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Consistency preservation in Quantity implicature: The case of *at least*[*]

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Abstract Spector (2006) and Fox (2007a) observed that a standard Neo-Gricean account of Quantity implicature, as articulated in Sauerland 2004, predicts that listeners draw inconsistent sets of Quantity inferences under certain configurations of asserted meaning and its alternatives. To properly assess the consequences of this observation, specific phenomena that can be argued to instantiate the relevant type of configuration need to be examined. This paper presents a case study on the superlative modifier *at least*, expanding on Büring's (2008) proposal that the ignorance implications that *at least* gives rise to are Gricean Quantity implicatures. It is argued that sentences with unembedded *at least* instantiate the relevant configuration, hence that the standard Neo-Gricean account incorrectly predicts inconsistent inference sets for those cases. It is then argued that a proper consistency preserving modification of the standard account must make reference to Fox's (2007a) notion of innocent exclusion. This argument for innocent exclusion, while embedded here in the Neo-Gricean setting, extends to any account of Quantity implications that, in the terminology of Sauerland (2004), posits strengthening of *primary* to *secondary* implications about the speaker's belief state.

Keywords: Quantity implicature, innocent exclusion, alternatives, superlative modifiers, *at least*

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1 Introduction

In the wake of Krifka 1999, a body of literature has emerged that deals with the meaning contribution of the superlative modifier *at least* in examples like (1).

(1) Al hired at least two cooks.

One central observation is that numerals modified by *at least* lack the upper-bounding implication typically associated with bare numerals (Horn 1972, Levinson 1983, Krifka 1999). Thus, while (2) below suggests that Al hired no more than two cooks, (1) has no such implication. Instead, and this is the second main observation, (1) carries an implication of speaker ignorance, suggesting that the speaker is uncertain about the exact number of cooks Al hired (Krifka 1999, Büring 2008).

(2) Al hired two cooks.

In a prominent view, articulated in Krifka 1999 and developed in Büring 2008, it is the very presence of an ignorance implication in cases like (1) that is responsible for the absence of an upper-bounding implication. Büring moreover suggested that ignorance implications with *at least* are Gricean conversational implicatures, viz., Quantity implicatures (Grice 1989). Büring specifically proposed that ignorance implications with *at least* can be likened to ignorance implications associated with disjunction. Relating examples like (1) to paraphrases like *Al hired two or more cooks*, Büring suggested that the two types of cases should receive parallel Neo-Gricean analyses, that is analyses where Gricean reasoning is taken to be regulated by grammatically determined sets of alternatives (Horn 1972). However, Büring stopped short of detailing the analysis of either disjunction or *at least* in a general account of Quantity implicature.

Much of the existing work on superlative modifiers either subscribes to a Neo-Gricean Quantity implicature analysis like Büring's (2008), without fully explicating the approach (Cummins & Katsos 2010, Westera & Brasoveanu 2014), or develops alternative analyses (Krifka 1999, Geurts & Nouwen 2007, Nouwen 2010, Penka 2010, Coppock & Brochhagen 2013, Cohen & Krifka 2014). The literature that does engage the Quantity implicature analysis of *at least* offers divergent opinions: Coppock & Brochhagen (2013) argue that Büring's (2008) likening of *at least* to disjunction resists a coherent theoretical interpretation; Schwarz & Shimoyama (2011) and Kennedy (2015) sketch proposals intended to implement Büring's idea; yet Mayr (2013) ends up questioning the feasibility of an analysis of *at least* in terms of Quantity implicature.

This paper offers a detailed appraisal of the Neo-Gricean approach to ignorance implications associated with *at least*. The main finding is that such an approach requires that the Neo-Gricean derivation of Quantity implicatures be regulated with reference to so-called innocent exclusion, a notion introduced in Fox 2007a. Hence the paper provides an argument for innocent exclusion as a necessary ingredient of Neo-Gricean pragmatics, an argument that is conditional on the assumption that ignorance implications with *at least* are indeed correctly analyzed as inferences grounded in Grice's Quantity maxim.

Innocent exclusion has not previously been proposed as an ingredient of Neo-Gricean pragmatics. Instead, developing the so-called Grammatical Theory of Implicature as an alternative to the Neo-Gricean approach, Fox (2007a) posited innocent exclusion in the lexical semantics of a covert operator *Exh*. This semantics is designed to properly preserve consistency of the meanings that *Exh* produces, by properly controlling the set of so-called alternatives that enter those meanings. Likewise, the motivation presented here for adopting innocent exclusion in a Neo-Gricean framework is the need for preserving consistency in the derivation of Quantity implicatures.

The argument presented, if successful, adjudicates between different elaborations of the Neo-Gricean approach to Quantity implicatures. On its own, it does not contribute to the relative assessment of the Neo-Gricean approach and the competing Grammatical Theory of Implicature. In particular, the proposed elaboration of the Neo-Gricean approach does not address the issues that have been presented in support of the Grammatical Theory.[1] At the same time, the argument for innocent exclusion, while embedded here in the Neo-Gricean setting, extends to any account of Quantity implications

1 The Grammatical Theory of Implicature, so termed in Chierchia, Fox & Spector 2011, is motivated in Fox 2007a, Magri 2009, Magri 2011, Chierchia, Fox & Spector 2011, Meyer 2013, Crnič 2013, Kilbourn-Ceron 2014, among other works. The arguments for the Grammatical Theory come from the analysis of free choice effects, so-called embedded implicature, as well as Quantity related oddness effects. The validity of the proposed arguments for the Grammatical Theory is debated in, for example, Geurts 2010, Schlenker 2012, and Lauer 2014. The present findings could conceivably be extended into an argument against the Neo-Gricean approach, viz., by presenting an independent case *against* reference to innocent exclusion in Neo-Gricean pragmatics. However, no such extension is attempted below.

that, in the terminology of Sauerland (2004), posits strengthening of *primary* to *secondary* implications about the speaker's belief state.

Section 2 reviews the standard Neo-Gricean account of Quantity implicature and the potential problem of inconsistency that it gives rise to. Section 3 argues that a standard Neo-Gricean implementation of Büring (2008)'s proposal about *at least* leads to an actual instance of the inconsistency problem: the derivation of ignorance implications brings with it the unwanted derivation of an inconsistent set of inferences. Section 4 presents two conceivable solutions to the inconsistency problem emerging from the literature (van Rooij & Schulz 2004, Spector 2006, Spector 2007), which are argued to predict inferences that, while avoiding inconsistency, are nevertheless too strong. Section 5 shows that restricting the derivation of Quantity implicatures with reference to Fox's (2007a) notion of innocent exclusion has the intended effect. Section 6 concludes with a few further comments on the argument presented.

2 The problem of inconsistency in Neo-Gricean pragmatics

2.1 The Standard Recipe

Recent literature (Gamut 1991, Sauerland 2004, Fox 2007a, Geurts 2010) presents an elaboration of the so-called Neo-Gricean account of Quantity implicature (Horn 1972) that serves as a common point of departure in subsequent work on this topic and that I will in the following refer to as the *Standard Recipe*.[2] Here I provide a compressed rendition of the Standard Recipe based on the definitions in (3), where $\Box\alpha$ conveys that the speaker believes α.

(3) i. $0_{p} = \{\Box p\}$

ii. $1_{p,A} = 0_{p} \cup \{\neg\Box q : q \in A \wedge q \subset p\}$

iii. $2_{p,A} = 1_{p,A} \cup \{\Box\neg q : \neg\Box q \in 1_{p,A}$ & $\Box\neg q$ is consistent with $1_{p,A}\}$

The Standard Recipe holds that an utterance of sentence ϕ with semantic content p and the so-called alternative set A will lead a listener to hold the set of assumptions $2_{p,A}$ in (3iii), which is is defined with reference to $1_{p,A}$ in (3ii), which is in turn defined with reference to $0_{p,A}$ in (3i). By (3ii), the set $1_{p,A}$ results from adding to 0_{p}, the singleton $\{\Box p\}$, the assumption $\neg\Box q$, for each

2 The term is borrowed from Geurts 2010.

q in A that is semantically stronger than p; according to (3iii), $2_{p,A}$ results from adding to $1_{p,A}$ the inference $\Box\neg q$, for each assumption $\neg\Box q$ in $1_{p,A}$ that meets the condition of being consistent with $1_{p,A}$, a condition that was first articulated in Sauerland 2004 and that, following Meyer 2013, I will sometimes refer to as a *consistency check*.[3] As noted, the Standard Recipe holds that an utterance ϕ of a statement with semantic content p will lead a listener to infer the set of assumptions in $2_{p,A}$. For the inferences in $1_{p,A}$, the Standard Recipe is grounded in familiar Gricean rationales based on the maxims of Quality and Quantity. I will in the following refer to $\Box p$ as a *Quality* inference (about p); adapting Sauerland's (2004) terminology, I will refer to $\neg\Box q$ as a *primary* (*Quantity*) inference (about q); and again adapting Sauerland's terminology, I will refer to $\Box\neg q$ as a *secondary* (*Quantity*) inference (about q). Note that a secondary inference $\Box\neg q$ is stronger than the corresponding primary inference $\neg\Box q$. Specifically, the secondary inference $\Box\neg q$ is equivalent to the conjunction of the primary inference $\neg\Box q$ with the assumption $\Box q\vee\Box\neg q$, which indicates, in the terminology of van Rooij & Schulz (2004), that the speaker is *competent* about q.

According to the Standard Recipe, then, the speaker's utterance of a statement ϕ with semantic content p will lead the listener to add to any prior assumptions about the speaker's beliefs, first, the Quality inference about p, and, second, primary Quantity inferences about the semantic meanings of all alternatives to ϕ that are semantically stronger than p. Moreover, the Standard Recipe holds that the listener will expand this set of inferences further by adding, for every primary inference about a proposition q already adopted, a secondary Quantity inference about q, provided this secondary inference is consistent with the Quality inference and the primary Quantity inferences triggered by the utterance of ϕ. The listener's addition of these secondary inferences amounts to the adoption, for each proposition about which the listener has already drawn a primary inference, the assumption that the speaker is competent about that proposition, provided that competence assumption is consistent with the Quality inference and the primary Quantity inferences associated with the utterance of ϕ. So the Standard

3 The alternatives in the set A referred to in (3) are propositions, that is, semantic objects encoding information content, rather than syntactic expressions in the object language. However, as will become clear in Section 3, semantic alternatives are the semantic values of alternate syntactic expressions. It will in fact be convenient to sometimes use the term *alternative* in a syntactic, rather than semantic sense. As well, while I assume the asserted meaning p to be a member of A, will sometimes use the term *alternative* informally as excluding p or the sentence expressing it. No confusion should arise from this.

Recipe effectively posits that the listener takes the speaker to be competent by default, a default that is overruled only by a failed consistency check.[4]

I will now illustrate the workings of the Standard Recipe by applying it to two archetypal examples, beginning with the bare numeral case (2), repeated below.

(2) Al hired two cooks.

In a classic view due to Horn (1972), statements with bare numerals are semantically weak in the sense that the numeral sets a lower bound but no upper bound. In this view, (2) expresses the proposition that Al hired more than one cook, here abbreviated as **[2,...)**. Suppose now, again following Horn (1972), that for the purposes of Quantity implicature, the alternative meanings to the proposition expressed by a bare numeral sentence are propositions whose content varies in the position of the numeral. In the case at hand, then, the set of alternative meanings is {**[n,...)**: n is a natural number}, hereafter abbreviated as $\{\mathbf{[n,...)}\}_{n\geq1}$. Introducing what I think is a helpful expository device, (4) depicts this set in a format that transparently reflects semantic strength relations between members, marking the asserted meaning with an asterisk.

(4)
$$\begin{array}{cccccl}
 & & & \ldots & \ldots & \\
 & & & \mathbf{[4} & \ldots & \mathbf{)} \\
 & & \mathbf{[3} & \mathbf{4} & \ldots & \mathbf{)} \\
 & \mathbf{[2} & \mathbf{3} & \mathbf{4} & \ldots & \mathbf{)}^{*} \\
\mathbf{[1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \ldots & \mathbf{)}
\end{array}$$

Assuming these alternatives, the three clauses of (3) give rise to the equalities in (5). I will sometimes refer to the three sets defined in such triplets of equalities as *set 0*, *set 1*, and *set 2*. According to the Standard Recipe, then, an utterance of (2) will lead the listener to draw all of the inferences in set 2 in (5).

4 The Standard Recipe as presented here is a simplification. First, the Standard Recipe should ultimately be restricted to derive inferences about alternatives that the listener considers relevant (e.g., Gamut 1991, Fox 2007a, Geurts 2010). The significance of this simplification in the present context is discussed briefly in Section 3. Second, the Standard Recipe as presented here (like its variants presented in Sections 4 and 5) leaves it open how the recipe applies in cases where a potential primary or secondary inference is inconsistent with the listener's assumptions prior to the speaker's utterance. I believe that the main issues I focus on below are independent of the answer to this question, and I will continue to set it aside in the following.

(5) i. $0_{[2,\ldots)} = \{\Box\mathbf{[2,\ldots)}\}$

ii. $1_{[2,\ldots),\{[n,\ldots)\}_{n\geq 1}} = \{\Box\mathbf{[2,\ldots)}\} \cup \{\neg\Box\mathbf{[n,\ldots)}\}_{n\geq 3}$

iii. $2_{[2,\ldots),\{[n,\ldots)\}_{n\geq 1}} = \{\Box\mathbf{[2,\ldots)}\} \cup \{\neg\Box\mathbf{[n,\ldots)}\}_{n\geq 3} \cup \{\Box\neg\mathbf{[n,\ldots)}\}_{n\geq 3}$

The conjunction of all the Quantity inferences in set 2 amounts to $\Box\neg\mathbf{[3,\ldots)}$, which together with the Quality inference $\Box\mathbf{[2,\ldots)}$ yields $\Box\mathbf{[2]}$. Assuming the listener considers the inferred beliefs of the speaker to be correct, the listener will moreover arrive at a set of *bottom line* inferences, viz., $\mathbf{[2\ldots)}$ and $\neg\mathbf{[3\ldots)}$, and hence $\mathbf{[2]}$.[5] So the Standard Recipe accounts for the finding that (2) is typically understood in the two-sided interpretation expressing that Al hired exactly two cooks.

As a second illustration of the Standard Recipe, I will apply it to the disjunctive sentence in (6), following in the footsteps of Sauerland (2004). Assuming *or* to be semantically inclusive, (6) semantically expresses $\mathbf{b}\vee\mathbf{c}$, the proposition that Bill or Sue or both called. Sauerland (2004) proposed that for the purposes of Quantity implicature, the alternatives to a disjunctive sentence include not only the corresponding conjunction (Horn 1972), but also the individual disjuncts. For the case at hand, the semantic meanings of these alternatives are displayed in (7) (where, as before, stronger alternatives are displayed above weaker alternatives).

(6) Bill applied or Carol applied.

(7)
$$\begin{array}{c} \mathbf{b}\wedge\mathbf{c} \\ \mathbf{b} \quad \mathbf{c} \\ \mathbf{b}\vee\mathbf{c}^{*} \end{array}$$

Given these alternatives, the Standard Recipe holds that based on an utterance of (6), the listener will draw the inferences in set 2 in (8).

