Semantics & Pragmatics Volume 2, Article 3: 1–78, 2009
doi: 10.3765/sp.2.3

# Local Contexts*

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Received 2008-02-08 / First Decision 2008-04-27 / Revised 2008-05-19 / Second Decision 2008-07-08 / Revised 2008-07-17 / Third Decision 2008-09-16 / Revised 2008-11-17 / Accepted 2008-12-05 / Published 2009-07-06

**Abstract** The dynamic approach posits that a presupposition must be satisfied in its local context. But how is a *local context* derived from the global one? Extant dynamic analyses must specify in the lexical entry of any operator what its 'Context Change Potential' is, and for this very reason they fail to be sufficiently explanatory. To circumvent the problem, we revise two assumptions of the dynamic approach: we take the update process to be derivative from a classical, non-dynamic semantics — which obviates the need for dynamic lexical entries; and we deny that a local context encodes what the speech act participants 'take for granted.' Instead, we take the local context of an expression $E$ in a sentence $S$ to be the *smallest domain that one may restrict attention to when assessing* $E$ without jeopardizing the truth conditions of $S$. To match the results of dynamic semantics, local contexts must be computed *incrementally*, using only information about the expressions that precede $E$. This version of the theory can be shown to be nearly equivalent to the dynamic theory of Heim 1983 — but unlike the latter, it is entirely predictive. We also suggest that local contexts can, at some cost, be computed *symmetrically*, taking into account information about all of $S$(except $E$); this leads to gradient predictions, whose assessment is left for future research.

* I wish to thank the following for discussions, critical comments and suggestions: Emmanuel Chemla, Danny Fox, Ben George, Ted Gibson, Vincent Homer, Nathan Klinedinst, Nick LaCasse, François Recanati, Daniel Rothschild, Benjamin Spector, Ed Stabler, and the participants to my seminars at UCLA in the Fall of 2007 and at NYU in the Fall of 2008. Special thanks to three anonymous referees and to both editors of *Semantics and Pragmatics*, who provided exceptionally detailed comments and criticisms. A shorter and less technical version of this theory is developed in Schlenker 2009; some expository parts are common to both papers.

**Keywords:** Presupposition, Local Context, Dynamic Semantics, Transparency Theory

The notion of 'local context' has played a key role in the analysis of presupposition projection since the pioneering works of Stalnaker (1974), Karttunen (1974) and Heim (1983). These authors took local contexts to result from a rational process of belief update (Stalnaker) or from an enriched compositional semantics (Karttunen and Heim). We argue that these accounts were insufficiently predictive, and we offer an alternative analysis that addresses this challenge. The intuition we develop is that the local context of an expression *E* represents the smallest domain of objects that the interpreter needs to consider when he assesses the contribution of *E* to the meaning of the discourse. We take the computation of the local context to be preferably done 'on the fly,' as soon as *E* is processed — with the result that information that comes before *E* is typically taken into account, while information that comes after *E* isn't. Our analysis is couched in a classical, non-dynamic semantics; it predicts in great generality the value of any local context once the syntax and bivalent semantics of a language have been specified. By adopting the traditional assumption that a presupposition must be satisfied in its local context, we obtain a predictive account of presupposition projection without recourse to dynamic semantics.

Our project is introduced in Section 1, its core is developed in Section 2, and some extensions are considered in Section 3. The analysis is compared with another non-dynamic account, the Transparency theory, in Appendix A; and Appendices B–D offer a detailed implementation of the framework.

## 1 Local Contexts and Dynamic Semantics

### 1.1 The Dynamic Approach

A powerful intuition behind much recent research is that *a presupposition must be satisfied in the context in which it is evaluated.* The relevant notion of context is, in Stalnaker's (1978) terminology, the 'context set,' which encodes what the speech act participants take for granted (we will often say 'context' for brevity).[1] But an unadorned version of this analysis faces immediate

1 In the literature on indexicals, the term 'context' refers to an object that determines the speaker, time and world of the utterance; the indexical notion should be clearly distinguished from the presuppositional one. A context set can sometimes be equated to a *set* of contexts in the indexical sense (e.g. Dekker 2000, Schlenker 2004).

difficulties with complex sentences: *John is incompetent and he knows that he is* does not require that the speech act participants already take for granted that John is incompetent, since this proposition is asserted, not presupposed. The dynamic approach solves the problem by postulating that the second conjunct is evaluated with respect to a *local* context, obtained by updating the global one with the content of the first conjunct; this explains why the presupposition of the second conjunct is in this case automatically satisfied. This analysis is captured by the dynamic rule stated in (1): the update of a context $C$ with a conjunction is the successive update of $C$ with each conjunct.

(1) $C[F \text{ and } G] = C[F][G]$

Despite its considerable appeal, standard versions of this analysis suffer from several well-known deficiencies. We will only discuss two versions here: Stalnaker's (1974) and Heim's (1983). In Stalnaker's pragmatic analysis, the dynamic approach takes the update to result from a rational process of information exchange. Stalnaker's theory works beautifully for unembedded conjunctions because the assertion of a conjunction can plausibly be equated with the successive assertion of each conjunct; on the assumption that the context is updated after each act of assertion, the update rule in (1) falls out naturally. But this analysis does not easily extend to environments in which an expression does not have assertive force, as happens when a presupposition trigger appears in the scope of other connectives or operators:

(2) a. John is not incompetent, or he is aware that he is.
b. None of my students is both incompetent and aware of it.

