] \text{people died in the crash}]]$

(49) $[[\text{up to}] \text{ten}] \lambda d [\diamond [[d \text{ many}_w] \text{people died in the crash}]]$

Now the derived degree predicates denote the sets in (25) (repeated from earlier) and (50).

(25) $\{d: \diamond \exists!X [|X| = d \ \& \ \llbracket \text{people} \rrbracket (X) = \llbracket \text{died in the crash} \rrbracket (X) = T]\}$

$$(50) \quad \{d: \diamond \exists X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\}$$

Suppose that according to the speaker's evidence, someone may have died in the crash and let p be the largest number of fatalities consistent with that evidence.¹⁶ Then (25) is a set whose maximal element is p and (50) is the set $[1, p]$. This entails once again that the sets in (25) and (50) have the same maximum element. For any cardinality n , the statements in (51) and (52) are equivalent, and so (41) and (49) have identical truth conditions.

$$(51) \quad \max(\{d: \diamond \exists! X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) \ \& \ \llbracket \text{died in the crash} \rrbracket(X) = T]\}) \\ = n$$

$$(52) \quad \max(\{d: \diamond \exists X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) \ \& \ \llbracket \text{died in the crash} \rrbracket(X) = T]\}) \\ = n$$

The addition of *many_w*, then, is innocuous for Nouwen's original account of upper-bound class B modifiers. Unfortunately, this welcome feature is not shared by the revised version we have presented above. While our account of the bottom-of-the-scale effect is preserved under the addition of *many_w*, our rationale for why *up to* must be accompanied by a possibility operator must be revised, and our proposed derivation of ignorance implications is lost. We dedicate the following three subsections to substantiating these points.

To facilitate the presentation, though, it will be useful to first highlight another pair of equivalencies. Recall that the sets in (46) and (50), the sets arising from LFs with *many_w*, necessarily denote intervals of the form $[1, n]$, sets consisting of a maximum n and all smaller cardinalities down to the bottom of the scale, 1. This means that the membership of these sets is fully determined by their maximal elements: For any cardinality n , the maximality statements in (48) and (52) are equivalent to the equalities in (53) and (54).

$$(53) \quad \{d: \exists X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\} = [1, n]$$

$$(54) \quad \{d: \diamond \exists X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\} = [1, n]$$

Having a name for these two equivalencies will simplify the exposition below, so we will collectively refer to them as the *range law*.

¹⁶ The sets in (25) and (50), even if non-empty, are not guaranteed to have maximal elements. However, we can safely focus on the case where they do, as this is a presupposition triggered by *up to* in Nouwen's account.

6.2 Bottom-of-the-scale effect revisited

Above we proposed that (22b), repeated below, is excluded because it only has a contradictory interpretation. To maintain this account, we now have to consider the LF with *many_w* in (55). We want to show that this LF shares the contradictory truth conditions of its counterpart with (strong) *many* (see (23) in section 5.1).

(22b) #Up to one person died in the crash.

(55) [[up to] one] $\lambda d[\diamond \llbracket d \text{ many}_w \rrbracket \text{ people died in the crash}]$

Given the semantics of *many_w*, the truth conditions we assign to (55) are given by the conjunction of (56a) and (56b).

- (56) a. $\max(\{d: \diamond \exists X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\}) = 1$
 b. $\text{range}(\{d: \diamond \exists X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\})$

Just as we saw in section 5.1 with (strong) *many*, a set of degrees whose maximum is the bottom of the scale is a singleton set containing that degree. It follows that (56a) is equivalent to (57). (This is also evident from the range law since $[1, 1] = \{1\}$.)

(57) $\{d: \diamond \exists X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) \llbracket \text{died in the crash} \rrbracket(X) = T]\} = \{1\}$

Therefore the conjunction of (56a) and (56b) is inconsistent, hence the LF in (55) is excluded in the same way as its variant with strong *many* in (23).