(8) i. $0_{\mathbf{b}\vee\mathbf{c}} = \{\Box\mathbf{b}\vee\mathbf{c}\}$

ii. $1_{\mathbf{b}\vee\mathbf{c},\{\mathbf{b}\vee\mathbf{c},\,\mathbf{b},\,\mathbf{c},\,\mathbf{b}\wedge\mathbf{c}\}} = \{\Box\mathbf{b}\vee\mathbf{c}, \neg\Box\mathbf{b},\ \neg\Box\mathbf{c},\ \neg\Box\mathbf{b}\wedge\mathbf{c}\}$

iii. $2_{\mathbf{b}\vee\mathbf{c},\{\mathbf{b}\vee\mathbf{c},\,\mathbf{b},\,\mathbf{c},\,\mathbf{b}\wedge\mathbf{c}\}} = \{\Box\mathbf{b}\vee\mathbf{c}, \neg\Box\mathbf{b},\ \neg\Box\mathbf{c},\ \neg\Box\mathbf{b}\wedge\mathbf{c},\ \Box\neg\mathbf{b}\wedge\mathbf{c}\}$

So the Standard Recipe derives the secondary inference $\Box\neg\mathbf{b}\wedge\mathbf{c}$ in set 2, and hence the familiar bottom line inference $\neg\mathbf{b}\wedge\mathbf{c}$. Set 2 also contains two primary inferences, viz., $\neg\Box\mathbf{b}$ and $\neg\Box\mathbf{c}$, without containing the corresponding secondary inferences, i.e., $\Box\neg\mathbf{b}$ and $\Box\neg\mathbf{c}$. This is in accordance with the definition of $2_{p,A}$ in (3iii), due to the entailments in (9): in conjunction with the

5 Bottom line inferences are what van Rooij & Schulz (2004) refer to as *factive inferences*.

Quality inference $\Box\mathbf{b}\vee\mathbf{c}$, $\neg\Box\mathbf{b}$ and $\neg\Box\mathbf{c}$ entail the *possibility* implications $\neg\Box\neg\mathbf{b}$ and $\neg\Box\neg\mathbf{c}$.

(9) $$\frac{\Box\mathbf{b}\vee\mathbf{c},\ \neg\Box\mathbf{b},\ \neg\Box\mathbf{c}}{\neg\Box\neg\mathbf{b},\ \neg\Box\neg\mathbf{c}}$$

Given this, each of the potential secondary inferences $\Box\neg\mathbf{b}$ and $\Box\neg\mathbf{c}$ is inconsistent with set 1. These potential secondary inferences, then, do not pass the consistency check in (3iii), and so the Standard Recipe does not predict that the listener will actually draw these inferences.

To be sure, (6) can not be understood as implying (that the speaker believes) that Bill did not apply or that Carol did not apply. Instead, the sentence suggests the speaker does not know whether Bill applied and also does not know whether Carol applied. This pair of ignorance implications is also accounted for under the Standard Recipe. It falls out from the primary inferences $\neg\Box\mathbf{b}$ and $\neg\Box\mathbf{c}$ in conjunction with the possibility entailments in (9): $\neg\Box\mathbf{b}\wedge\neg\Box\neg\mathbf{b}$ conveys that the speaker fails to know whether Bill applied, and likewise for $\neg\Box\mathbf{c}\wedge\neg\Box\neg\mathbf{c}$. Note that an ignorance implication $\neg\Box\mathbf{q}\wedge\neg\Box\neg\mathbf{q}$ is the negation of the competence assumption $\Box\mathbf{q}\vee\Box\neg\mathbf{q}$. For the individual disjuncts of a disjunctive sentence, then, the Standard Recipe derives ignorance implications that preempt the competence assumptions that would amount to the corresponding secondary inferences.

Note that (9) holds by virtue of the logical relation between the relevant alternatives: $\mathbf{b}$ and $\mathbf{c}$ are both stronger than $\mathbf{b}\vee\mathbf{c}$ and jointly exhaust the logical space carved out by $\mathbf{b}\vee\mathbf{c}$, in the sense that $\mathbf{b}\vee\mathbf{c}$ entails the disjunction of $\mathbf{b}$ and $\mathbf{c}$. It is indeed a general feature of the Standard Recipe that it derives an ignorance implication about an alternative stronger than the asserted meaning (if and) only if that alternative pairs up with another strong alternative to jointly exhaust the logical space given by the semantic meaning of the assertion. Adopting Fox's (2007a) terminology I refer to any pair of alternatives that relate to the asserted meaning in this way as *symmetric*. This sort of symmetry, or its absence, plays a central role in the following.

Before proceeding, it will be useful to summarize, and further extend, the terminological conventions introduced above. I have used the terms (*Gricean*) *inference* and *implicature* interchangeably, and I will continue to assume that either term applies to information content that is assumed to have a pragmatic derivation grounded in Gricean maxims. I will continue to talk more specifically about *Quality* inferences in the sense introduced above, and likewise about *primary* (Quantity) inferences (inferences of the form

$\neg\Box\alpha$), *secondary* (*Quantity*) inferences ($\Box\neg\alpha$), and inferences of *possibility* ($\neg\Box\neg\alpha$), *competence* ($\Box\alpha\vee\Box\neg\alpha$) and *ignorance* ($\neg\Box\alpha\wedge\neg\Box\neg\alpha$). I will refer to a given information content as an *implication* to remain neutral about whether it is to be analyzed as a Gricean inference or as part of semantic meaning. I will furthermore refer to an implication as an *epistemic implication* if it is of the form $\Box\alpha$ (or if it is a conjunction or disjunction of implications of this form) and as a *bottom line implication* otherwise. Finally, I will also use *Quantity implication* as a neutral term to refer to information content that under the Standard Recipe is credited to a Quantity inference or its bottom line, and I will take the terms *primary* and *secondary implication* etc. to be understood accordingly.

2.2 Inconsistency under the Standard Recipe

An observation reported in Spector 2006 and Fox 2007a is that there are conceivable configurations of asserted meaning and alternatives for which the Standard Recipe delivers inconsistent sets of inferences. Spector (2006) illustrates this point with the hypothetical case in (10), where **b**, **c**, and **d** are to be understood as above (hence, in particular, are taken not to be related by entailment). For this case, (3) produces the set of inferences in (11).

(10) $\mathbf{b}\quad \mathbf{c}\quad \mathbf{d}$

$\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}^{*}$

(11) i. $0_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}\}$

ii. $1_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\,\mathbf{b},\,\mathbf{c},\,\mathbf{d}\}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}, \neg\Box\mathbf{c}, \neg\Box\mathbf{d}\}$

iii. $2_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\,\mathbf{b},\,\mathbf{c},\,\mathbf{d}\}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}, \neg\Box\mathbf{c}, \neg\Box\mathbf{d}, \Box\neg\mathbf{b}, \Box\neg\mathbf{c}, \Box\neg\mathbf{d}\}$

Here set 2 includes a secondary inference about each of the three alternatives. This is in accordance with (3) because each of the three inferences individually passes the consistency check in (3iii). The reason for this is that none of the alternatives in question has a symmetric partner, hence no ignorance inference are generated to preempt the relevant secondary inferences. However, despite the lack of symmetry and the concomitant absence of ignorance inferences, the inferences in set 1 jointly entail a family of possibility implications, as recorded in (12).

(12)
$$\frac{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\ \neg\Box\mathbf{b},\ \neg\Box\mathbf{c},\ \neg\Box\mathbf{d}}{\neg\Box\neg\mathbf{c}\vee\mathbf{d},\ \neg\Box\neg\mathbf{b}\vee\mathbf{d},\ \neg\Box\neg\mathbf{b}\vee\mathbf{c}}$$

Each of the three entailed possibility implications is inconsistent with a two-membered subset of the secondary inferences listed in set 2, viz., {$\Box\neg\mathbf{b}$, $\Box\neg\mathbf{c}$}, {$\Box\neg\mathbf{c}$, $\Box\neg\mathbf{d}$}, and {$\Box\neg\mathbf{b}$, $\Box\neg\mathbf{d}$}, respectively. Set 2 as a whole, therefore, turns out to be inconsistent as well. In fact, inconsistency in set 2 arises even when disregarding primary inferences, since {$\Box\neg\mathbf{b}$, $\Box\neg\mathbf{c}$, $\Box\neg\mathbf{d}$}, the full set of secondary inferences in set 2, is inconsistent with the Quality inference $\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}$.

The finding that the Standard Recipe derives contradictory inferences for seemingly inconspicuous configurations like (10) may be disconcerting. But in order to develop this finding into an empirical challenge to the Standard Recipe, it would be necessary to identify an actual utterance that can be argued to instantiate the problematic configuration and yet shows no sign of contradiction. Indeed, Spector (2006) portrays the case in (10) as a conceptual, rather than empirical, problem for the Standard Recipe. In particular, as Spector notes, while the obviously non-contradictory (13) plausibly has the semantic meaning $\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}$, this observation could be rendered consistent with the Standard Recipe by arguing that the alternative set in this case is not the set of propositions expressed by the three individual disjuncts. Such a proposal has been presented in Sauerland 2004, and will be reviewed in the next section.

(13) Bill applied or Carol applied or Dan applied.

However, Fox 2007a presents a possible instantiation of a case much like (13), introducing the issue of inconsistency as an actual empirical challenge to the Standard Recipe. Fox suggests that in a dialogue like (14), the alternative set for B's response in the context of A's question is the set of propositions {that x applied: x is a person or a set of people}.

(14) A: Who applied?
B: Some cook.

Assuming for purposes of illustration that Bill, Carol, and Dan are all the people in the domain and are all cooks, (13) instantiates the configuration in (15), which can be obtained from (11) through closure under conjunction.[6]

6 As Fox (2007a) notes (with respect to a parallel example), the answer in (14) is odd under the assumption that there are no non-cooks, presumably because in that case the answer merely states a presupposition of the question. However, as in Fox's exposition, this assumption is added here merely to limit the size of the alternative set under consideration, and of the diagram in (15). The point made here does not change if alternatives based on non-cooks are added to the picture.

(15)
$$\begin{array}{c} \mathbf{b{\wedge}c{\wedge}d} \\ \mathbf{b{\wedge}c} \quad \mathbf{b{\wedge}d} \quad \mathbf{c{\wedge}d} \\ \mathbf{b} \quad \mathbf{c} \quad \mathbf{d} \\ \mathbf{b{\vee}c{\vee}d}^{*} \end{array}$$

Given that the set of alternative in (15) includes the set in (11) without introducing any symmetry, the set of inferences that (3) derives for (15) includes the set derived for (11); and since the latter was shown to be inconsistent, it follows that the former is inconsistent as well. To be sure, however, no inconsistency is actually perceived in the exchange in (14). Therefore, if such cases indeed fall under the purview of the theory of Quantity implicature, they present an empirical challenge to the Standard Recipe.

Notably, existing literature does not seem to explore the consequences of this type of challenge for Neo-Gricean pragmatics on the basis of empirical evidence. Fox (2007a) solves the problem of inconsistency presented by (14) and (15) within the so-called Grammatical Theory of Implicature, which is designed to replace Neo-Gricean pragmatics. However, Fox's move to the Grammatical Theory is not actually motivated by the problem of inconsistency in cases like (14) (but by the independent problem of free choice effects). The question therefore remains open how within the Neo-Gricean setting, the Standard Recipe might be amendable so as to solve the problem of inconsistent inference sets.

The case study presented below addresses this question. It does so with reference to the interpretation of *at least*, rather than indefinites in question-answer dialogues such as (14). In a first step, it will be established that a proper derivation of ignorance implications with *at least* under the Standard Recipe again runs into the problem of inconsistency. In a second step, it is argued that within the Neo-Gricean approach, the meaning contribution of *at least* calls for an amendment of the Standard Recipe that makes reference to the notion of innocent exclusion, a notion that for independent reasons, Fox 2007a introduced in the context of the Grammatical Theory of Implicature.

3 *At least* and inconsistency

As noted in the introduction, *at least* introduces an implication of speaker ignorance (Krifka 1999, Büring 2008). Sentence (1) conveys that the speaker is uncertain about the exact number of cooks Al hired.

(1) Al hired at least two cooks.

Under the Standard Recipe, ignorance implications are due to symmetry in the set of alternatives. The Standard Recipe, then, offers a straightforward approach to the meaning of *at least*, based on the postulation of suitable symmetric alternatives, an account explored in Schwarz & Shimoyama 2011 and Mayr 2013. However, the approach is shown below to lead to another instance of the problem of inconsistent inference sets.

3.1 Symmetric alternatives stipulated

Suppose that *at least* modifying a numeral does not affect the semantic meaning of its host sentence, and suppose again that numerals yield weak propositions that do not impose upper bounds. Sentence (1) then has the semantic meaning **[2,...)**. Suppose moreover that, as depicted in (16), the alternatives to **[2,...)** are **[2]** and **[3,...)**, the proposition that Al hired exactly two cooks and the proposition that he hired more than two. Crucially, the two alternatives are not only stronger than **[2,...)**, but they also form a symmetric pair, as **[2,...)** entails the disjunction of **[2]** and **[3,...)**.

(16)
$$\begin{array}{lllll} [2] & [3 & 4 & \ldots &) \\ [2 & 3 & 4 & \ldots &)^{*} \end{array}$$

Assuming (16), the application of the definitions in (3) to sentence (1) is analogous to the case of disjunction reviewed in Section 2. The resulting inferences are shown in (17). Due to symmetry, the set 1 inferences entail the pair of possibility implications shown in (18) below, hence entail the ignorance implications $\neg\Box\mathbf{[2]}\wedge\neg\Box\neg\mathbf{[2]}$ and $\neg\Box\mathbf{[3,\ldots)}\wedge\neg\Box\neg\mathbf{[3,\ldots)}$, the inference that the speaker does not know whether Al hired exactly two cooks and the inference that the speaker does not know whether Al hired more than two cooks. Notably, these are precisely the ignorance implications that Büring (2008) posited for cases like (1), and that are shared by disjunctive sentences like *Al hired two or more cooks*. The two ignorance implications are inconsistent with the competence assumptions $\Box\mathbf{[2]}\vee\Box\neg\mathbf{[2]}$ and $\Box\mathbf{[3,\ldots)}\vee\Box\neg\mathbf{[3,\ldots)}$, whose adoption would be tantamount to adopting the secondary inferences $\Box\neg\mathbf{[2]}$ and $\Box\neg\mathbf{[3,\ldots)}$. These potential secondary inferences are accordingly not actually derived, hence absent from set 2.