In (2a) *aware* is in the scope of a disjunction, which the speaker can assert without being committed to either disjunct; it is unclear how an assertion-based analysis of local contexts can work in this case. In (2b), the presupposition of the second conjunct is somehow satisfied by the first conjunct, but an assertion-based analysis cannot easily explain this fact: first, because the context set would in the present case need to have predicative rather than propositional type (and it isn't clear what it means to believe something of predicative type); second, because in any event both conjuncts are in the scope of a negative quantifier, and thus neither of them is asserted.

In fact, even in the most favorable cases (unembedded conjunctions, or sequences of sentences in discourse), there is little reason to assume that the addressee must necessarily *grant F* after he has heard the speaker assert it — after all, the speaker might well be wrong, and the addressee might have every reason *not* to believe him.[2] A proponent of Stalnaker's theory might argue that all that matters is that the addressee *pretends* to accept the speaker's claim; but even fictional acceptance leads to difficulties. Analyzed in terms of common belief, a context (whether real or fictional) is intrinsically symmetric between the beliefs of the speaker and those of the addressee. But this very symmetry makes it difficult to explain why (3a) is Moore-paradoxical while (3b) isn't:

(3) a. #It is raining but I (still) don't believe it.
b. It is raining but you (still) don't believe it.

If the context set is really updated with the first conjunct before the second one is processed, both sentences should be equally deviant: after the first update, the context set entails that it is raining; this means in particular that the speaker believes that it is raining, and that the addressee believes it too. When we come to the second conjunct, we should obtain exactly the same deviance in both cases. But in fact there is a clear difference between (3a) and (3b): the former is Moore-paradoxical, the latter isn't; it seems that in (3b) the purported update process need not apply. A natural account can be given if the context set is *not* updated with the content of the first conjunct (but at most with the information that the speaker *believes* the first conjunct): due to the first conjunct, (3a) can only be asserted if the speaker believes that conjunct — but this is contradicted by the content of the second conjunct; no such problem arises in (3b), since the speaker and the addressee need not hold the same beliefs.[3]

2 This discussion hinges on the (Stalnakerian) assumption that the context set encodes what the speech act participants take for granted. See Simons 2003 for refinements and extensions of this analysis, and Thomason et al. 2006 for a rather different picture.

3 See Gillies 2001 for a recent analysis of Moore's paradox within dynamic semantics (Gillies (2001) does not discuss the asymmetry in (3), however). As the Editors have suggested, within Stalnaker's approach one could argue that (3b) is coherent because the present tense refers to a time that *precedes* the point at which the context set is updated. This would yield a meaning akin to: *It is raining at time t and you don't believe it at time t*, with the evaluation of *you don't believe it ⟨at time t⟩* taking place at a later time — say $t + 300\,\text{ms}$. This would make it possible to preserve Stalnaker's view of context update: at $t + 300\,\text{ms}$, the addressee adds to his belief the fact that *at t* he didn't believe that it was raining — and no contradiction ensues. But this line of argument won't work in (i):

(i) It is raining, and despite the fact that I have just told you so for the second time, you still don't believe it — you really are stubborn.

In its semantic incarnation (Heim 1983), the dynamic approach makes the update process part and parcel of the compositional semantics. Karttunen 1974 had already provided rules of compositional context update for each propositional connective and operator. However his analysis had two drawbacks: it did not establish any relation between the truth-conditional contribution of a connective and its context update behavior; and it did not extend to quantifiers. Heim (1983) addressed both problems. First, she took the meaning of any expression to be a Context Change Potential, and she showed that the truth-conditional contribution of an operator could be derived from it — which established a clear relation between the two notions. Second, she developed an account that extended to quantifiers.

In Heim's (1983) theory, the rule for *and* in (1) is preserved, but it is interpreted in semantic rather than pragmatic terms — which avoids the technical and conceptual problems raised by Stalnaker's analysis. It is possible to recover the standard meaning of *and* from this rule: in general, we can take a non-presuppositional sentence $H$ to be true at a world $w$ just in case $w$ 'survives' the update with $H$, or in other words if $\{w\}[H] = \{w\}$. For $H = F \textit{ and } G$, we do get the result that $H$ is true at $w$ just in case $F$ and $G$ are both true at $w$. But although truth conditions can be recovered from Context Change Potentials, the converse is not true. As was noted early on, there are a variety of dynamic connectives that are compatible with the truth conditions of *and* — for instance the deviant conjunction *and**, defined in (4) from (Soames 1989):[4]

(4) $C[(F \textit{ and* } G)] = C[G][F]$

(4) predicts that *John is incompetent and* he knows that he is* should come out as a presupposition failure, whereas *John knows that he is incompetent and* he is* should be entirely acceptable; this is of course the opposite of

Here *I have just told you so for the second time* is naturally understood as making reference to the first conjunct; and it is the addressee's skepticism after this reiteration which is taken as a sign of stubbornness. Interpreting (i) as *It is raining at t, and despite the fact that I have just told you so at* $t' < t$*, you still don't believe it at t* would yield the wrong truth conditions. By contrast, with the assumption that there is a 300 ms interval between the assessment of the conjuncts, we obtain the desired truth conditions if we understand (i) to mean: *It is raining at t, and despite the fact that at t I have just told you so for the second time, you still don't believe it at* $t + 300$ ms. However, if the addressee updates his beliefs with the first conjunct before assessing the second one, a pragmatic contradiction should be obtained, contrary to fact.