It should moreover be clear that omitting the possibility operator from (55) does not change the relevant semantic relations: The maximum of the relevant set is still the bottom of the scale (and the range law will apply as before), and so a contradiction will again be derived. The addition of *many_w*, then, does not save bottom-of-the-scale numerals from yielding contradictory truth conditions, and so we conclude that our account of the bottom-of-the-scale effect still applies correctly.

6.3 Blocking revisited

Nouwen (2010) proposes that unless they contain a possibility modal (in the immediate scope of the degree phrase), LFs with upper-bound class B

modifiers are blocked by less-marked LFs with bare numerals. In our revised account, only positing strong *many*, we proposed that instead of being blocked by markedness, the relevant LFs are excluded by virtue of being contradictory. Having added *many_w*, we now have to revisit this account. We will focus here on numerals above the bottom of the scale, since we just discussed how bottom-of-the-scale numerals give rise to contradictions.

Consider again example (21b), repeated below. We need to examine the predicted interpretation of the LF in (45), repeated below, which features *many_w* but no possibility modal.

(21b) Up to ten people died in the crash.

(45) $[[\text{up to}] \text{ten}] \lambda d[[d \text{ many}_w] \text{ people died in the crash}]$

Under our semantics for *up to*, the truth conditions of (45) are the conjunction of (58a) and (58b).

- (58) a. $\max\{d: \exists X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\}$
 $= 10$
 b. $\text{range}\{d: \exists X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\}$

Note that according to the range law, (58a) is equivalent to (59). Since in this case the numeral is above the bottom of the scale, the set of cardinalities it determines according to the range law is necessarily a range.

(59) $\{d: \exists X[|X| = d \ \& \ \llbracket \text{people} \rrbracket(X) = \llbracket \text{died in the crash} \rrbracket(X) = T]\} = [1, 10]$

Accordingly, the maximality statement in (58a) entails the range requirement in (58b). It follows that the conjunct in (58b) is effectively redundant: The truth conditions of (45) are given by (58a) alone.

So our revised semantics for *up to* assigns to (45) the very same truth conditions as Nouwen's original semantics: The LF is true just in case exactly ten people died in the crash. Since these truth conditions are contingent, our rationale for why *up to* requires a possibility operator in its scope is not preserved under the addition of *many_w*.

As things stand, then, we are in danger of predicting an unattested interpretation of (21b), according to which exactly ten people died in the crash. Evidently, we must abandon the idea that the presence of a possibility operator is always motivated by the need to prevent contradiction.

Fortunately, though, given that we assign to (45) the very same truth conditions as Nouwen, a natural modification suggests itself. To force the

presence of a possibility operator in an LF with *many_w*, we can adopt Nouwen’s notion of blocking by markedness, which after all is not inconsistent with anything we have proposed. So, following Nouwen, we could assume that (45) is excluded by virtue of being more marked than the equivalent LF in (38b), repeated from earlier.

(38b) [[ten many] people died in the crash]

In the resulting account, then, there would be two different reasons why *up to* requires the presence of a possibility operator (with numerals above the bottom of the scale): In LFs with strong *many*, the possibility operator renders an otherwise contradictory LF contingent; in LFs with *many_w*, the possibility operator prevents the LF from being blocked by a less-marked competitor.

6.4 Ignorance implications revisited

We have proposed to credit ignorance implications to the range requirement introduced by the lexical semantics of *up to*. Once again, we need to go back and examine the effect of the addition of *many_w*. Let us return to the LF in (49), repeated below.

(49) [[up to] ten] $\lambda d[\diamond \text{[[d many}_w\text{] people died in the crash}}]$

Under our account, the truth conditions of (49) are given by the conjunction of (60a) and (60b). According to the range law, we moreover have the equivalence of (60a) and (61).