(17) i. $0_{[2,\ldots)} = \{\Box\mathbf{[2,\ldots)}\}$

ii. $1_{[2,\ldots),\ \{[2,\ldots),\ [2],\ [3,\ldots)\}} = \{\Box\mathbf{[2,\ldots)},\ \neg\Box\mathbf{[2]},\ \neg\Box\mathbf{[3,\ldots)}\}$

iii. $2_{[2,\ldots),\ \{[2,\ldots),\ [2],\ [3,\ldots)\}} = \{\Box\mathbf{[2,\ldots)},\ \neg\Box\mathbf{[2]},\ \neg\Box\mathbf{[3,\ldots)}\}$

(18) $$\frac{\Box\mathbf{[2,\ldots)},\ \neg\Box\mathbf{[2]},\ \neg\Box\mathbf{[3,\ldots)}}{\neg\Box\neg\mathbf{[2]},\ \neg\Box\neg\mathbf{[3,\ldots)}}$$

So, with the alternatives in (16) stipulated, the Standard Recipe straightforwardly accounts for the two central observations about (1) reported at the outset: (1) lacks the upper-bounding bottom line implication $\neg\mathbf{[3,\ldots)}$ typically associated with bare numerals; and (1) instead conveys that the speaker is uncertain about the exact number of cooks Al hired. Moreover, under the Standard Recipe the absence of an upper-bounding implication and the presence of ignorance implications are intimately related, the former being preempted by the latter, as proposed in Büring 2008. The analysis can also be viewed as explicating Büring's proposal that for the purposes of Quantity implicature calculation, *at least* sentences behave like disjunctions, even though the analysis does not require that *at least* statements are in any way disjunctive at a syntactic level.[7]

Finally, the analysis makes a prediction, highlighted in Schwarz & Shimoyama 2011 and Mayr 2013, about cases where *at least* appears under a universal operator, such as (19). Given the alternatives in (20) (where $\forall\mathbf{[2,\ldots)}$ is the proposition that every manager hired hired more than one cook, etc.), (3) derives for this example the equalities in (21).

(19) Every manager hired at least two cooks.

(20) $\forall\mathbf{[2]}\quad \forall\mathbf{[3\quad 4\quad \ldots\ )}$
$\forall\mathbf{[2\quad 3\quad 4\quad \ldots\ )}^{*}$

(21)
i. $0_{\forall[2,\ldots)} = \{\Box\forall\mathbf{[2,\ldots)}\}$
ii. $1_{\forall[2,\ldots),\{\forall[2,\ldots),\ \forall[2],\ \forall[3,\ldots)\}} = \{\Box\forall\mathbf{[2,\ldots)},\ \neg\Box\forall\mathbf{[2]},\ \neg\Box\forall\mathbf{[3,\ldots)}\}$
iii. $2_{\forall[2,\ldots),\{\forall[2,\ldots),\ \forall[2],\ \forall[3,\ldots)\}} = \{\Box\forall\mathbf{[2,\ldots)},\ \neg\Box\forall\mathbf{[2]},\ \neg\Box\forall\mathbf{[3,\ldots)},\ \Box\neg\forall\mathbf{[2]},\ \Box\neg\forall\mathbf{[3,\ldots)}\}$

Crucially, the propositions $\forall\mathbf{[2]}$ and $\forall\mathbf{[3,\ldots)}$ are not symmetric relative to $\forall\mathbf{[2,\ldots)}$. That is, $\forall\mathbf{[2,\ldots)}$ does not entail the disjunction of $\forall\mathbf{[2]}$ and $\forall\mathbf{[3,\ldots)}$. After all, it is possible for $\forall\mathbf{[2,\ldots)}$ to be true by virtue of the fact that that some managers hired exactly two cooks, while the others hired more than two. That is, the universal operator breaks the symmetry that would otherwise obtain. As a consequence, the propositions in set 1 in this case do not entail

7 The analysis thereby addresses a concern raised in Coppock & Brochhagen 2013, who note that there is nothing disjunctive about the syntactic shape of *at least*, and who reject Büring's approach by arguing that his likening of *at least* to a disjunction cannot be developed into a coherent theory.

ignorance implications about the alternatives $\forall$**[2]** and $\forall$**[3,...)**. Hence nothing preempts the competence assumptions about them, and so set 2 includes the corresponding secondary inferences $\Box\neg\forall$**[2]** and $\Box\neg\forall$**[3,...)**.

So the Standard Recipe does not derive ignorance inferences for (19), hence it predicts an utterance of (19) to be consistent with the speaker having full information about the number of cooks every manager hired; moreover, (19) is predicted to yield the inferences that not not every manager hired exactly two cooks and that not every manager hired more than two cooks. As noted in Schwarz & Shimoyama 2011 and Mayr 2013, these predictions seem correct.[8]

The finding that ignorance implications with *at least* are obviated under universal operators, with secondary implications emerging instead, establishes a notable parallel with disjunction, for which Fox (2007a) reports analogous effects correctly predicted under the Standard Recipe. Obviation of ignorance implications under universal operators can in fact be considered a signature of ignorance effects due to Quantity implications. Under an analysis of *at least* in terms of the Standard Recipe, it is therefore significant that such obviation is indeed attested for *at least*. Obviation of ignorance implications under universals also sets an analysis of *at least* in terms Quantity implications apart from certain alternative accounts. In particular, the attested interpretation (19) is hard to reconcile with the analyses of *at least* offered in Geurts & Nouwen 2007, Nouwen 2010, and Penka 2010. This suggests, on the one hand, that an analysis of *at least* in terms of Quantity implications is at least a contender, and on the other hand, that the repercussions of such an analysis for theory of Quantity implications are indeed worth exploring.

However, as presented so far, the analysis remains incomplete. While the Standard Recipe has been shown to have the intended effect once the alternatives in (16) are stipulated, what is required for a complete account is a general theory of alternatives that would derive a proper alternative set for (1). This is the issue that I turn to next.

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8 Büring (2008) presented analogous effects for cases where *at least* appears embedded below a deontic necessity modal, permitting what Büring dubbed an *authoritative reading*. Schwarz & Shimoyama (2011) suggested that ignorance implications are more generally obviated under all universal operators, and they note that this effect is indeed predicted under the Standard Recipe. Mayr (2013) presents data showing that ignorance implications with *at least* are also obviated under a range of non-universal operators. All of the operators that Mayr discusses break symmetry, and hence their obviating effect, too, falls out from the Standard Recipe.

3.2 Deriving the alternative set

Horn (1972) proposed that alternatives for the purposes of Quantity implicature are generated by a method of syntactic substitution. In Horn's approach, alternatives are generated from the logical form of the asserted statement by replacing certain lexical elements in that logical form with other lexical elements. The space of possible substitutions is moreover regulated based on a stipulated set of families of lexical items, the so-called Horn scales: a substitution of one lexical item for another is only permitted if the two are members of the same Horn scale.

One of the Horn scales that Horn posited is the set of numerals in (22). Given this Horn scale, substitution of scale mates for *two* in sentence (2) delivers the set $\{\mathbf{[n,\ldots)}\}_{n\geq 1}$ displayed in (4), the infinite set of alternatives to $\mathbf{[2,\ldots)}$ assumed in the analysis of (2) in Section 2.

(22) Horn scale: {*one*, *two*, *three*, ...}

(2) Al hired two cooks.

(4)
$$\begin{array}{cccccc}
 & & & \ldots & \ldots & \\
 & & & \mathbf{[4} & \ldots & \mathbf{)} \\
 & & \mathbf{[3} & \mathbf{4} & \ldots & \mathbf{)} \\
 & \mathbf{[2} & \mathbf{3} & \mathbf{4} & \ldots & \mathbf{)}^{*} \\
\mathbf{[1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \ldots & \mathbf{)}
\end{array}$$

Turning to example (1), Krifka (1999) suggested that *at least* affects the alternative set for its host sentence by blocking the projection of alternatives from its scope. Krifka proposed that, as a consequence, *at least* sentences like (1) are not associated with alternatives at all, and that this absence of alternatives is responsible for the absence of upper bounding inferences.

(1) Al hired at least two cooks.

However, as Büring (2008) noted, this analysis leaves ignorance implications unaccounted for. It is also challenged by the observation that *at least* sentences under certain circumstances give rise to other Quantity inferences as well, as was demonstrated with reference the universal quantifier case in (19) above.

A conceivable amendment to the analysis in Krifka 1999 emerges from proposals in Cummins & Katsos 2010, Kennedy 2015, and Mayr 2013, who suggest that *at least* is itself a scalar element, and in particular is available for substitution by the comparative operator *more than*. Substitution of *more than* for *at least*, as in (23a), then straightforwardly delivers for (1) the intended alternative **[3,...)**. Similarly, Mayr (2013) proposes that *at least* can also be replaced with the numeral modifier *exactly*, as in (23b), which accounts for the alternative **[2]**. So the Horn scale in (24) straightforwardly delivers for (1) the intended alternative set in (16).

(23) a. Al hired more than two cooks.
b. Al hired exactly two cooks.

(24) Horn scale: {*at least*, *exactly*, *more than*}

(16) **[2] [3 4 ...)**
[2 3 4 ...)*

However, while such a "one-scale" analysis derives the intended set of alternatives and inferences for examples like (1) without reference to the Horn scale of numerals, it fails to have the intended effect in the general case. As emphasized in Krifka 1999 and Coppock & Brochhagen 2013, *at least* is focus sensitive and its syntactic distribution resembles that of other focus sensitive particles. One particular manifestation of this, which will serve to illustrate the limitations of the one-scale approach, is the ability of *at least* to associate with a (focused) numeral at a distance. This is illustrated by (25), where *at least* can be read as associating with the numeral across a possibility modal.

(25) Al is at least allowed to hire two cooks.

In analogy to (1), an utterance of (25) can be read as conveying that the speaker is uncertain about whether Al is allowed to hire more than two cooks. Writing ◇ to express deontic possibility, under the Standard Recipe this ignorance implication indicates that ◇**[2,...)**, the semantic meaning of (25), has the symmetric alternatives ◇**[2]**∧¬◇**[3,...)** and ◇**[3,...)**, the propositions that Al is only allowed to hire two cooks and that he is allowed to hire more than two. A problem for a one-scale analysis relying only on (24) therefore arises from the fact, reported in Geurts & Nouwen 2007, that the distribution of *at least* differs greatly from that of comparative operators like *more than*. In particular, the result of substituting *more than* for *at least* in (25), shown in (26), is ungrammatical. A one-scale analysis based on (24), then, seems to leave the alternative ◇**[3,...)** unaccounted for.

(26) *Al is more than allowed to hire two cooks.

So the one-scale analysis examined here fails to generate the full set of alternatives needed to consistently capture the meaning contribution of *at least*. There also does not seem to be any other potential Horn scale mate for *at least* that could be posited to recover the intended effect of *more than* in cases where *more than* is unavailable. Therefore, the one-scale analysis does not seem viable.[9]

By way of elimination, this leads to a "two-scale" analysis of the sort envisioned in Schwarz & Shimoyama 2011 and Mayr 2013. This analysis abandons Krifka's (1999) proposal that *at least* blocks the projection of alternatives. Based on the Horn scale of numerals in (22), the alternative ◇**[3,...)** can then be attributed to the grammatical statement in (27), obtained from (25) by substituting the numeral *three* for its scale mate *two*. Likewise, (28) replaces (23a) in the analysis of (1).

(27) Al is at least allowed to hire three cooks.

(28) Al hired at least three cooks.

The two-scale analysis, then, relies on substitutions from the Horn scales of both *at least* and the numeral. Accordingly, since the numeral Horn scale covers the intended effect of the comparative operator *more than*, that operator can now safely be expunged from the Horn set for *at least*, so that (24) is replaced by (29).

(29) Horn scale: {*at least*, *exactly*}

For sentence (1), the two-scale analysis accounts for the pair of symmetric alternatives intended to derive the ignorance implications posited in Büring 2008 and derived in Section 3.1. However, it delivers many additional alternatives, given the assumption that substitutions from different scales can combine in the derivation of alternatives, an assumption made explicit and motivated in Sauerland 2004. That is, the predicted set of alternatives propositions is not {**[2,...)**, **[2]**, **[3,...)**}, displayed in (16), but the larger set $\{\mathbf{[n]}, \mathbf{[n{+}1,...)}\}_{n\geq 1}$, shown in (30). This is the type of alternative set for *at least* sentences derived in Schwarz & Shimoyama (2011), and scrutinized in Mayr (2013).

9 The assumption underlying this conclusion is that unacceptability, at least unacceptability of sort attested in (26), excludes the corresponding logical form from the set of alternatives. See Meyer (2015) for a defense of the assumption that "unassertable" logical forms do not serve as alternatives for the purposes of implicature calculation.

(30)
$$\begin{array}{llllll}
 & & & \textbf{...} & \textbf{...} & \\
 & & \textbf{[3]} & \textbf{[4} & \textbf{...} & \textbf{)} \\
 & \textbf{[2]} & \textbf{[3} & \textbf{4} & \textbf{...} & \textbf{)} \\
\textbf{[1]} & \textbf{[2} & \textbf{3} & \textbf{4} & \textbf{...} & \textbf{)}^{*} \\
\textbf{[1} & \textbf{2} & \textbf{3} & \textbf{4} & \textbf{...} & \textbf{)}
\end{array}$$

The set of alternatives in (30) that are stronger than the asserted meaning **[2,...)** properly includes the set of strong alternatives in (16). The obvious question, then, is what the expected observable effect of this surplus of alternatives might be under the Standard Recipe. I will address this question in the next subsection.

Before moving on, though, I attend to a challenge to the particular two-scale analysis presently entertained, a challenge that parallels the objection levelled above against the one-scale analysis. It turns out that the syntactic distribution of *exactly* tracks that of *at least* no more than *more than* does. In particular, the result of substituting *exactly* for *at least* in (25), shown in (31), is no more acceptable than (26) above is. Therefore, even a two-scale analysis based on (22) and (29) faces the problem that some alternatives, here ⟐**[2]**∧¬⟐**[3,...)**, remain unaccounted for.

(31) *Al is exactly allowed to hire two cooks.

However, a promising solution to this problem presents itself within the two-scale approach to alternatives for *at least* sentences: *exactly* can be replaced with the exclusive particle *only*, substituting (32) for (29).

(32) Horn scale: {*at least*, *only*}

This substitution has the intended effect for all the examples discussed so far. The proposition **[2]**, so far attributed to (23b), can be credited to the sentence (33) instead. And, solving the problem just identified, the alternative ⟐**[2]**∧¬⟐**[3,...)** in the analysis of (25) can be attributed to (34), which is indeed judged to have the requisite interpretation.[10]

(33) Al only hired two cooks.

10 In the classic analysis of Horn (1969), the implication of (33) that Al hired more than one cook is presupposed, rather than asserted. If so, (33) does not recover the exact semantic content of (23b). However, the purported presuppositional content posited under Horn's analysis often fails to impose the expected conditions on the felicitous use of sentences with *only* (see e.g., Beaver & Clark 2008). This seems to be true in particular for cases where *only* associates with a numeral: felicitous uses of (33) do not seem to be restricted to common grounds that entail that Al hired more than one cook. So it is not clear that the purported

(34) Al is only allowed to hire two cooks.

This amendment to the two-scale analysis renders the unacceptability of (31) inconsequential. I conclude that the proposed two-scale analysis is not threatened by the mere observation that *at least* can associate with a numeral at a distance.

3.3 Inconsistency

The conclusion reached above is that there is no proper theory of alternatives that for sentence (1) delivers the target configuration in (16), deriving the intended pair of symmetric alternatives and no others. A proper theory of alternatives instead derives the larger set in (30).