4 Heim (1992) attributes a similar observation to Mats Rooth, in a letter written in 1986.

what we find in natural language. Heim's account fails to be predictive in the following sense: if we are given the syntax and classical truth-conditional behavior of an operator, we cannot thereby predict how it will transmit presuppositions — a point that was repeatedly made by Heim (1990b, 1992) herself. To put it differently, any classical operator can be 'dynamicized' in a variety of ways; its dynamic extensions make the same predictions with respect to non-presuppositional sentences, but they make conflicting predictions about presupposition projection. But Heim's analysis does not provide a general recipe for choosing the 'right' dynamic connectives.

The problem isn't academic. For although there is general agreement about the dynamic entry of *and*, there are significant disagreements about other operators. Thus Beaver 2001 provides a dynamic entry for *or* equivalent to that in (5a), which predicts that $(G \textit{ or } H)$ inherits the presuppositions of $G$, as well as a conditional presupposition that *if G does not hold, the presupposition of H is satisfied.* By contrast, Geurts 1999 defines a dynamic entry that predicts that a disjunction inherits the presuppositions of each of its component parts, as indicated in (5b) (see also Krahmer 1998 for discussion); and there are other conceivable choices (throughout this discussion, we use # to encode semantic failure).[5]

5 The rule in (i) is the mirror image of that in (5a):

(i) $C[(G \textit{ or}_3 H)] = C[(H \textit{ or}_1 G)]$

In case one takes (i) to be 'perverse' because, in some sense to be made clear, it executes the update operations in the 'wrong' order, one should note that (ii) below is empirically quite reasonable: the utterance of $(G \textit{ or } H)$ only requires that either *not G entails the presupposition of H*, or *not H entails the presupposition of G* (see Rothschild 2008c for a defense of such an analysis).

(ii) $C[(G \textit{ or}_4 H)] \neq \#$ iff $(C[G] \neq \# \textit{ and } C[(\textit{not } G)][H] \neq \#)$
or $(C[H] \neq \# \textit{ and } C[(\textit{not } H)][G] \neq \#)$.
If $\neq \#$, $C[(G \textit{ or}_4 H)] = C[G] \cup C[(\textit{not } G)][H]$ if $C[G] \neq \#$ and $C[(\textit{not } G)][H] \neq \#$;
otherwise, $C[(G \textit{ or}_4 H)] = C[H] \cup C[(\textit{not } H)][G]$.

(5) a. $C[(G\ or_1\ H)] = \#$ iff $C[G] = \#$ or $C[(not\,G)][H] = \#$.
If $\neq \#$, $C[(G\ or_1\ H)] = C[G] \cup C[(not\,G)][H]$.
b. $C[(G\ or_2\ H)] = \#$ iff $C[G] = \#$ or $C[H] = \#$.
If $\neq \#$, $C[(G\ or_2\ H)] = C[G] \cup C[H]$.

Similarly, Heim 1983 predicted for quantified sentences of the form *No student has stopped smoking* a universal presupposition that *every student used to smoke*. By contrast, Beaver 1994 predicted an existential presupposition, namely that *at least one student used to smoke*. But there was nothing in either theory to force one choice over the other. Finally, when it comes to a connective like *unless*, dynamic theories make essentially no predictions. More precisely, even if we tell these theories that *unless F, G* means the same thing as *if not F, G* when *F* and *G* contain no presupposition triggers, they are still unable to predict that *unless F, G* should display the same projective behavior as *if not F, G*, as is suggested by (6).[6]

(6) a. If John didn't leave, Mary will know that he is here.
b. Unless John left, Mary will know that he is here.
Presupposition of (6): *If John didn't leave, he is here.*

In both cases, *not F* can serve to justify the presupposition of *G*. But from the observation that *if not F, G* and *unless F, G* have the same classical semantics (i.e. the same semantic behavior in non-presuppositional cases), it does not follow within dynamic semantics that that they should also transmit presuppositions in the same way.[7]

6 See Soames 1982 and Beaver 2001 for a discussion of similar data.

7 See Schlenker 2007 for a slightly more detailed discussion. To illustrate the formal problem, it can be observed that the dynamic entries in (i) make the same predictions when F and G are non-presuppositional, but that they disagree in the presuppositional case.

(i) a. $C[unless_1\ F, G] = \#$ iff $C[F] = \#$ or $C[(not\,F)][G] = \#$.
If $\neq \#$, $C[unless\ F, G] = C - C[(not\,F)][(not\,G)]$.
b. $C[unless_2\ F, G] = \#$ iff $C[F] = \#$ or $C[F][G] = \#$.
If $\neq \#$,... (as in (a)).
c. $C[unless_3\ F, G] = \#$ iff $C[F] = \#$ or $C[G] = \#$.
If $\neq \#$,... (as in (a)).
d. $C[unless_4\ F, G] = \#$ iff $C[G] = \#$ or $C[(not\,G)][F] = \#$.
If $\neq \#$, $C[unless\ F, G] = C - C[(not\,G)][F]$.

To my knowledge, no general solution has been offered to this problem in the published literature.[8] For clarity, it might be useful to distinguish two aspects of the difficulty.