(60) a. $\max(\{d: \diamond \exists X[|X| = d \ \& \ \text{[[people]]}(X) = \text{[[died in the crash]]}(X) = T]\}) = 10$

b. $\text{range}(\{d: \diamond \exists X[|X| = d \ \& \ \text{[[people]]}(X) = \text{[[died in the crash]]}(X) = T]\})$

(61) $\{d: \diamond \exists X[|X| = d \ \& \ \text{[[people]]}(X) = \text{[[died in the crash]]}(X) = T]\} = [1, 10]$

This means that much like in the case studied in the previous subsection, the range requirement ends up being redundant, as it is entailed by (60a).¹⁷ So in

¹⁷ Despite initial appearance, the statement in (61) does not imply speaker ignorance. Specifically, it is consistent with the speaker being certain that exactly ten people died in the crash. So (61) does not entail that the set of worlds consistent with the speaker’s evidence varies in terms of exact number of fatalities. In contrast, equating the set in (25) with a range like

this case, too, our semantics for *up to* derives the very same truth conditions as Nouwen's.

This is certainly not a welcome result. The range requirement being redundant, our account of ignorance implications is lost. We showed that the range requirement derives ignorance implications for LFs with strong *many*. However, if we assume that *many_w* is freely available as well, we admit LFs where the range requirement does not actually strengthen the truth conditions in any way, and so we no longer predict that *up to* is perceived to always introduce these implications.

So if one admits *many_w*, an alternative derivation of ignorance implications needs to be sought. But insisting on deriving ignorance implications through the range requirement, by excluding *many_w*, has its price as well. As reported above, *many_w* is motivated by weak readings of bare numerals, so an alternative account of those would be needed.¹⁸

7 Summary and conclusion

What we consider the main result of this commentary is that so-called upper-bound class B numeral modifiers come in two types. This is shown, first, by NPI licensing facts and intuitions on corresponding inferences. The class B numeral modifier *at most* licenses NPIs and is downward entailing, according to intuitions on inferences. In contrast, the class B modifier *up to* has neither of these properties. Second, while *at most* can combine with bottom-of-the-scale numerals, *up to* cannot. These basic semantic contrasts suffice to establish that *at most* and *up to* cannot share the same semantic interpretation.

More specifically, NPI licensing data and the corresponding inferences demonstrate that Nouwen's (2010) semantics for upper-bound class B modifiers is not correct for *at most*. In attempting to apply it to *up to*, we identified

[1, 10] does imply speaker ignorance.

¹⁸ The literature provides possible starting points for either direction of analysis. One might hope to derive ignorance implications as conversational implicatures, perhaps along the lines of Büring 2008, which provides an analysis of *at least* (see also Cummins & Katsos 2010 and Schwarz & Shimoyama 2011). Conversely, one might follow Kratzer 2003 in employing situation semantics to derive apparent weak readings from strong lexical meanings, such as the strong *many* we have been discussing. A third possibility is that *many_w* is an available lexical item with a suitably limited distribution: Specifically, it cannot combine with a trace of a numeral modified by *up to*; unfortunately, however, we don't see what principle of grammar such a restriction might follow from.

two shortcomings. First, the analysis fails to derive the bottom-of-the-scale effect; second, it surprisingly fails to derive the very ignorance implications that Nouwen identified as a defining feature of class B modifiers.

We presented an amendment of Nouwen's account that adds the range requirement to the truth conditional content of *up to*. This amendment captures the bottom-of-the-scale effect. Under the additional assumption that the counting quantifier *many* is always strong, but not without this assumption, we also derive ignorance implications. So the extent to which our proposal improves upon Nouwen's original account depends on whether a weak counting quantifier is assumed to be available.

Apart from the issue raised by *many_w*, one additional concern about our proposed derivation of ignorance implications with *up to* is that it might be insufficiently general, as it is unclear how it would carry over to *at most*. However, in view of the semantic differences between *up to* and *at most* that we have identified, it is not clear to us that the ignorance implications associated with the two expressions should even have the same source. Thus, our discussion raises the possibility that there are two types of ignorance implications, and hence that this common feature of class B numeral modifiers is not necessarily indicative of a shared analysis.

To be sure, there may well be subclasses of class B modifiers which do form semantic and pragmatic natural classes. If morphology is a guide, we expect *at least* to pattern with *at most* (see Penka 2010). But there are other numeral modifiers characterized as class B by Nouwen, such as *minimally* and *maximally*, whose place in the typology remains to be determined.

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