(30)
$$\begin{array}{llllll}
 & & & \textbf{...} & \textbf{...} & \\
 & & \textbf{[3]} & \textbf{[4} & \textbf{...} & \textbf{)} \\
 & \textbf{[2]} & \textbf{[3} & \textbf{4} & \textbf{...} & \textbf{)} \\
\textbf{[1]} & \textbf{[2} & \textbf{3} & \textbf{4} & \textbf{...} & \textbf{)}^{*} \\
\textbf{[1} & \textbf{2} & \textbf{3} & \textbf{4} & \textbf{...} & \textbf{)}
\end{array}$$

I now turn to examining the inferences that the Standard Recipe predicts a listener to draw on the basis of this larger set. The definitions in (3) that underly the under the Standard Recipe can be shown to give rise to the equalities in (35). These equalities are to be compared with those in (17) above.

(35) i. $0_{\mathbf{[2,...)}} = \{\Box\mathbf{[2,...)}\}$

ii. $1_{\mathbf{[2,...)},\{\mathbf{[n,...)},\,\mathbf{[n]}\}_{n\geq 1}} = \{\Box\mathbf{[2,...)},\ \neg\Box\mathbf{[2]},\ \neg\Box\mathbf{[3,...)}\} \cup \{\neg\Box\mathbf{[n]},\ \neg\Box\mathbf{[n+1,...)}\}_{n\geq 3}$

iii. $2_{\mathbf{[2,...)},\{\mathbf{[n,...)},\,\mathbf{[n]}\}_{n\geq 1}} = \{\Box\mathbf{[2,...)},\ \neg\Box\mathbf{[2]},\ \neg\Box\mathbf{[3,...)}\} \cup \{\neg\Box\mathbf{[n]},\ \neg\Box\mathbf{[n+1,...)}\}_{n\geq 3} \cup \{\Box\neg\mathbf{[n]},\ \Box\neg\mathbf{[n+1,...)}\}_{n\geq 3}$

presuppositionality of *only* prevents it from serving the role assigned to it in the proposed two-scale analysis. At the same time, the particular Horn scale in (32) is not a necessary component of a proper two-scale analysis. In an alternative version, *only* is replaced with the silent exhaustivity operator *Exh* posited in Fox 2007a and much subsequent work. Yet another option, hinted at in Krifka 1999 and recently spelled out in Kennedy 2015, is that the upper bounding implication contributed by *only* in (33) could instead be attributed to the semantics of the numeral itself. (Schwarz & Shimoyama (2011) pursue a version of Kennedy's approach in the analysis of Japanese data.) However, it is unclear how Kennedy's proposal might extend to cases where *at least* associates with non-numerals, such as *Al fired at least the cook* or *Al is at least an Assistant Professor.*

The first point to be made is that the sets of primary Quantity inferences in (35) include the corresponding sets of primary inferences in (17). This crucially ensures that, given the entailment in (18), the ignorance implications **¬□[2]∧¬□¬[2]** and **¬□[3,...)∧¬□¬[3,...)** continue to be accounted for.

(18) $$\frac{\square[2,\ldots),\ \neg\square[2],\ \neg\square[3,\ldots)}{\neg\square\neg[2],\ \neg\square\neg[3,\ldots)}$$

But (35) evidently also lists Quantity inferences not listed in (17). Apart from the Quality implicature **□[2,...)** and the primary inferences **¬□[2]** and **¬□[3,...)**, set 1 in (35) includes primary inferences that do not appear in (17). There is one additional primary inference in (35) for each of the additional alternatives stronger than **[2,...)**, forming the set $\{\neg\square\mathbf{[n]}, \neg\square\mathbf{[n+1,...)}\}_{n\geq 3}$. These additional primary inferences, though, are not expected to be directly detectable, as they are already entailed by the the primary inference **¬□[3,...)**.

Again in contrast to (17), set 2 in (35) also includes secondary inferences. These form the set $\{\square\neg\mathbf{[n]}, \square\neg\mathbf{[n+1,...)}\}_{n\geq 3}$, which includes one secondary inference for each of the additional alternatives stronger than **[2,...)**. The presence of these secondary inferences stands in contrast with the absence of the potential secondary inference **□¬[2]** and **□¬[3,...)**. This contrast relates to the fact that (3) fails to produce ignorance implications that would preempt the competence assumptions corresponding to the secondary inferences in $\{\square\neg\mathbf{[n]}, \square\neg\mathbf{[n+1,...)}\}_{n\geq 3}$. And the reason for the failure to generate ignorance implications has to do with the way the additional alternative meanings relate to the asserted meaning **[2,...)** and to each other: none of the alternatives in $\{\mathbf{[n]}, \mathbf{[n+1,...)}\}_{n\geq 3}$ forms a symmetric pair relative to **[2,...)** together with another alternative. So, the fact that the Standard Recipe derives a secondary inference about each of the alternatives in $\{\mathbf{[n]}, \mathbf{[n+1,...)}\}_{n\geq 3}$ is a direct consequence of the fact that none of these alternatives is symmetric to any other alternative.

To illustrate this lack of symmetry for a concrete case, note that the disjunction of **[4,...)** with any one of the alternatives in (30) stronger than **[2,...)** is again stronger than **[2,...)**, hence not entailed by **[2,...)**. The definitions in (3) therefore fail to generate the ignorance implication **¬□[4,...)∧¬□¬[4,...)**, hence fail to preempt the competence assumption **□[4,...)∨□¬[4,...)**. So the secondary inference **□¬[4,...)** passes the consistency check in (3iii), and therefore the Standard Recipe derives this secondary inference and its bottom line **¬[4,...)**.

So given the set of alternatives in (30) and given the asserted meaning **[2,...)**, the Standard Recipe predicts (1) to imply that Al hired exactly two or exactly three cooks. In fact, in Mayr's (2013) assessment (transferred to the case at hand), (1) is predicted to mean that Al hired exactly two or exactly three cooks. Mayr judges this interpretation to be unavailable, and this leads Mayr to abandon an analysis of *at least* within the Standard Recipe, and to pursue an alternative account.[11]

Mayr is certainly correct that under the Standard Recipe, the alternatives in (30) support an unwanted inference. However, Mayr's characterization of the problem is incomplete, in view of the fact that the above argument for **[4,...)** can be replicated for any other alternative in $\{$**[n]**, **[n+1,...)**$\}_{n\geq 3}$, and that therefore, as noted, the full set of secondary inferences derived under (3) is $\{\Box\neg$**[n]**, $\Box\neg$**[n+1,...)**$\}_{n\geq 3}$. Crucially, this set contains many subsets whose members jointly entail $\Box\neg$**[3,...)**, such as, for example, $\{\Box\neg$**[3]**, $\Box\neg$**[4,...)**$\}$ or $\{\Box\neg$**[n]**$\}_{n\geq 3}$. However, $\Box\neg$**[3,...)** contradicts the ignorance implications that arise from (18), specifically the possibility implication $\neg\Box\neg$**[3,...)**. The relevant subsets of secondary implicatures, then, are inconsistent with set 1 in (35), and hence set 2 as a whole is inconsistent as well.

To be sure, while it is certainly conceivable for the listener to draw an inconsistent set of inferences, the prediction that this should be prompted by an utterance of (1) is clearly not supported by intuitions. An analysis of *at least* in terms of the Standard Recipe, then, gives rise to another instantiation of the problem of inconsistency that Fox (2007a) exemplified with reference to the example in (14) discussed in Section 2.

(14) A: Who applied?
B: Some cook.

Within the Neo-Gricean approach to *at least*, I will in the following explore possible solutions to this new instantiation of the inconsistency problem. Based on the discussion of two failed attempts in Section 4, I will propose in Section 5 that a Neo-Gricean analysis calls for a revision of the Standard Recipe that makes reference to Fox's (2007a) notion of innocent exclusion.

But before turning to these arguments, I should acknowledge that a conceivable solution to the inconsistency problem with *at least* might be available within the Standard Recipe, or rather, within a more complete elaboration

11 Mayr (2013) attempts an account within the Grammatical Theory of Implicature (Fox 2007a) that assumes that Horn scale {*at least*, *at most*}, but observes that it too generates inadequate implications for a certain class of cases. Mayr for this reason concludes that "the ultimate account of the puzzle thus remains to be determined". The account Mayr explores moreover remains silent on the source of ignorance implications with *at least*.

of the account. In the version of the Standard Recipe whose application to *at least* I examined, I have omitted reference to an ingredient that is no doubt critical for the overall empirical adequacy of a theory of Quantity inferences, viz., the notion of *relevance*. According to the Standard Recipe as presented above, the listener draws inferences on the basis of all alternatives that are semantically stronger than the asserted meaning. However, in an empirically more adequate elaboration, in order for an alternative stronger than the assertion to give rise to a Quantity inference, the listener must also consider this alternative *relevant* (e.g., Gamut 1991, Fox 2007a, Geurts 2010). Under a conceivable approach to preventing inconsistency, then, certain alternatives routinely fail to give rise to Quantity inferences in virtue of being considered irrelevant. Applied to the case of (1), this approach hypothesizes a systematic contrast in relevance between two types of alternatives: the symmetric alternatives **[2]** and **[3,...)** are invariably relevant when (1) is uttered, hence invariably give rise to the attested Quantity inferences, viz., ignorance implications; in contrast, any alternative in $\{\mathbf{[n]}, \mathbf{[n{+}1,...)}\}_{n \geq 3}$ is invariably irrelevant, hence never gives rise to a Quantity inference. However, while such an approach is surely conceivable, and in the spirit of Geurts (2010), I am unaware of an independently motivated notion of relevance that would have the intended effect. Nevertheless, it is to be kept in mind that the conclusions drawn below could in principle be undermined by a novel notion of relevance that solves the inconsistency problem that *at least* introduces under the Standard Recipe.[12]

4 Preserving consistency: two inadequate attempts

I will motivate a solution of the inconsistency problem for *at least* in terms of innocent exclusion via a process of elimination. In this section, I examine two conceivable revisions of the Standard Recipe that succeed at preserving consistency, but which I will argue to nevertheless be empirically inadequate. Both revisions will be shown to derive inadequately strong inferences, either

12 Another conceivable approach to the problem questions the assumption that primary inferences are strengthened to secondary inferences by default. However, this approach would need to be reconciled with the general pervasiveness of secondary Quantity implications that has been taken to motivate such default strengthening (see, e.g., Geurts 2010). Notably, it seems that such strengthening can be observed even in examples with *at least*, viz., in cases that include another scalar expression. For example, it seems that *At least one student was interested in some of the topics* is naturally read as suggesting that no student was interested in all of the topics.

at the primary or the secondary level. Echoing the comment above about the Standard Recipe, I note at the outset that in both cases, a suitable notion of relevance could in principle preempt the unwanted inferences, by excluding the underlying alternatives as irrelevant. In the absence of an independently motivated notion of relevance that has the intended effect, the actual prospects of such an approach are unclear. However, this caveat is to be kept in mind in the final assessment of the conclusions drawn below.

For expository reasons, I will introduce each of the two revisions of Standard Recipe with reference to the simple hypothetical configuration in (10), shown in Section 2 to yield an inconsistent inference set under the Standard Recipe, before applying it to *at least*.

4.1 Closure under disjunction

One conceivable amendment to the analysis of (1) in the previous section extrapolates from Sauerland's (2004) analysis of three-part disjunctions like (13) within the Standard Recipe. In Sauerland's analysis, the substitution method for generating alternatives applied to (13) does not deliver the configuration in (10), but rather the one in (36).[13] Given this set of alternatives, the Standard Recipe derives for (13) the set of inferences in (37).

(13) Bill applied or Carol applied or Dan applied.

(10)
$$\begin{array}{ccc} \mathbf{b} & \mathbf{c} & \mathbf{d} \\ & \mathbf{b}\vee\mathbf{c}\vee\mathbf{d}^{*} & \end{array}$$

(36)
$$\begin{array}{ccc} \mathbf{b} & \mathbf{c} & \mathbf{d} \\ \mathbf{b}\vee\mathbf{c} & \mathbf{b}\vee\mathbf{d} & \mathbf{c}\vee\mathbf{d} \\ & \mathbf{b}\vee\mathbf{c}\vee\mathbf{d}^{*} & \end{array}$$

(37)
i. $0_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}\}$

ii. $1_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\, \mathbf{b}\vee\mathbf{c},\, \mathbf{b}\vee\mathbf{d},\, \mathbf{c}\vee\mathbf{d},\, \mathbf{b},\, \mathbf{c},\, \mathbf{d}\}} =$
$\{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}\vee\mathbf{c}, \neg\Box\mathbf{b}\vee\mathbf{d}, \neg\Box\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}, \neg\Box\mathbf{c}, \neg\Box\mathbf{d}\}$

iii. $2_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\, \mathbf{b}\vee\mathbf{c},\, \mathbf{b}\vee\mathbf{d},\, \mathbf{c}\vee\mathbf{d},\, \mathbf{b},\, \mathbf{c},\, \mathbf{d}\}} =$
$\{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}\vee\mathbf{c}, \neg\Box\mathbf{b}\vee\mathbf{d}, \neg\Box\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}, \neg\Box\mathbf{c}, \neg\Box\mathbf{d}\}$

Notably, this set does not include any secondary inferences. The reason is that each of the alternatives in (36) participates in a pair of symmetric alternatives that jointly exhaust the asserted meaning $\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}$. Symmetry results in

13 Actually, the set of alternatives that Sauerland assumes also includes the conjunctions $\mathbf{b}\wedge\mathbf{c}$ etc., but these are not relevant for the point that (36) is intended to illustrate.

the entailments shown in (38): the Quality implicature and the primary implicatures jointly entail the negation of each potential secondary implicatures, and hence entail an ignorance implication about each alternative. Under the Standard Recipe, this preempts the competence assumption and the corresponding secondary inference for each alternative. The inconsistency that arises under the smaller alternative set in (10) is thereby obviated.

(38)
$$\frac{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\ \neg\Box\mathbf{b}\vee\mathbf{c},\ \neg\Box\mathbf{b}\vee\mathbf{d},\ \neg\Box\mathbf{c}\vee\mathbf{d},\ \neg\Box\mathbf{b},\ \neg\Box\mathbf{c},\ \neg\Box\mathbf{d}}{\neg\Box\neg\mathbf{b}\vee\mathbf{c},\ \neg\Box\neg\mathbf{b}\vee\mathbf{d},\ \neg\Box\neg\mathbf{c}\vee\mathbf{d},\ \neg\Box\neg\mathbf{b},\ \neg\Box\neg\mathbf{c},\ \neg\Box\neg\mathbf{d}}$$

Conceivably, Sauerland's analysis of three-part disjunctions could serve as a model for an analysis of (1) in a modification of the Standard Recipe. Such an analysis would effectively augment the alternative set in (30) so as to provide each alternative with a symmetric partner. In analogy to the case in (36), this would ensure that all secondary inferences are preempted. The resulting inference set would accordingly be consistent.

(1) Al hired at least two cooks.