First, is the dynamic analysis *predictive*? As usually stated, it is not, in the sense that it fails to predict how operators whose presuppositional behavior is not initially stipulated are to transmit presuppositions. It is sometimes thought that independent evidence about the 'right' lexical entries can be provided by other phenomena, in particular by anaphora resolution. And indeed, it is generally accepted that anaphora and presupposition resolution share many properties — so that anaphoric data might well suffice to decide which are the right dynamic lexical entries. But since an entire line of analysis, the 'E-type approach' (e.g. Evans 1980, Ludlow 1994, Heim 1990a), treats pronouns as concealed definite descriptions, which are themselves presupposition triggers, it is not obvious that anaphoric data count as 'independent' evidence for an analysis of presuppositions; they might well be presuppositional data in pronominal clothing.[9] So it is currently an open question whether non-presuppositional evidence can help choose among various dynamic operators.

Second, is the dynamic analysis *parsimonious*? Even if one could find independent evidence for positing certain dynamic operators rather than others, it would still seem that a theory that needs as many axioms as there are lexical entries is less desirable than one which provides a general recipe by which the presuppositional behavior of *any* connective is predicted once its classical semantics has been specified.

Our goal in the present paper is to offer a solution to (both aspects of) the problem faced by Heim's analysis. Specifically, we will address the following challenge:

(7) **Explanatory Challenge**
Find an algorithm that predicts how any operator transmits presuppositions once its syntax and its classical semantics have been specified.

We solve the problem by developing a modular theory: the semantic module is non-dynamic and fully classical; the pragmatic (or at least post-semantic) module yields an algorithm that computes the value of all relevant local

8 See, however, LaCasse 2008 and Rothschild 2008c for very interesting attempts to constrain dynamic semantics.

9 Other researchers — in particular van der Sandt 1993 and Geurts 1999 — unify the two phenomena in a different way, by reducing presupposition projection to anaphora resolution.

contexts on the basis of the classical semantics. In this way, once the semantic behavior of an operator with respect to non-presuppositional (and non-anaphoric) data is specified, its presuppositional behavior is thereby predicted as well.

The pragmatic module incorporates an incremental component, which for instance accounts for the asymmetric behavior of conjunction. We take this incremental feature to stem from the fact that local contexts are preferably computed 'early' in the processing of a sentence, without waiting for the entire sentence to be processed. We will briefly suggest, however, that this is just a preference, and that local contexts may, at some cost, be computed 'late', using semantic information about the entire sentence (this derives new predictions, whose assessment is left for future research).

### 1.2 A New Analysis of Local Contexts

The explanatory problem outlined above has led some to throw the dynamic baby out with its lexicalist bathwater (Schlenker 2007, 2008a; George 2008a,b; Chemla 2008b). But this measure was premature: it is possible to reconstruct a notion of 'local context' which is extremely close to that of dynamic semantics, but is derived from a fully predictive algorithm. In order to achieve this result, however, we depart from both sides of the dynamic tradition. Against the pragmatic line, we deny that local contexts result from an update of the beliefs of the speech act participants. Against the semantic line, we deny that they are the product of intrinsically dynamic meanings.

In a nutshell, we take the local context of an expression $E$ to be the minimal domain of objects that the interpreter needs to consider when he attempts to compute the meaning of a sentence. How can this notion of 'minimal domain' be motivated? The interpreter's task is to determine which worlds of the context set are compatible with the speaker's claim; in other words, he must compute a function from worlds in the context set to truth values. To do so, he has access to the context set $C$, and to the meaning of the words, which we take to be functions of various types. Now we will assume (i) that it is easier to perform the steps of the computation when part of the domain of a function can be disregarded, (ii) that the interpretation is performed incrementally, and (iii) that before processing any expression, the interpreter tries to simplify his task as much as possible given what he already knows about the meaning of the sentence. From these assumptions, it follows that the interpreter will try to decide in advance of interpreting any expression $E$ what is the smallest domain that he needs to consider when he assesses the meaning of $E$; this 'smallest domain' is our notion of a local context.

Let us make the intuition clear with an example. We are in a context $C$, and we have heard the speaker say: *If John used to smoke, E*. We set out to assess the value of the consequent $E$ of this conditional, which we analyze for simplicity as a material implication. A possible strategy would be to check the value of $E$ in *all* possible worlds. But for the purposes of the conversation, we are solely interested in those worlds that lie in $C$, because all other worlds are excluded by the shared assumptions of the conversation partners. For this reason, it won't hurt to replace $E$ with ${}^{c'}E$, where $c'$ denotes $C$ and ${}^{c'}E$ is interpreted as the conjunction of $c'$ and $E$; this makes it possible to only consider the value of $E$ in the $C$-worlds, without paying attention to the value it may have outside of $C$. Since we will repeatedly use the notation ${}^{c'}E$, it is worth introducing explicitly right away:

(8) **Local Context Notation**
If $E$ is of a type that 'ends in $t$' (in particular, if $E$ is of propositional or predicative type), and if $c'$ is of the same type as $E$, ${}^{c'}E$ is interpreted as the (generalized) conjunction of $c'$ and $E$.[10]

In our example, when $c'$ denotes $C$, it will not affect the computation of the meaning of the sentence relative to the context set; for this reason, we will say that $c'$ is an 'innocuous' restriction on $E$. But we can find a smaller restriction which is equally innocuous by only considering those $C$-worlds in which John smoked, since all worlds in which John never smoked will make the conditional true no matter what the value of $E$ turns out to be. We take the local context of $E$ to be the *smallest* restriction of this sort that one can make without jeopardizing the computation of the truth conditions of the entire sentence. In the case at hand, the smallest such restriction is just the set of those $C$-worlds in which John smoked. For if any such world $w$ were excluded from $c'$, we *would* take a risk by computing ${}^{c'}E$ instead of $E$. To see this, suppose that $E$ is true at w. Then of course *if John used to smoke*, $E$ is true as well at w; but *if John used to smoke*, ${}^{c'}E$ is false because $c'$ excludes w — an undesirable result.