(30)
$$\begin{array}{cccccc}
 & & & \mathbf{...} & \mathbf{...} & \\
 & & \mathbf{[3]} & \mathbf{[4} & \mathbf{...} & \mathbf{)} \\
 & \mathbf{[2]} & \mathbf{[3} & \mathbf{4} & \mathbf{...} & \mathbf{)} \\
\mathbf{[1]} & \mathbf{[2} & \mathbf{3} & \mathbf{4} & \mathbf{...} & \mathbf{)}^{*} \\
\mathbf{[1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{...} & \mathbf{)}
\end{array}$$

A principled way of providing each alternative in (30) with a symmetric partner is suggested by the observation that (36) can be obtained from (10) through closure under disjunction: the generalized disjunction of any non-empty subset of (36) is itself an element of (36). Spector (2007) in fact entertains the possibility that the sets of proposition that feed the calculation of Quantity inferences are always closed under disjunction. In a possible implementation of this suggestion, the definitions in (3) are replaced with those in (39), where the set A in the definition of the sets of primary inferences is replaced with CUD(A), the closure under disjunction of A. This substitution transforms the Standard Recipe into what I will refer to as the *Closure Based Recipe*.

(39) i. $0_p = \{\Box p\}$

ii. $1_{p,A} = 0_p \cup \{\neg\Box q\text{: } q\in \mathrm{CUD(A)}\ \&\ q\subset p\}$

iii. $2_{p,A} = 1_{p,A} \cup \{\Box\neg q\text{: } \neg\Box q\in 1_{p,A}\ \&\ \Box\neg q \text{ is consistent with } 1_{p,A}\}$

When applying the Closure Based Recipe to example (1) and the configuration in (30), note that CUD({**[n,...)**, **[n]**}$_{n\geq 1}$) is equal to CUD({**[n]**}$_{n\geq 1}$). The definitions in (39) can then be seen to give rise to the following equalities.

(40) i. $0_{[2,...)} = \{\Box\mathbf{[2,...)}\}$

ii. $1_{[2,...),\{[n,...),[n]\}_{n\geq 1}} = \{\Box\mathbf{[2,...)}\} \cup \{\neg\Box\mathbf{q}\}_{q\in CUD(\{[n]\}_{n\geq 1})\ \&\ q\subset [2,...)}$

iii. $2_{[2,...),\{[n,...),[n]\}_{n\geq 1}} = \{\Box\mathbf{[2,...)}\} \cup \{\neg\Box\mathbf{q}\}_{q\in CUD(\{[n]\}_{n\geq 1})\ \&\ q\subset [2,...)}$

Note that (40) preserves the main result captured under the Standard Recipe: given that closure under disjunction retains the crucial symmetric alternatives **[2]** and **[3,...)**, the inferences in (40) are guaranteed to entail the attested ignorance inferences ¬□**[2]**∧¬□¬**[2]** and ¬□**[3,...)**∧¬□¬**[3,...)**. Crucially, moreover, (40) does not feature any secondary inference. For a concrete demonstration, I return to the case of the proposition **[4,...)** discussed above. The set CUD($\{$**[n]**$\}_{n\geq 1}$) does not only contain **[4,...)** (= $\bigcup\{$**[n]**$\}_{n\geq 4}$), but also its symmetric partner **[2,3]** (= $\bigcup\{$**[n]**$\}_{2\leq n\leq 3}$). Due to this symmetry, the Closure Based Recipe will deliver ignorance implications about these two propositions, thereby preempting secondary inferences about them. Similarly, CUD($\{$**[n]**$\}_{n\geq 1}$) furnishes **[3]** (= $\bigcup\{$**[n]**$\}_{n=3}$) with the symmetric partner **[2,4,...)** (= $\bigcup\{$**[n]**$\}_{n=2 \text{ or } n\geq 4}$), again preempting a secondary inference. This generalizes to all other propositions in CUD($\{$**[n]**$\}_{n\geq 1}$), and that is why set 2 in (40) does not contain any secondary inferences, and hence remains consistent. The problem of inconsistency has found a principled solution.

However, this solution comes with an additional commitment regarding the meaning of (1). After all, closure under disjunction removes unwanted secondary inferences by virtue of introducing additional ignorance implications. For example, due to the particular cases of symmetry identified above, the set 1 inferences will jointly entail an ignorance implication about each of the propositions **[4,...)**, **[2,3]**, **[3]**, and **[2,4,...)**. More generally, given that there is a symmetric partner for every proposition in CUD($\{$**[n]**$\}_{n\geq 1}$), the Closure Based Recipe predicts inferences which, modulo the Quality implicature □**[2,...)**, jointly imply *total* speaker ignorance regarding the number of cooks Al hired.

I have been unable to detect any evidence for such an implication of total ignorance of contributed by *at least*. It seems obvious, in fact, that (1) does not carry speaker ignorance implications about very large numbers. The sentence certainly does not suggest that the speaker considers it possible that Al hired, say, millions of cooks. But more generally, it appears that the only ignorance implications consistently detectable for *at least n* are those about **[n]** and

[n+1,...), the propositions expressed by the alternative statements with *only n* and *at least n+1*.

This is illustrated by the contrast in (41). The sequence in (41a) is expectedly judged to be incoherent. Just as predicted, the first sentence is judged to imply that the speaker considers it possible that the quintet has more than two German members. In conjunction with this implication, the asserted content of the second sentence is perceived to be in conflict with the common knowledge that a quintet has exactly five members.

(41) a. #At least two members of the quintet were born in Germany. Exactly three were born in Canada.
b. At least one member of the quintet was born in Germany. Exactly three were born in Canada.

If indeed *at least n* triggered an inference of total ignorance regarding the number of German quintet members modulo the assumption that it exceeds n-1, then the incoherence detectable in (41a) should likewise be perceived in (41b). Here, too, the first sentence should imply that the speaker considers it possible that the quintet has more than two German members, and so the asserted content of the second sentence is again predicted to clash with common knowledge. This prediction is incorrect. In contrast to (41a), (41b) is judged to be coherent.[14,15]

I conclude that while for a sentence with *at least n*, the ignorance implications about the symmetric alternatives **[n]** and **[n+1,...)** are consistently attested, there appears to be no evidence for corresponding inferences about the equally symmetric pair **[n, n+1]** and **[n+2,...)**. More generally, just as implied by Büring's (2008) original characterization of the meaning of *at least*, there appears to be no sign of the additional ignorance implications predicted by closure under disjunction. I conclude that closure under disjunction is not an adequate solution to the inconsistency problem.[16]

14 In an earlier version of this paper, I attempted to make the sort of argument just presented with reference to examples like (i).

(i) # At least five members of the quintet are Canadian.

The argument presupposed that under the Standard Recipe, the oddness of (i) can be attributed to ignorance implications about a pair of symmetric alternatives. As Benjamin Spector pointed out to me, however, no symmetry actually obtains if the semantic strength relation operative in the Standard Recipe is relativized to contextual information, as it should be in a (Neo-)Gricean account. Let **q** be the proposition that any quintet has exactly five members, and **[n]**/**[n,...)** the proposition that exactly n/at least n members of the relevant quintet are Canadian. Given that **q**∧**[5,...)** = **[5]** = **q**∧**[5]**, substitution of *exactly* (or *only*) for *at least* in (i) yields a statement that is contextually equivalent to (i). Relative to common knowledge, then, (i) does not in fact have symmetric alternatives that would derive ignorance implications explaining the oddness of (i). In other words, (i) behaves as though the strength relation applicable in the Standard Recipe was after all not contextual strength, but logical strength. In recent literature, a range of phenomena have been identified that raise this very issue for Neo-Gricean pragmatics (e.g., Heim 1991, Percus 2006, Sauerland 2008, Magri 2009, Magri 2011, Meyer 2013). The question whether the phenomena in question are ultimately amenable to a Neo-Gricean treatment is currently being debated (Schlenker 2012, Lauer 2014), but the present study does not contribute to this debate.

15 Benjamin Spector points out that an *at least* sentence can invite a stronger ignorance inference when interpreted as a matching answer to a wh-question. For example, in the context of the question *How many members of the quintet were born in Germany?*, the answer *At least two* is likely to convey that the speaker has no information about the number of German quintet members other than that it is no less than two and no more than five. This is consistent with the observations and conclusions above under the assumption that a question can force an answer to be interpreted relative to an alternative set that strictly includes the alternative set generated by Horn scales and the substitution method alone.

16 In its effect for *at least* cases like (1), the Closure Based Recipe resembles the *Basic Grice* theory articulated in Fox 2007a, under which the alternative set is not grammatically regulated in the first place (via Horn scales or otherwise), but only by relevance. Assuming that relevance is preserved under conjunction and negation, every relevant alternative has a relevant symmetric partner, and so Basic Grice produces an ignorance implication about any relevant alternative stronger than the asserted meaning, and it produces no secondary inferences. If successful, the argument presented here against the Closure Based Recipe at the same time establishes that ignorance implications with *at least*, whose content *does*

4.2 Maximizing competence

The Closure Based Recipe considered and discarded above solves the inconsistency problem in Neo-Gricean pragmatics by preempting the derivation of any of the secondary inferences that jointly give rise to the inconsistency, but it does so at the cost of an inadequate strengthening of the set of primary inferences. Another approach to the problem emerges from two closely related proposals in van Rooij & Schulz 2004 and Spector 2007, which build on Groenendijk & Stokhof 1984. Rather than positing radical preemption of secondary inferences, and hence competence assumptions, this approach effectively assumes that listeners ascribe to speakers a maximal degree of competence about alternatives that is consistent with primary inferences.

In van Rooij & Schulz 2004 and Spector 2007, the approach is given impressively elegant implementations, in terms of algorithms that model the derivation of Quantity implicatures by directly defining sets of possible belief states, rather than sets of inferences that characterize such belief states. A set of belief states so defined is to be interpreted as set of belief states that the listener thinks the speaker might be in.

I now present a rendition of this approach that I will refer to as the *Exhaustivity Based Recipe*. I begin with two preliminaries. First, following Spector 2007, I assume that possible belief states are propositions, and so can be said to entail, or fail to entail, other propositions, such as the semantic meaning expressed by an asserted sentence and its alternatives. Second, I introduce the pair of auxiliary definitions in (42) below.

(42) i. $\mathrm{POS}_{s,A} = \{q \in A: s \subseteq q\}$
ii. $\mathrm{NEG}_{s,A} = \{q \in A: s \subseteq \neg q\}$

$\mathrm{POS}_{s,A}$ is the set of alternatives in A that s entails to be true; and $\mathrm{NEG}_{s,A}$ is the set of alternatives in A that s entails to be false. I will also refer to $\mathrm{POS}_{s,A}$ and $\mathrm{NEG}_{s,A}$ as the *positive* and *negative yield* of s in A, respectively.

My rendition of the Exhaustivity Based Recipe, guided by Spector 2007, closely follows the format of the presentations of the Standard Recipe and the Closure Based Recipe above. In (43) below, the variable u stands for set of the belief states that, prior to the utterance in question, the listener considers it possible that the speaker is in. The three clauses of (43) describe increasingly

seem regulated by grammar and not just relevance, cannot be properly understood in terms of Basic Grice. Meyer (2013) makes much the same argument with respect to ignorance implications associated with disjunction, leading Meyer to reject Basic Grice.

restricted subsets of u. The Exhaustivity Based Recipe holds that an utterance of sentence ϕ with semantic content p and alternatives A will lead a listener to reduce this set to $2_{p,A,u}$ in (43iii), which is defined as a subset of the set $1_{p,A,u}$ in (43ii), which in turn is defined as a subset of the set $0_{p,A,u}$ in (43i).

(43) i. $0_{p,u} = \{s \in u\text{: } s \subseteq p\}$
ii. $1_{p,A,u} = \{s \in 0_{p,u}\text{: } \neg\exists s' \in 0_{p,u}[POS_{s',A} \subset POS_{s,A}]\}$
iii. $2_{p,A,u} = \{s \in 1_{p,A,u}\text{: } \neg\exists s' \in 1_{p,A,u}[NEG_{s,A} \subset NEG_{s',A}]\}$

According to (43i), $0_{p,u}$ is the set of all those states in u that entail p; (43ii) states that $1_{p,A,u}$ retains from $0_{p,A,u}$ only those states that have a minimal positive yield in A; and according to (43iii), $2_{p,A,u}$ is comprised of only those states in $1_{p,A,u}$ that have a maximal negative yield in A.

Given (43i), the Exhaustivity Based Recipe posits that the listener infers that the speaker's beliefs include p. Clause (43i), then, is the obvious state based equivalent of clause (3i) under the Standard Recipe. For the stock examples of Quantity implicatures, such as those covered in Section 2.1, (43ii) also has the very same effect as clause (3ii) in the Standard Recipe, effectively adding a primary inference about each of the stronger alternatives. The examples discussed below will illustrate this as well. As for (43iii), this clause again replicates the effect of (3iii) under the Standard Recipe for the cases covered in Section 2.1, which I will leave for the reader to verify. Here I will focus on cases where (43iii) has a different effect than (3iii), demonstrating how the Exhaustivity Based Recipe solves the inconsistency problem that arises under the Standard Recipe.[17]

Consider, then, once again the hypothetical configuration (10), for which the Standard Recipe derives an inconsistent inference set. To facility presentation, I will focus on the idealized case where u comprises all possible belief states, that is, the case where the listener has no prior assumptions regarding the speaker's beliefs. To aid readability, I will in fact omit the u

17 It is not claimed here that the definitions in (43) preserve consistency in all cases. Set 2 will end up empty in case the alternative set does not contain any subset that is a maximal negative yield of a state in set 1. This can arise in cases where an infinite alternative set is densely ordered by semantic strength, cases of the sort discussed in Fox & Hackl 2006, Fox 2007b, and Gajewski 2009. (In Fox & Hackl's analysis, such dense orderings of alternatives are isomorphic to a dense ordering of degrees expressed by numerals and other degree phrases. Under present assumptions, where the Horn scale of numerals is discrete, the sort of case Fox & Hackl discuss does not arise.) Crucially, in Fox & Hackl's cases, the prediction of inconsistency is argued to be correct, rather than problematic.

parameter in the following. With this simplification, the definitions in (43) can be shown to support the equalities in (44).

(10) **b** **c** **d**
b∨**c**∨**d***

(44) i. $0_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}} = \{s\text{: } s\subseteq\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}\}$

ii. $1_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\, \mathbf{b},\, \mathbf{c},\, \mathbf{d}\}} = \{s\text{: } s\subseteq\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}\ \&\ s\nsubseteq\mathbf{b}\ \&\ s\nsubseteq\mathbf{c}\ \&\ s\nsubseteq\mathbf{d}\}$

iii. $2_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\, \mathbf{b},\, \mathbf{c},\, \mathbf{d}\}} = \{s\text{: } s\subseteq\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}\ \&\ s\nsubseteq\mathbf{b}\ \&\ s\nsubseteq\mathbf{c}\ \&\ s\nsubseteq\mathbf{d}\ \&$
$(s\subseteq\neg\mathbf{b} \text{ or } s\subseteq\neg\mathbf{c} \text{ or } s\subseteq\neg\mathbf{d})\}$

By (43i), set 0 is the set of states that entail **b**∨**c**∨**d**, as noted in (44i). By (43ii), set 1 selects from set 0 those states that have a minimal positive yield in (10), which is the set of states whose positive yield is {**b**∨**c**∨**d**}; so, as recorded in (44ii), set 1 is the set of states that entail **b**∨**c**∨**d** but not any of the three alternatives **b**, **c**, and **d**. This fully replicates the effect of the primary implicatures derived for this case under the Standard Recipe. In particular, just like under the Standard Recipe, the lack of symmetry in (10) ensures that no ignorance implications arise. However, again replicating the effect of the Standard Recipe, the entailments in (45) ensure that there are no states in set 1 that entail the negations of more than one of the alternatives **b**, **c**, and **d**.

(45) $$\frac{s\subseteq\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\ s\nsubseteq\mathbf{b},\ s\nsubseteq\mathbf{c},\ s\nsubseteq\mathbf{d}}{s\nsubseteq\neg(\mathbf{c}\vee\mathbf{d}),\ s\nsubseteq\neg(\mathbf{b}\vee\mathbf{d}),\ s\nsubseteq\neg(\mathbf{b}\vee\mathbf{c})}$$

The entailments in (45) are to be kept in mind when calculating the membership of set 2, which by (43iii) comprises those members of set 1 that have a maximal negative yield in (10); since there are no states in set 1 that entail the negation of more than one of the alternatives **b**, **c**, and **d**, the maximal negative yields in (10) of states in set 1 are the three singletons {**b**}, {**c**}, and {**d**}; therefore, set 2 selects from set 1 those sets that entail the negation of one of the three alternatives **b**, **c**, and **d**; so set 2 can be described as stated in (44iii). Equivalently, and perhaps more transparently, set 2 can be described as in (46).

(46) $\{s\text{: } s\nsubseteq\mathbf{b}\ \&\ s\nsubseteq\mathbf{c}\ \&\ s\nsubseteq\mathbf{d}\ \&\ (s\subseteq(\mathbf{c}\vee\mathbf{d})\wedge\neg\mathbf{b} \text{ or } s\subseteq(\mathbf{b}\vee\mathbf{d})\wedge\neg\mathbf{c} \text{ or } s\subseteq(\mathbf{b}\vee\mathbf{c})\wedge\neg\mathbf{d})\}$

Under (43) and the Exhaustivity Based Recipe, then, an utterance with the semantic meaning **b**∨**c**∨**d** and the alternatives in (10) should lead the listener to infer that the speaker's beliefs do not include any of the propositions **b**, **c**, and **d**, but do include for any two of **b**, **c**, and **d**, the proposition that one of the two is true while the third is false. So this set permits three different types of candidate belief states. Each type effectively maximizes the assumed competence of the speaker by excluding one of the three alternatives as false. The three types of states differ with regard to which of the three alternatives they exclude as false.

So under the Exhaustivity Based Recipe, based on an utterance with semantic meaning $\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}$ and the alternatives in (10), the listener is predicted to infer that the speaker is competent about one of the three alternatives, considering it false, but without being led to an assumptions about which of the three it is that the speaker is competent about. And because of this uncertainty regarding the speaker's competence, the listener will only be able to infer a weak bottom line inference, viz., the inference that among the three propositions **b**, **c**, and **d**, there is one that is false. So, in contrast to the Standard Recipe, the inferences that the Exhaustivity Based Recipe derives for (10) remain consistent.

I will now proceed to showing that for the target case in (1) under the the alternative set (30), the predictions of the Exhaustivity Based Recipe differ from those of the Standard Recipe in much the same way they do for the hypothetical case in (10), again obviating inconsistency. This will be demonstrated by establishing and interpreting the equalities in (47).

(1) Al hired at least two cooks.

(30)
$$\begin{array}{cccccc}
 & & & \textbf{...} & \textbf{...} & \\
 & & \textbf{[3]} & \textbf{[4} & \textbf{...} & \textbf{)} \\
 & \textbf{[2]} & \textbf{[3} & \textbf{4} & \textbf{...} & \textbf{)} \\
\textbf{[1]} & \textbf{[2} & \textbf{3} & \textbf{4} & \textbf{...} & \textbf{)}^{*} \\
\textbf{[1} & \textbf{2} & \textbf{3} & \textbf{4} & \textbf{...} & \textbf{)}
\end{array}$$

(47) i. $0_{\mathbf{[2,...)}} = \{s\text{: } s\subseteq\mathbf{[2,...)}\}$

ii. $1_{\mathbf{[2,...)},\{\mathbf{[n,...)},\,\mathbf{[n]}\}_{n\geq 1}} = \{s\text{: } s\subseteq\mathbf{[2,...)}\ \&\ s\not\subseteq\mathbf{[2]}\ \&\ s\not\subseteq\mathbf{[3,...)}\}$

iii. $2_{\mathbf{[2,...)},\{\mathbf{[n,...)},\,\mathbf{[n]}\}_{n\geq 1}} = \{s\text{: } s\subseteq\mathbf{[2,...)}\ \&\ s\not\subseteq\mathbf{[2]}\ \&\ s\not\subseteq\mathbf{[3,...)}\ \&$
$(s\subseteq\neg\mathbf{[4,...)} \text{ or } s\subseteq\neg\mathbf{[3,5,...)} \text{ or } s\subseteq\neg\mathbf{[3,4,6,...)} \text{ or } \dots)\}$

By (43i), set 0 is the set of states that entail **[2,...)**. By (43ii), set 1 selects from set 0 those states that have a minimal positive yield in (30); these are the states in set 0 whose positive yield in (30) is {**[1,...)**, **[2,...)**}, hence set 1 is the set of states that entail **[2,...)** (and therefore **[1,...)**), but do not entail any of the other alternatives in (30); as shown in (47ii), that set can be described as the set of states that entail **[2,...)** but not **[2]** or **[3,...)**. The membership of set 1 is shaped by symmetry, which gives rise to the entailments in (48), in analogy to corresponding entailments under the Standard Recipe.

(48)
$$\frac{s\subseteq\mathbf{[2,...)},\ s\not\subseteq\mathbf{[2]},\ s\not\subseteq\mathbf{[3,...)}}{s\not\subseteq\neg\mathbf{[2]},\ s\not\subseteq\neg\mathbf{[3,..)}}$$

Given (48), all states in set 1 entail neither **[2]** or **[3,...)** nor ¬**[2]** or ¬**[3,...)**. Like the Standard recipe, then, relative to the alternatives in (30), the Exhaustivity Based Recipe predicts (1) to support an ignorance implication, viz., the inference that the speaker does not know whether Al hired exactly two cooks or more than two.

By (43iii), set 2 selects from set 1 those states that have a maximal negative yield in (30). Since every state in sets 1 entails **[2,...)**, there is no state in set 1 whose negative yield includes **[2,...)** or **[1,...)**, while every negative yield of a state in set 1 includes **[1]**. Further, because of (48), no negative yield of a state in set 1 includes **[2]** or **[3,...)**. Moreover, again because of (48), no negative yield of a state in set 1 includes any alternatives whose negations jointly entail ¬**[3,...)**, or equivalently, whose generalized disjunction is entailed by **[3,...)**. The maximal negative yields of states in set 1, then, are maximal subsets of the alternatives in (30) that include **[1]**, do not include **[1,...)**, **[2,...)**, **[2]**, or **[3,...)**, and do include a maximal subset of $\{\mathbf{[n]}, \mathbf{[n+1,...)}\}_{n\geq 3}$ whose generalized disjunction is not entailed by **[3,...)**. There are many maximal subsets of the latter kind. Three of them are displayed in (49) below, where alternatives that are not members of the relevant maximal negative yield are shown in gray. The corresponding maximal negative yields of set 1 states in (30) are shown in (50).

(49) a.

			...	**...**	
		[5]	**[6**	**...**	**)**
	[4]	**[5**	**...**	**...**	**)**
[3]	**[4**	**5**	**...**	**...**	**)**

b.

			...	**...**	
		[5]	**[6**	**...**	**)**
	[4]	**[5**	**...**	**...**	**)**
[3]	[4	5	...	...	)

c.				...	...	
			[5]	**[6**	**...**	**)**
		[4]	[5	...	...	)
	[3]	[4	5	...	...	)

(50) a. {**[1]**, **[4,...)**, **[4]**, **[5,...)**, **[5]**, **[6,...)**, ...}

b. {**[1]**, **[3]**, **[5,...)**, **[5]**, **[6,...)**, ...}

c. {**[1]**, **[3]**, **[4]**, **[6,...)**, ...}

More generally, apart from **[1]**, a maximal negative yield of a set 1 state is comprised of, for some m≥ 3, all the elements of $\{$**[n]**, **[n+1,...)**$\}_{n\geq3}$ except for those entailed by **[m]**. Therefore, as recorded in (47iii), set 2 is comprised of states that entail **[2,...)** but not **[2]** or **[3,...)**, and also entail one of the propositions ¬**[4,...)**, ¬**[3,5,...)**, ¬**[3,4,6,...)**, and so on. This set can also be described as in (51), as the set of states that entail neither **[2]** nor **[3,...)**, but entail the disjunction of **[2]** with one of the alternatives in $\{$**[m]**$\}_{m\geq3}$.

(51) {s: s⊈**[2]** & s⊈**[3,...)** & (s⊆**[2,3]** or s⊆**[2,4]** or s⊆**[2,5]** or ...)}

So according to the Exhaustivity Based Recipe, assuming the alternatives in (30), an utterance of (1) will lead the listener to maximize the assumed competence of the speaker by inferring that for some n≥3, the speaker believes Al to have hired exactly 2 or exactly n cooks. This purported inference is non-contradictory, establishing that the Exhaustivity Based Recipe indeed solves the inconsistency problem that arises under the Standard Recipe. The purported inference is moreover unobjectionable as far as the concomitant bottom line inference is concerned. Since the listener is predicted to infer that the speaker is in one of the states in (51), but without knowing which type, the strongest bottom line inference that can be drawn is that the Al either hired exactly two cooks or hired more than two. This expected bottom line inference is unobjectionable, as it is already entailed by (in fact, equivalent to) the assumed semantic meaning of (1).

The question that remains, however, is whether the specific content of the non-contradictory inference about the speaker's beliefs that (51) encodes is in accordance with intuitions. Notice that the effect of the Exhaustivity Based Recipe is in a sense diametrically opposed to the effect of the Closure Based Recipe discussed above. The Closure Based Recipe derives for (1) an inference of total ignorance about the alternatives, modulo the assumption that the speaker believes the asserted meaning that Al hired more than one cook. In contrast, the Exhaustivity Based Recipe derives for (1) an inference of total competence about the alternatives, modulo the ignorance inference that the speaker does not know whether Al hire exactly two cooks or more than two cooks.

The latter prediction is surely no more adequate than the former. I take it to be evident that sentence (1) does not support the inference of strong competence encoded by (51). If (1) had this interpretation, it could not be used sincerely by a speaker who had no assumptions about the number of cooks Al hired beyond the belief that the number is above one and below some reasonable upper bound imposed by contextual knowledge, say ten. But the use of (1) is not actually constrained in this way. To be sure, in this regard (1) sharply contrasts with (2), for which the relevant intuitions are in accordance with the predictions under both the Exhaustivity Based Recipe and the Standard Recipe. Sentence (2) could not normally be used sincerely by a speaker who did not believe that Al hired exactly two cooks.

(2) Al hired two cooks.

I conclude that like the Standard Recipe and the Closure Based Recipe, the Exhaustivity Based Recipe fails to apply correctly to (1) relative to the alternative set (30), and therefore does not constitute a viable solution to the problem of inconsistency in Neo-Gricean pragmatics.

5 Innocent exclusion in Neo-Gricean pragmatics

The two failed attempts in Section 4 of solving the inconsistency problem for (1) help sharpen the profile of an adequate solution to the problem: the lesson from the discussion of the Closure Based Recipe is that an adequate solution retains the effect of the Standard Recipe with regard to primary inferences; and the discussion of the Exhaustivity Based Recipe shows that an adequate solution does not merely weaken the effect of the Standard Recipe with regard to secondary inferences, but does not in fact derive any inferences based an assumptions of speaker competence.

The blueprint for a revision of (3) and the Standard Recipe that meets these conditions is provided in Fox 2007a, who introduced a notion of so-called innocent inclusion as a means to prevent inconsistencies of the sort arising under the Standard Recipe. Fox did not actually devise innocent inclusion as an ingredient of a Neo-Gricean account of Quantity implicature, but his definition can be adapted to the purposes at hand. The effect of revising the Standard Recipe in this way will be that a secondary inference is added to the set inferences only if this can be done without restricting the range of further possible consistent additions of secondary inferences. This has the intended effect for the case of (1) relative to the alternative set assumed. It ensures not only that the set of secondary inferences remains consistent, but it correctly fails to derive any secondary inferences.

In the implementation of this account, call it the *Innocent Exclusion Based Recipe*, the definitions in (3) under the Standard Recipe are replaced with those in (52a); (52a) refers to the definition of *innocently excludable* in (52b).

(52) a. i. $0_p = \{\Box p\}$

ii. $1_{p,A} = 0_p \cup \{\neg\Box q\text{: } q \in A \;\&\; q \subset p\}$

iii. $2_{p,A} = 1_{p,A} \cup \{\Box\neg q\text{: } \neg\Box q \in 1_{p,A} \;\&$ q is innocently excludable relative to $1_{p,A}\}$

b. p is *innocently excludable* relative to S :⇔
$\Box\neg p$ is an element of every maximal subset of $\{\Box\neg q\text{: } \neg\Box q \in S\}$ consistent with S

The difference between (3) and (52a) resides in the second condition in the description of the set of secondary inferences in set 2. According to (52a), in order for a potential secondary inference to actually be included in $2_{p,A}$, it must not merely be consistent with $1_{p,A}$, but it must be innocently excludable relative to $1_{p,A}$; by (52b), this means that the secondary inference must be a member of every maximal set of potential secondary inferences that is consistent with $1_{p,A}$.

Note that any alternative that is symmetric to another relative to the asserted content, and hence gives rise to ignorance implications entailed by set 1, fails to be innocently excludable relative to set 1. This is so because the secondary inference about such an alternative will not be an element of any set of potential secondary inferences consistent with set 1, let alone every maximal set of this kind.[18] Crucially, as illustrated shortly, non-symmetric alternatives can fail to be innocently excludable as well. So the condition for strengthening primary inferences under the Innocent Exclusion Based Recipe is more stringent than in the Standard Recipe.

18 More accurately, this only holds as long as *there are* such maximal consistent sets of potential secondary inferences, a condition that is met in all the cases examined in this paper, but that is not met in certain other cases (Gajewski 2009). Relatedly, a reviewer suggests that the conjunct "$\neg\Box q \in 1_{p,A}$" in (52a.iii) is redundant and could safely be omitted. The conjunct is indeed redundant for the cases studied here, where maximal consistent sets of potential secondary inferences exist. In the absence of such maximal sets, however, the conjunct strictly strengthens the condition on set membership in (52a.iii). (See also footnote 19.)

Like the Standard Recipe, the Innocent Exclusion Based Recipe posits that the listener by default takes the speaker to be competent about alternatives stronger than the asserted content. That default, however, is now overruled for a broader range of alternatives, not just for symmetric pairs of alternatives that yield ignorance implications, but also for alternatives that merely fail to be innocently excludable.