10 Importantly, this notation applies when $E$ is of propositional type, but also when it is of predicative type. In the latter case, we want the local context of $E$ to be itself predicative, which will prove crucial when we develop an account of presupposition projection in quantified structures.

We assume that the computation of the local context of an expression *E* is preferably 'done on the fly', on the basis of the information that has been heard before one processes *E*. This gives rise to an asymmetry: information that comes before *E* is known when the local context is computed, but information that comes after *E* isn't, and the interpreter must therefore ensure that *no matter how the sentence ends* the local context will indeed be innocuous. Let us illustrate this incremental procedure by considering the interpretation of $(p\ and\ q)$ uttered in a context set $C$. To minimize clutter in our notations, we adopt the following convention:

(9) If *E* is an expression, we write its semantic value as **E**.
If w a world and s an assignment function, $\mathbf{E}^{w}$ is the value of *E* at w, and $\mathbf{E}^{w,s}$ is the value of *E* at w evaluated with respect to s.

We can now illustrate how the computation of meaning is performed for a simple sentence $(p\ and\ q)$, first without local contexts (in figure 1a), and then with local contexts (in figure 1b). We take the context set to be $C = \{w_1, w_2, w_3\}$, and we write $\wedge$ for (generalized) conjunction.

In figure 1a, the meaning of $(p\ and\ q)$ relative to $C$ is computed without making use of local contexts: the interpreter accesses all of the function **p**, all of the function **q**, and computes the meaning of $(p\ and\ q)$ over the entire domain of possible worlds, which he conjoins to the context set $C$ to obtain the final result on the right-hand side. In figure 1b, by contrast, the interpreter makes use of the local context $c'$ of $p$, and of the local context $c''$ of $q$. When he sets out to interpret $p$, he knows that the context set is $C$, but he does not know anything else about the rest of the sentence. So the local context of the first conjunct $p$ is just $\mathbf{c}' = \{w_1, w_2, w_3\} = C$. This allows him to restrict attention to that part of the domain of **p** which is in $\mathbf{c}'$ by uniformly assigning the value 0 to all other worlds. Making use of this information, he computes $^{\mathbf{c}'}\mathbf{p}$ instead of **p**, knowing that this simplification won't affect the semantic result he is after. Before he starts interpreting $q$, he already has access to **p** (or rather to $^{\mathbf{c}'}\mathbf{p}$), and he can decide to disregard all worlds that make the first conjunct false, since these will make the entire sentence false no matter what the second conjunct turns out to be. By systematizing this reasoning, we can determine that $\mathbf{c}''$ is just $\{w_1, w_2\}$ (i.e. those $C$-worlds in which $p$ is true); making use of this information, the interpreter then computes $^{\mathbf{c}''}\mathbf{q}$. The final result on the right-hand side is exactly the same as in figure 1a; but in figure 1b the interpreter has made optimal use of information available at each step by uniformly assigning the value 0 to as many worlds as possible

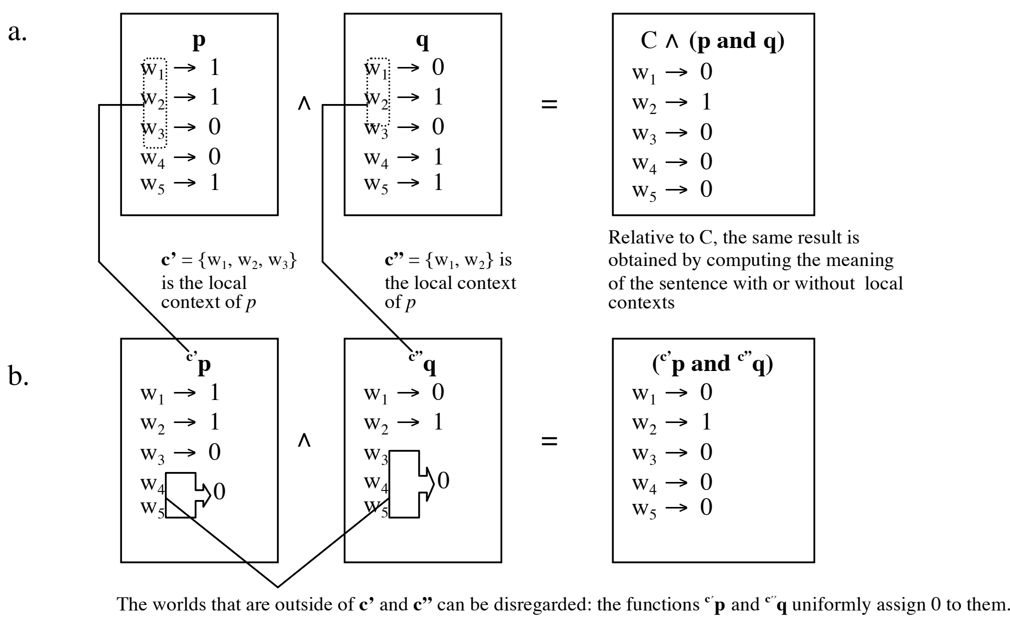


**Figure 1** Computation of the meaning of $(p\ and\ q)$ relative to $C = \{w_1, w_2, w_3\}$

before interpreting the next expression (this procedure may remind some readers of the 'Strong Kleene' or 'supervaluationist' trivalent logics, but there are important conceptual, technical and empirical differences between these frameworks and the present analysis; a brief comparison is offered in Section 3.4).

## 2 Reconstructing Local Contexts

### 2.1 Formal Preliminaries

Before we define local contexts and apply them to the analysis of presupposition, let us introduce our formal framework. We work within a bivalent semantics, and we assume that the presupposition $d$ of an expression $\underline{d}d'$ in a syntactic context $a\_b$ must be entailed by its local context given the global context set. Following the spirit of Stalnaker's approach, we take this requirement to be pragmatic in nature; as a result, presupposition failure need not be encoded in the semantics itself. Thus we treat the semantics as bivalent, and we interpret $\underline{d}d'$ as the conjunction of $d$ and $d'$; the fact that $\underline{d}$ is underlined has no *semantic* import, but it is crucial to indicate to the *pragmatics* that $d$ must be entailed by its local context (so $\underline{p_i}p_k$ will in the end represent a proposition with a presupposition $p_i$ and an assertive content $p_k$; $\underline{P_i}P_k$ has the same interpretation, except that each element is predicative rather than propositional).

In the rest of this discussion, we provide formal details as are needed to offer a self-contained presentation of the theory and of a few examples; systematic definitions and general results can be found in Appendix C. In order to keep the discussion manageable, we assume in most of the discussion a simplified formal syntax, summarized in (10), in which constituency is encoded by parentheses: conjunctions and disjunctions have the form $(F\ \mathit{and}\ G)$ and $(F\ \mathit{or}\ G)$, negations have the form $(\mathit{not}\ F)$, and generalized quantifiers and conditionals appear as $(Q\,F.\,G)$ and $(\mathit{if}\ F.\,G)$ respectively. Since the system is intensional, propositions are of type $\langle s,t\rangle$ and predicates are of type $\langle s,\langle e,t\rangle\rangle$ (for brevity, we will speak of the type of an expression, but also of the set-theoretic object it corresponds to).

(10) **Syntax**

a. Generalized Quantifiers: $Q ::= Q_i$
b. Predicates: $P ::= P_i \mid \underline{P_i}P_k$ (Type: $\langle s, \langle e, t\rangle\rangle$)
c. Propositions: $p ::= p_i \mid \underline{p_i}p_k$ (Type: $\langle s, t\rangle$)
d. Formulas: $F ::= p \mid (not\, F) \mid (F\; and\; F) \mid (F\; or\; F) \mid (if\; F.F) \mid (Q_i\, P.P)$

The 'official' object language is supplemented with the local context notation introduced in (8).[11] As mentioned, we view the local context of an expression $E$ in a sentence $S$ relative to a context set $C$ as the smallest set-theoretic object (of the type determined by $E$) that one can restrict attention to when assessing the contribution of $E$ to the truth conditions of $S$ relative to $C$. To implement this idea, we must decide what 'small' and 'restrict attention to' mean. Both notions can easily be defined if $E$ is of a type that 'ends in $t$', for instance $\langle s, t\rangle$, $\langle s, \langle e, t\rangle\rangle$, etc. In this case, 'smaller' will mean 'entails', with a generalized notion of entailment; and one may 'restrict attention to' $x$ when evaluating $E$ if when $c'$ denotes $x$, $c'$ is an innocuous restriction on $E$ — in other words, $E$ can be replaced with ${}^{c'}E$ in $S$ without risk that this might affect the truth conditions of $S$ relative to $C$. We will use the symbol $\leq$ to denote generalized entailment both in the object language and in the meta-language, and — as mentioned — we write $\wedge$ in the meta-language for generalized conjunction. It is essential that entailment and conjunction be 'generalized' because the local context of a propositional element is propositional, whereas the local context of a predicative expression is itself predicative; and we want our definitions to apply to both cases.[12] We remind the reader of the relevant definitions in (11) and (12), but these are entirely standard.

(11) **Generalized Entailment**

a. If $x$ and $x'$ are two objects of a type $\tau$ that 'ends in $t$', and can take at most $n$ arguments, $x \leq x'$ just in case whenever $y_1, \ldots, y_n$ are objects of the appropriate type, if $x(y_1)\ldots(y_n) = 1$, then $x'(y_1)\ldots(y_n) = 1$.

11 We will sometimes extend the object language with predicate conjunctions, which are written as $(P\; and\; P')$ and receive the natural interpretation.

12 We use notations from Boolean Algebra to emphasize the commonalities between the predicate and the propositional case. In both domains, $x \wedge y$ represents the 'greatest lower bound' of $x$ and $y$, and $x \leq y$ indicates that $x \wedge y = x$ (in fact, this could be taken as a definition of $\leq$ in terms of $\wedge$).

b. If $E$ and $E'$ are two expressions of a type $\tau$ that 'ends in $t$', $w \vDash^{s} (E \leq E')$ iff $\mathbf{E}'^{w,s} \leq \mathbf{E}^{w,s}$.