The Innocent Exclusion Based Recipe preserves all the intend effects of the Standard Recipe illustrated in Section 2, as the reader is invited to verify. At the same time, the Innocent Exclusion Based Recipe obviates the derivation of inconsistent inference sets for the relevant cases, as I will now demonstrate. I begin by returning to the hypothetical case in (10). To establish the requisite background for the application of (52), it will be useful to first review the predictions under the Standard Recipe presented in Section 2, where the definitions in (3) were shown to support the equalities in (11).

(10)
$$\begin{array}{ccc} \mathbf{b} & \mathbf{c} & \mathbf{d} \\ & \mathbf{b}\vee\mathbf{c}\vee\mathbf{d}^{*} & \end{array}$$

(11)
i. $0_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}\}$

ii. $1_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\,\mathbf{b},\,\mathbf{c},\,\mathbf{d}\}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}, \neg\Box\mathbf{c}, \neg\Box\mathbf{d}\}$

iii. $2_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\,\mathbf{b},\,\mathbf{c},\,\mathbf{d}\}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}, \neg\Box\mathbf{c}, \neg\Box\mathbf{d}, \Box\neg\mathbf{b}, \Box\neg\mathbf{c}, \Box\neg\mathbf{d}\}$

Set 2 is inconsistent. $\{\Box\neg\mathbf{b}, \Box\neg\mathbf{c}, \Box\neg\mathbf{d}\}$, the set of all secondary inferences in set 2 is inconsistent with the Quality inference $\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}$, and hence with set 1. In fact, the entailments in (12) ensure that even each of the three doubleton sets $\{\Box\neg\mathbf{b}, \Box\neg\mathbf{c}\}$, $\{\Box\neg\mathbf{b}, \Box\neg\mathbf{d}\}$, and $\{\Box\neg\mathbf{c}, \Box\neg\mathbf{d}\}$ is inconsistent with set 1. With this background from Section 2, I now turn to applying (52) to (10).

(12)
$$\frac{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\ \neg\Box\mathbf{b},\ \neg\Box\mathbf{c},\ \neg\Box\mathbf{d}}{\neg\Box\neg(\mathbf{b}\vee\mathbf{c}),\ \neg\Box\neg(\mathbf{c}\vee\mathbf{d}),\ \neg\Box\neg(\mathbf{b}\vee\mathbf{d})}$$

Each of the three singletons $\{\Box\neg\mathbf{b}\}$, $\{\Box\neg\mathbf{c}\}$, and $\{\Box\neg\mathbf{d}\}$ is consistent with set 1, and so these singletons are the maximal subsets of $\{\Box\neg\mathbf{b}, \Box\neg\mathbf{c}, \Box\neg\mathbf{d}\}$ consistent with set 1. Those three singletons have an empty intersection, and so according to (52b), none of the alternatives **b**, **c**, and **d** is innocently excludable relative to set 1. According to (52), this has the effect that no secondary inferences are added to set 1 in the formation of set 2. Therefore, under (52), the triple of equalities in (11) is to be replaced with the one in (53). In (53), set 2 no longer contains any of the secondary inferences responsible for the inconsistency in (11). The Innocent Exclusion Based Recipe, then, steers clear of the inconsistency that the Standard Recipe derives for the hypothetical case in (10).

(53) i. $0_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}\}$

ii. $1_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\,\mathbf{b},\,\mathbf{c},\,\mathbf{d}\}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}, \neg\Box\mathbf{c}, \neg\Box\mathbf{d}\}$

iii. $2_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\,\mathbf{b},\,\mathbf{c},\,\mathbf{d}\}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}, \neg\Box\mathbf{c}, \neg\Box\mathbf{d}\}$

I will now demonstrate that the account likewise preserves consistency for the central case of (1) under the alternative set (30). Section 3 showed that for this case the definitions in (3) derive the equalities in (35).

(1) Al hired at least two cooks.

(30)

			...	...	
		[3]	[4	...	)
	[2]	[3	4	...	)
[1]	[2	3	4	...	)*
[1	2	3	4	...	)

(35) i. $0_{[2,...)} = \{\Box[\mathbf{2,...}]\}$

ii. $1_{[2,...),\{[n,...),\,[n]\}_{n\geq1}} = \{\Box\mathbf{[2,...)}, \neg\Box\mathbf{[2]}, \neg\Box\mathbf{[3,...)}\} \cup \{\neg\Box\mathbf{[n]}, \neg\Box\mathbf{[n+1,...)}\}_{n\geq3}$

iii. $2_{[2,...),\{[n,...),\,[n]\}_{n\geq1}} = \{\Box\mathbf{[2,...)}, \neg\Box\mathbf{[2]}, \neg\Box\mathbf{[3,...)}\} \cup \{\neg\Box\mathbf{[n]}, \neg\Box\mathbf{[n+1,...)}\}_{n\geq3} \cup \{\Box\neg\mathbf{[n]}, \Box\neg\mathbf{[n+1,...)}\}_{n\geq3}$

In this case, too, set 2 turned out to be inconsistent. This inconsistency is due to the entailments in (18). Specifically, while (18) shows the inferences in set 1 to jointly entail $\neg\Box\neg\mathbf{[3,...)}$, the secondary inferences in set 2, the members of the set $\{\Box\neg\mathbf{[n]}, \Box\neg\mathbf{[n+1,...)}\}_{n\geq3}$, and in fact the members of many of its subsets, jointly entail $\Box\neg\mathbf{[3,...)}$. Keeping in mind these observations from Section 3, I now turn to applying the definitions in (52) to (30).

(18) $$\frac{\Box\mathbf{[2,...)}, \neg\Box\mathbf{[2]}, \neg\Box\mathbf{[3,...)}}{\neg\Box\neg\mathbf{[2]}, \neg\Box\neg\mathbf{[3,...)}}$$

To be sure, $\{\Box\neg\mathbf{[n]}, \Box\neg\mathbf{[n+1,...)}\}_{n\geq3}$ has many subsets that *are* consistent with $\neg\Box\neg\mathbf{[3,...)}$, and hence with set 1, and it in particular has many maximal subsets of this kind. Three such maximal subsets are visualized in the diagram in (49), recycled from Section 4. Here alternatives for which the corresponding secondary inferences are not members of the consistent subset in question are shown in gray. The sets of secondary inferences represented by (49) are shown in (54).

(49) a.

			...	...	
		[5]	[6	...	)
	[4]	[5	...	...	)
[3]	[4	5	...	...	)

b.

			...	...	
		[5]	[6	...	)
	[4]	[5	...	...	)
[3]	[4	5	...	...	)

c.

			...	...	
		[5]	[6	...	)
	[4]	[5	...	...	)
[3]	[4	5	...	...	)

(54) a. $\{\Box\neg\mathbf{[4,...)}, \Box\neg\mathbf{[4]}, \Box\neg\mathbf{[5,...)}, \Box\neg\mathbf{[5]}, \Box\neg\mathbf{[6,...)}, ...\}$

b. $\{\Box\neg\mathbf{[3]}, \Box\neg\mathbf{[5,...)}, \Box\neg\mathbf{[5]}, \Box\neg\mathbf{[6,...)}, ...\}$

c. $\{\Box\neg\mathbf{[3]}, \Box\neg\mathbf{[4]}, \Box\neg\mathbf{[6,...)}, ...\}$

More generally, maximal subsets of the set $\{\Box\neg\mathbf{[n]}, \Box\neg\mathbf{[n+1,...)}\}_{n\geq 3}$ consistent with set 1 can be constructed by subtracting from $\{\Box\neg\mathbf{[n]}, \Box\neg\mathbf{[n+1,...)}\}_{n\geq 3}$, for any o≥3, all and only those elements that entail $\Box\neg\mathbf{[o]}$. This means that for any o≥3, there is some maximal subset of $\{\Box\neg\mathbf{[n]}, \Box\neg\mathbf{[n+1,...)}\}_{n\geq 3}$ consistent with set 1 that does not include either $\Box\neg\mathbf{[o]}$ or $\Box\neg\mathbf{[o,...)}$. By (52b), therefore, neither **[o]** nor **[o,...)** is innocently excludable for any o≥3. In other words, none of the alternatives in $\{\mathbf{[n]}, \mathbf{[n+1,...)}\}_{n\geq 3}$ is innocently excludable. Under (52), then, the family of equalities in (35) is to be replaced with the one in (55).

(55) i. $0_{[2,...)} = \{\Box\mathbf{[2,...)}\}$

ii. $1_{[2,...),\{[n,...),\,[n]\}_{n\geq 1}} = \{\Box\mathbf{[2,...)}, \neg\Box\mathbf{[2]}, \neg\Box\mathbf{[3,...)}\} \cup \{\neg\Box\mathbf{[n]}, \neg\Box\mathbf{[n+1,...)}\}_{n\geq 3}$

iii. $2_{[2,...),\{[n,...),\,[n]\}_{n\geq 1}} = \{\Box\mathbf{[2,...)}, \neg\Box\mathbf{[2]}, \neg\Box\mathbf{[3,...)}\} \cup \{\neg\Box\mathbf{[n]}, \neg\Box\mathbf{[n+1,...)}\}_{n\geq 3}$

So, for the central example (1), the Innocent Exclusion Based Recipe steers clear of inconsistency when applied to the alternative set in (30).[19]

19 Parallel to the comment about (43) in footnote 17, and directly extending an observation in Gajewski 2009, I note that the definitions in (52) are not claimed to preserve consistency in all cases. Consider a case where there are no maximal consistent sets of potential secondary implicatures. In that case, all potential secondary inferences will trivially be innocently excludable, and so set 2 will contain a secondary inference corresponding to each primary implicature in set 1. This can result in inconsistency in cases of the sort discussed in Fox & Hackl 2006, Fox 2007b, and Gajewski 2009.

Specifically, as intended, the Innocent Exclusion Based Recipe accounts for the intended ignorance implications without deriving any secondary inferences. The Innocent Exclusion Based Recipe, then, provides an elaboration of a Neo-Gricean account of Quantity implicature that reconciles the observed interpretation of (1) with a two-scale derivation of the alternatives for *at least* sentences.[20]

6 Conclusions

The considerations presented above lead to the main conclusion that a Neo-Gricean account of ignorance implications associated with *at least* must make reference to a version of Fox's (2007a) notion of innocent exclusion, limiting the derivation of secondary inferences to alternatives that are innocently excludable. These considerations amount to an argument for innocent exclusion in Neo-Gricean pragmatics. The argument is conditional on the assumption that the meaning contribution of *at least* is indeed properly analyzed in the Neo-Gricean setting, as originally envisioned in Büring 2008.[21] As well, the argument is conditional on the absence of alternate solutions within the Neo-Gricean approach to the specific version of the inconsistency problem that *at least* gives rise to, in particular on the unavailability of a solution based on an independently motivated notion of relevance.

Irrespective of the final verdict on the argument presented here, it is worth emphasizing that the present case study deals with just one pos-

20 The above calculation under the Innocent Exclusion Based Recipe shows transparent parallels to the calculation under the Exhaustivity Based Recipe presented in Section 4. Both calculations make reference, directly or indirectly, to maximal sets of secondary inferences consistent with the Quality inference and primary inferences. This parallel invites detailed study of the precise formal relationship between these recipes. I will leave such a study to future work, which would extend the project of Spector (2016), who compares the two corresponding operators that Spector (2006) and Fox (2007a) define to derive bottom line inferences as truth conditional entailments.

21 There are two types of competitors to a Neo-Gricean analysis of *at least*. One type of account radically departs from Büring's (2008) approach by rejecting the proposed parallel between the meaning of *at least* sentences and Quantity implications found in disjunctions and elsewhere (Krifka 1999, Geurts & Nouwen 2007, Nouwen 2010, Penka 2010, Coppock & Brochhagen 2013, Cohen & Krifka 2014). Another type of account retains Büring's (2008) proposal that *at least* gives rise to Quantity implications, but rejects a Neo-Gricean account of such implications. With regard to this second type of competitor, I believe the effects of the Innocent Exclusion Based Recipe presented here can be replicated in Meyer's (2013) radical Grammatical Theory of Implicature, but I will not attempt to demonstrate this here.

sible manifestation of the general inconsistency problem for Neo-Gricean pragmatics discovered by Spector (2006) and Fox (2007a). To be sure, the consequences of this general problem for the Neo-Gricean framework remain to be explored relative to a broader empirical base, which will include question-answer dialogues of the type presented in Fox 2007a and mentioned in Section 2. The present case study, then, broaches a general issue that so far does not seem to have been properly investigated in the literature.

I will conclude with a few further observations about the Innocent Exclusion Based Recipe and the argument presented in its support. To begin, note that the Innocent Exclusion Based Recipe is sensitive to a three-way typology of alternatives stronger than the asserted meaning: (i) alternatives that are symmetric to another alternative (relative to the asserted content); (ii) alternatives that are innocently excludable (relative to the Quality inference and primary inferences); and (iii) alternatives that are neither symmetric nor innocently excludable. The Innocent Exclusion Based Recipe agrees with the Standard Recipe in its effect on alternatives of type (i) and (ii). The difference between the two recipes lies in the treatment of alternatives of type (iii): the Standard Recipe, but not the Innocent Exclusion Based Recipe, derives secondary inferences about alternatives of this type.

Like the Standard Recipe, though, the Innocent Exclusion Based Recipe derives primary inferences about type (iii) alternatives. Moreover, is worth noting that, while the Innocent Exclusion Based Recipe *by itself* does not strengthen such primary inferences, it does allow for any one of them to be strengthened consistently in context, that is, by assumptions about the speaker's beliefs that the listener might be making in an utterance situation. To illustrate, I return once more to the hypothetical case in (10) and the predicted inferences in (53).

(10) $\begin{array}{ccc} \mathbf{b} & \mathbf{c} & \mathbf{d} \\ & \mathbf{b}\vee\mathbf{c}\vee\mathbf{d}^{*} & \end{array}$

(53) i. $0_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}\}$

ii. $1_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\,\mathbf{b},\,\mathbf{c},\,\mathbf{d}\}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}, \neg\Box\mathbf{c}, \neg\Box\mathbf{d}\}$

iii. $2_{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\{\mathbf{b}\vee\mathbf{c}\vee\mathbf{d},\,\mathbf{b},\,\mathbf{c},\,\mathbf{d}\}} = \{\Box\mathbf{b}\vee\mathbf{c}\vee\mathbf{d}, \neg\Box\mathbf{b}, \neg\Box\mathbf{c}, \neg\Box\mathbf{d}\}$

The Innocent Exclusion Based Recipe derives a primary inferences about each of the three alternatives to the asserted meaning. Any one of these primary inferences could be strengthened contextually into an ignorance implication or a secondary implication, while preserving consistency. For example, the listener might assume in a context that $\neg\Box\neg\mathbf{b}$, arriving at an ignorance implication about **b**; symmetrically, the context might lead the listener to make the competence assumption $\Box\mathbf{b}\vee\Box\neg\mathbf{b}$, thereby supporting the secondary implication about **b**.