(12) **Generalized Conjunction**

a. If $x$ and $x'$ are two objects of a type $\tau$ that 'ends in $t$', and can take at most $n$ arguments, of types $\tau_1, \ldots, \tau_n$ respectively, then $x \wedge x' = \lambda y_{1\tau_1} \lambda y_{n\tau_n} x(y_1)\ldots(y_n) = x'(y_1)\ldots(y_n) = 1$.

b. If $E$ and $E'$ are two expressions of a type $\tau$ that 'ends in $t$', $({}^{\mathbf{E}'}\mathbf{E})^{w,s} = (\mathbf{E}' \textit{ and } \mathbf{E})^{w,s} = \mathbf{E}'^{w,s} \wedge \mathbf{E}^{w,s}$.

The rest of this section is devoted to a theory of incremental contexts and incremental satisfaction, which is the closest counterpart in our system of the local contexts of dynamic semantics (a symmetric version of these notions is defined in Section 4).

## 2.2 Local Contexts and Local Satisfaction

### 2.2.1 Introduction

Let us start with an example. As we informally saw in figure 1, when we evaluate in $C$ a sentence that starts with $(p \textit{ and } q \ldots$, we only need to be concerned with the value that $q$ has in those $C$-worlds that satisfy $p$ — all other words will be irrelevant, either because they lie outside of $C$, or because they make $p$ false, and thus make the entire conjunction false as well no matter what the second conjunct is. In other words, we can be certain that no matter what the end of the sentence — call it $b'$ — turns out to be, the restriction to the worlds in $C \wedge \mathbf{p}$ will be innocuous. Calling 'good final' a string that turns the beginning of a sentence into a complete sentence,[13] we can thus assert (13):

(13) For every constituent $d'$, for every good final $b'$,
$C \vDash^{c' \to C \wedge \mathbf{p}} [(p \textit{ and } {}^{c'} d' b'] \Longleftrightarrow [(p \textit{ and } d' b']$

Here we employ standard notations from modal logic: $C \vDash^{c' \to \mathbf{p}} F$ means that under an assignment function in which $c'$ denotes $\mathbf{p}$, every world $w$ in $C$ makes $F$ true (i.e. every world $w$ in $C$ guarantees that $w \vDash^{c' \to \mathbf{p}} F$). Importantly, $d'$ and $b'$ are meta-variables over strings, which explains why $(p \textit{ and } {}^{c'} d' b'$ does not seem to contain the same number of left and of right

13 Good finals were called 'sentence completions' in Schlenker (2007). Thanks to Ed Stabler for pointing out that the term 'good final' belongs to established terminology.

parentheses: if the string denoted by $b'$ really is a good final, it will contain enough right parentheses to turn the beginning of the sentence into a well-formed formula.[14] With these conventions, (13) means that if $c'$ denotes $C \wedge \mathbf{p}$, for any good final $b'$ the formula $(p\ and\ ^{c'}d'\,b'$ is equivalent (relative to $C$) to the formula $(p\ and\ d'\,b'$: the restriction to $c'$ is innocuous.

To show that $C \wedge \mathbf{p}$ is the local context of $q$, we need to show that it is the *smallest* innocuous restriction that one can find; in other words, we need to establish that $C \wedge \mathbf{p}$ is a subset of every innocuous restriction. So we suppose, for contradiction, that the denotation of $c'$ excludes a $C$-world $w$ that satisfies $p$. If the sentence turns out to be $(p\ and\ t)$, where $t$ is true at $w$, we will have the unfortunate result that $w$ makes $(p\ and\ t)$ true, but that it makes $(p\ and\ ^{c'}t)$ false. Thus $c'$ is not innocuous in the end. This shows that $C \wedge \mathbf{p}$ is indeed the local context of the second conjunct.

### 2.2.2 Definitions

In the general case, local contexts are best defined in two steps. First, we find the set of denotations that make $c'$ truth-conditionally innocuous; we say in such cases (following the terminology of Schlenker (2007, 2008a)) that (the value of) $c'$ is 'transparent', or that it is a 'transparent restriction'. We then ask whether this set has a bottom element, i.e. one that entails all others; if so, it is the incremental local context of the expression (the case in which there is no bottom element is discussed in Appendix B). A notion of presupposition satisfaction is then easily defined: a propositional or predicative expression $\underline{d}d'$ is acceptable just in case $d$ is entailed by its local context (which will itself be propositional or predicative, as the case may be). In this part of our discussion, we stick to the intuition that restrictions must be innocuous *no matter what the end of the sentence* turns out to be. For this reason, our theory is incremental; analogous notions will be defined in Section 4 for a symmetric version of the theory.

The first step, then, is to define the set of denotations that make $c'$ (incrementally) transparent (or 'innocuous'). We write as $tr(C, d, a\_b)$ the set of transparent restrictions on the expression $d$ in a sentence of the form $a\ d\ b$ uttered in a context set $C$ (thus $a\_b$ is the syntactic environment in

14 In the case at hand, $(p\ and\ q\ldots$ could only have been produced by an application of the rule that creates conjunctions, and thus the entire sentence can only be $(p\ and\ q)$.

which the token $d$ occurs).

(14) $tr(C, d, a\_b) = \{x : x$ is an object of the type specified by $d$ and for every constituent $d'$ of the same type as $d$, for every good final $b'$, $C \vDash^{c' \to x} a\, {}^{c'}d\, b' \Longleftrightarrow a\, d'\, b'\}$

It can be observed that the string $b$ occurs on the left-hand side of the identity (14), but not on the right-hand side. This is as it should be: because the analysis is incremental, it can only access information which is available before $d$, which makes the precise identity of the string $b$ irrelevant.