So the postulation of alternatives of type (iii) can yield testable empirical predictions about the overall utterance meaning. Such predictions can in principle be used for putting an assumed theory of alternatives to the test. One might hope, in particular, that intuitions on meaning can confirm the two-scale theory of alternatives for (1) and the alternative set in (30) that it generates.

(1) Al hired at least two cooks.

(30)
$$
\begin{array}{cccccc}
 & & & \mathbf{...} & \mathbf{...} & \\
 & & \mathbf{[3]} & \mathbf{[4} & \mathbf{...} & \mathbf{)} \\
 & \mathbf{[2]} & \mathbf{[3} & \mathbf{4} & \mathbf{...} & \mathbf{)} \\
\mathbf{[1]} & \mathbf{[2} & \mathbf{3} & \mathbf{4} & \mathbf{...} & \mathbf{)}^{*} \\
\mathbf{[1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{...} & \mathbf{)}
\end{array}
$$

However, in this particular case, the primary inferences supported by type (iii) alternatives are not actually expected to be detectable. Each of the type (iii) alternatives, that is, each member of $\{\mathbf{[n]}, \mathbf{[n{+}1,...)}\}_{n\geq 3}$, is stronger than the symmetric alternative **[3,...)**; hence the primary inferences about the members of $\{\mathbf{[n]}, \mathbf{[n{+}1,...)}\}_{n\geq 3}$ are entailed by the primary inference about **[3,...)** (as noted in Section 3.3). So the alternatives in $\{\mathbf{[n]}, \mathbf{[n{+}1,...)}\}_{n\geq 3}$ have a nul effect: the Innocent Exclusion Based Recipe derives the very same overall meaning on the basis of the full set of strong alternatives in (30) that it derives on the basis of the symmetric alternatives **[2]** and **[3,...)** alone. Therefore, intuitions on the meaning of (1), while consistent with the alternative set in (30) and the two-scale account that derives it, cannot provide direct evidence for the presence of the type (iii) alternatives included in (30). The characteristic effect of type (iii) alternatives predicted under the Innocent Exclusion Based Recipe, then, will have to be tested with regard to a other sorts of cases, such as question-answer dialogues of the sort discussed in Fox 2007a and in Section 2.

I conclude with a brief look beyond the Neo-Gricean approach to Quantity implicature. The general issue of consistency preservation is not confined to this approach. In particular, as noted in Section 1, it also arises in the so-called Grammatical Theory of Implicature (Fox 2007a, Chierchia, Fox & Spector 2011, Sauerland 2012, Meyer 2013). This theory posits an object language operator *Exh* intended to deliver bottom line inferences as truth conditional entailments, with reference to the same sorts of alternatives also posited in Neo-Gricean pragmatics. Spector (2006) (building on van Rooij & Schulz 2004) and Fox (2007a) consider two different ways of defining the semantics of such an operator, proposing (56) and (57), respectively.

(56) $[\![\text{Exh}]\!](A)(p) = \lambda w.p(w)\ \&\ \neg\exists v[p(v)\ \&\ \{q{\in}A{:}\ q(v)\} \subset \{q{\in}A{:}\ q(w)\}]$

(57) a. $[\![\text{Exh}]\!](A)(p) = \lambda w.p(w)\ \&$
$\forall q[q \text{ is innocently excludable relative to } p \text{ and } A \rightarrow \neg q(w)]$

b. q is *innocently excludable* relative to p and A :⇔
¬q is an element of every maximal subset of {¬r: r∈A} consistent with p

The two definitions differ in the method for consistency preservation that they employ, and the two methods transparently mirror the Exhaustivity Based Recipe and the Innocent Exclusion Based Recipe presented above. A question that arises, then, is whether the choice between (56) and (57) makes a difference for the interpretation of (1) relative to (30), mirroring the different effects of the two recipes.

It turns out that for the case of (1) and (30), (56) and (57) have the exact same effect. This follows almost immediately from a general result suggested in Fox 2007a and formally established in Spector 2016. Spector shows that (56) and (57) have the same truth conditional effect for all cases where the alternative set is closed under conjunction. Notice now that it is only the absence of the contradictory proposition that keeps (30) from being closed under conjunction. But adding the contradictory proposition to the alternative set does not change the overall information content of the Quantity implications derived. Therefore, given Spector's result, (56) and (57) derive the same truth conditions for *Exh [Al hired at least two cooks]*.

Specifically, as the reader is invited to verify, under both (56) and (57), the application of *Exh* to (1) is vacuous, in the sense that *Exh [Al hired at least two cooks]* is equivalent to *Al hired at least two cooks*. Note that the vacuity of *Exh* in this case reproduces the effects of the Exhaustivity Based Recipe and the Innocent Exclusion Based Recipe. While the two recipes were shown to derive different overall meanings for (1) relative to (30), they turned out to agree in terms of the bottom line inferences derived (see (47) and (55)). Mirroring the vacuous application of *Exh*, in both cases all the bottom line inferences are already entailed by the asserted meaning. The difference between the two recipes as applied to (1) and (30) resides entirely in the epistemic inferences derived. It is unsurprising that this difference is neutralized in an account that derives only bottom line inferences, such as the Grammatical Theory of Implicature. Given this neutralization, the argument for innocent exclusion present here does not extend to the Grammatical Theory of Implicature.[22]

At the same time, the argument *does* extend to any account that, like the Neo-Gricean approach and unlike the Grammatical Theory, posits primary implications about the speaker's belief state that are strengthened to secondary implications. That is, the impact of the argument is independent of whether primary implications indeed arise as pragmatic inferences, and likewise of whether strengthening of primary to secondary implicatures is a matter of further pragmatic reasoning. Even if primary implications are viewed as semantic entailments, and even if strengthening of primary implicatures is attributed to grammatically encoded meaning, the argument presented suggests that this strengthening will need to be constrained with reference to innocent exclusion so as to preempt the derivation of unattested implications about the speaker's beliefs.[23]

Finally, the specific argument that a *Neo-Gricean* account requires reference to innocent exclusion could potentially be developed into an argument *against* that approach to Quantity implications, as it is unclear to what extent it is conceptually plausible to assume that listeners calculate innocent exclusion in their reasoning about speakers' beliefs. I will leave this important question as a topic for future work.

References

Beaver, David & Brady Clark. 2008. *Sense and sensitivity: How focus determines meaning*. Oxford: Wiley-Blackwell. http://dx.doi.org/10.1002/9781444304176.

Büring, Daniel. 2008. The least *at least* can do. *West Coast Conference on Formal Linguistics (WCCFL)* 26. 114–120. http://www.lingref.com/cpp/wccfl/26/paper1662.pdf.

Chierchia, Gennaro, Danny Fox & Benjamin Spector. 2011. The grammatical view of scalar implicatures and the relationship between semantics and

22 I am indebted to Benjamin Spector for helping me see this point.

23 I thank a reviewer for insisting that the novel argument for innocent exclusion is not specific to the Neo-Gricean approach.

pragmatics. In Paul Portner, Claudia Maienborn & Klaus von Heusinger (eds.), *Handbook of semantics*. Berlin: de Gruyter.

Cohen, Ariel & Manfred Krifka. 2014. Superlative quantifiers and meta-speech acts. *Linguistics and Philosophy* 37(1). 41–90. http://dx.doi.org/10.1007/s10988-014-9144-x.

Coppock, Elizabeth & Thomas Brochhagen. 2013. Raising and resolving issues with scalar modifiers. *Semantics and Pragmatics* 6(3). 1–57. http://dx.doi.org/10.3765/sp.6.3.

Crnič, Luka. 2013. Focus particles and embedded exhaustification. *Journal of Semantics* 30(4). 533–558. http://dx.doi.org/10.1093/jos/ffs018.

Cummins, Chris & Napoleon Katsos. 2010. Comparative and superlative quantifiers: Pragmatic effects of comparison type. *Journal of Semantics* 27(3). 271–305. http://dx.doi.org/10.1093/jos/ffq006.

Fox, Danny. 2007a. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and implicature in compositional semantics*, 71–120. London: Palgrave MacMillan. http://dx.doi.org/10.1057/9780230210752. http://web.mit.edu/linguistics/people/faculty/fox/free_choice.pdf.

Fox, Danny. 2007b. Too many alternatives: Density, symmetry, and other predicaments. *Semantics and Linguistic Theory (SALT)* 17. 89–111. http://dx.doi.org/10.3765/salt.v17i0.2967.

Fox, Danny & Martin Hackl. 2006. The universal density of measurement. *Linguistics and Philosophy* 29(5). 537–586. http://dx.doi.org/10.1007/s10988-006-9004-4.

Gajewski, Jon. 2009. Innocent exclusion is not contradiction free. Unpublished manuscript, University of Connecticut. http://www.gajewski.uconn.edu/papers/IEnotCF.pdf.

Gamut, L.T.F. 1991. *Logic, language, and meaning, volume 1: Introduction to meaning*. Chicago, IL: University of Chicago Press.

Geurts, Bart. 2010. *Quantity implicatures*. Cambridge: Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511975158.

Geurts, Bart & Rick Nouwen. 2007. *At least* et al.: The semantics of scalar modifiers. *Language* 83(3). 533–559. http://dx.doi.org/10.1353/lan.2007.0115.

Grice, H.P. 1989. *Studies in the way of words*. Cambridge, MA: Harvard University Press.

Groenendijk, Jeroen & Martin Stokhof. 1984. *Studies on the semantics of questions and the pragmatics of answers*. University of Amsterdam dissertation. http://dare.uva.nl/record/1/392528.

Heim, Irene. 1991. Artikel und definitheit. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantik: Ein Internationales Handbuch der Zeitgenössischen Forschung*, 487–535. Berlin: de Gruyter.

Horn, Laurence. 1969. A presuppositional analysis of *only* and *even*. *Chicago Linguistics Society (CLS)* 5. 98–107.

Horn, Laurence. 1972. *On the semantic properties of logical operators in English*. University of California, Los Angeles dissertation.

Kennedy, Christopher. 2015. A "de-Fregean" semantics (and neo-Gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics* 8(10). 1–44. http://dx.doi.org/10.3765/sp.8.10.

Kilbourn-Ceron, Oriana. 2014. *Almost*: scope and covert exhaustification. *West Coast Conference on Formal Linguistics (WCCFL)* 32. 121–130. http://www.lingref.com/cpp/wccfl/32/paper3163.pdf.

Krifka, Manfred. 1999. At least some determiners aren't determiners. In Ken Turner (ed.), *The semantics/pragmatics interface from different points of view, vol. 1*, 257–291. Amsterdam: Elsevier.

Lauer, Sven. 2014. Mandatory implicatures in Gricean pragmatics. In Judith Degen, Michael Franke & Noah Goodman (eds.), *Formal and Experimental Pragmatics Workshop*, 21–28. Tübingen. https://sites.google.com/site/fepesslli2014/proceedings/proceedings.pdf.

Levinson, Stephen C. 1983. *Pragmatics*. Cambridge: Cambridge University Press.

Magri, Giorgio. 2009. *A theory of individual-level predicates based on blind mandatory implicatures. Constraint promotion for optimality theory*. Cambridge, MA: Massachusetts Institute of Technology dissertation. http://dspace.mit.edu/handle/1721.1/55182.

Magri, Giorgio. 2011. Another argument for embedded scalar implicatures based on oddness in downward entailing environments. *Semantics and Pragmatics* 4(6). 1–51. http://dx.doi.org/10.3765/sp.4.6.

Mayr, Clemens. 2013. Implicatures of modified numerals. In Ivano Caponigro & Carlo Cecchetto (eds.), *From grammar to meaning: The spontaneous logicality of language*, 139–171. Cambridge: Cambridge University Press. http://dx.doi.org/10.1017/CBO9781139519328.009.

Meyer, Marie-Christine. 2013. *Ignorance and grammar*. Cambridge, MA: Massachusetts Institute of Technology dissertation. http://hdl.handle.net/1721.1/84420.

Meyer, Marie-Christine. 2015. Generalized free choice and missing alternatives. *Journal of Semantics*. http://dx.doi.org/10.1093/jos/ffv010. http://semanticsarchive.net/Archive/zQyNDczZ/meyer_2014.pdf.

Nouwen, Rick. 2010. Two kinds of modified numerals. *Semantics and Pragmatics* 3(3). 1–41. http://dx.doi.org/10.3765/sp.3.3.

Penka, Doris. 2010. A superlative analysis of superlative scalar modifiers. Unpublished talk handout for *Sinn und Bedeutung* 15. http://www.uni-saarland.de/fak4/fr41/reich/conf/sub15/program/abstracts/penka-sub15.pdf.

Percus, Orin. 2006. Antipresuppositions. In Ayumi Ueyama (ed.), *Theoretical and empirical studies of reference and anaphora: Toward the establishment of generative grammar as an empirical science*, 52–73. Report of the Grant-in-Aid for Scientific Research (B), Project No. 15320052, Japan Society for the Promotion of Science. http://semanticsarchive.net/Archive/GI3YzhlM/AntipresuppositionsVersion1.pdf.

van Rooij, Robert & Katrin Schulz. 2004. Exhaustive interpretation of complex sentences. *Journal of Logic, Language and Information* 13(4). 491–519. http://dx.doi.org/10.1007/s10849-004-2118-6.

Sauerland, Uli. 2004. Scalar implicatures in complex sentences. *Linguistics and Philosophy* 27(3). 367–391. http://dx.doi.org/10.1023/B:LING.0000023378.71748.db.

Sauerland, Uli. 2008. Implicated presuppositions. In Anita Steube (ed.), *The discourse potential of underspecified structures*, 581–600. Berlin: Mouton de Gruyter. http://dx.doi.org/10.1515/9783110209303.4.581.

Sauerland, Uli. 2012. The computation of scalar implicatures: Pragmatic, lexical or grammatical? *Language and Linguistics Compass* 6(1). 36–49. http://dx.doi.org/10.1002/lnc3.321.

Schlenker, Philippe. 2012. *Maximize presupposition* and Gricean reasoning. *Natural Language Semantics* 20(4). 391–429. http://dx.doi.org/10.1007/s11050-012-9085-2.

Schwarz, Bernhard & Junko Shimoyama. 2011. Negative Islands and obviation by *wa* in Japanese degree questions. *Semantics and Linguistic Theory (SALT)* 20. 702–719. http://dx.doi.org/10.3765/salt.v20i0.2563.

Spector, Benjamin. 2006. *Aspects de la pragmatique des opérateurs logiques*. Université Paris Diderot - Paris 7 dissertation.

Spector, Benjamin. 2007. Scalar implicatures: Exhaustivity and Gricean reasoning. In Maria Aloni, Alastair Butler & Paul Dekker (eds.), *Questions in dynamic semantics*, 225–250. Elsevier. http://www.semanticsarchive.net/Archive/WU2MGJkN/Exhaustivity&Gricean_Reasoning.pdf.

Spector, Benjamin. 2016. Comparing exhaustivity operators. *Semantics and Pragmatics* 9(11). http://dx.doi.org/10.3765/sp.9.11.

Westera, Matthijs & Adrian Brasoveanu. 2014. *Ignorance in context: The interaction of modified numerals and QUDs*. Semantics and Linguistic Theory (SALT) 24. http://dx.doi.org/10.3765/salt.v24i0.2436.

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