Using the same notations, we can define the (incremental) local context of $d$ as the bottom element of $tr(C, d, a\_b)$, if it has one:

(15) $$lc(C, d, a\_b) = \begin{cases} \text{the bottom element}^{15} \text{ of } tr(C, d, a\_b), & \text{if it exists} \\ \# & \text{otherwise} \end{cases}$$

To illustrate, the reasoning we developed at the beginning of this section shows that $lc(C, q, (p\ and\ \_)) = C \wedge \mathbf{p}$.

Restricting attention to the case in which all local contexts exist, we can say that in a global context $C$ the presupposition $d$ of an expression $\underline{d}d'$ in a syntactic environment $a\_b$ is (incrementally) satisfied in its local context just in case it is entailed by it. We write this as $Sat(C, \underline{d}d', a\_b)$:

(16) $Sat(C, \underline{d}d', a\_b)$ just in case $lc(C, \underline{d}d', a\_b) \le \mathbf{d}$

For a sentence $S$ to be acceptable in $C$, it should be the case that for *every* expression of the form $\underline{d}d'$ that appears in $S$, $d$ is entailed by its local context. We write this as $Sat(C, S)$:

(17) $Sat(C, F)$ just in case for all expressions $\underline{d}d'$, for all strings $a, b$, if $F = a\ \underline{d}d'\ b$, then $Sat(C, \underline{d}d', a\_b)$

### 2.3 Examples

We can now apply these definitions to some traditional examples; in each case, we derive on the basis of a classical semantics the results that Karttunen

15 As mentioned, by 'bottom element' of $tr(C, d, a\_b)$, we mean an element $e$ such that for all $e' \in tr(C, d, a\_b)$, $e \le e'$. It is immediate that if a bottom element exists, it is unique: if $e_1$ and $e_2$ are both bottom elements, $e_1 \le e_2$ and $e_2 \le e_1$, so $e_1 = e_2$ (this is the case because $e_1$ and $e_2$ are set-theoretical objects rather than formulas).

1974, Heim 1983, and Beaver 2001 had to stipulate in the update rules of their operators.

Since we compute local contexts by quantifying over good finals, the details of our formal syntax become rather important. If we seek to determine, say, the incremental context of $F$ in the formula $(F\ and\ G)$, we must — metaphorically — put ourselves in the position of someone who has seen a left bracket, and asks himself what is the smallest restriction that will be innocuous when he assesses the meaning of $F$, which is not known yet. Now any formula that starts with $(F \ldots$ can be turned into a full sentence in a variety of ways, for instance by adding $and\ H)$, or for that matter by adding $or\ H)$, to obtain the formulas $(F\ and\ H)$ and $(F\ or\ H)$ respectively. And *all* these possibilities must be taken into account to determine whether a restriction $c'$ on $F$ is or is not transparent (i.e. innocuous). We will further assume that the language is extremely expressive, and that every proposition or property can be denoted.

### 2.3.1 Propositional Examples

**(i) ($\underline{p}p'$ *and q*) and ($\underline{p}p'$ *or q*) both presuppose *p***

To start with a particularly simple example, let us show that the incremental context of $p$ in any formula that starts with $(p \ldots$ is just the context set $C$. It is clear that this restriction will be innocuous. Furthermore, if $c'$ denotes a set $x$ that excludes a world $w$ of $C$, it will fail to guarantee that $w \vDash^{c' \to x} (c'\ d'\ and\ t) \iff (d'\ and\ t)$ in case $d'$ is true in $w$ and $t$ is a tautology: in this case, the left-hand side is false but the right-hand is true. Thus any value for $c'$ which excludes any world of $C$ will fail to be transparent; in other words, any transparent restriction must include every world of $C$. This means that $C$ entails every transparent value for $c'$, and thus that $C$ is the incremental context of $p$.

(18) $lc(C, p, (\_\ and\ q)) = lc(C, p, (\_\ or\ q)) = C$

This result explains why both (19a) and (19b) are understood to presuppose that *John is incompetent*:

(19) a. John knows that he is incompetent, and he is depressed.
b. John knows that he is incompetent, or he is depressed.

Since $p$ must be entailed by its local context, we obtain the desired result: $C$

must entail $p$.

(20) $Sat(C, (\underline{p}p' \; and \; q))$ iff $Sat(C, (\underline{p}p' \; or \; q))$ iff $C \vDash p$

**(ii) (*not* $\underline{\text{p}}$p′) presupposes *p***

The incremental context of $\underline{p}p'$ in the formula $(not \; \underline{p}p')$ is also the context set $C$. First, a restriction to $C$ is certainly innocuous. Second, by the same reasoning as in (i), we can see that if $c'$ denotes a set $x$ that excludes any world $w$ of $C$, we will fail to guarantee that $w \vDash^{c' \to x} (not \; {}^{c'}d') \iff (not \; d')$ in case $d'$ is true in $w$: in this case, the left-hand side is true because ${}^{c'}d'$ is false, while the right-hand side is false. This shows again that the incremental context of $\underline{p}p'$ is $C$.

(21) $lc(C, \underline{p}p', (not \; \_)) = C$

This result explains why (22) presupposes that *John is incompetent*, which motivates the standard conclusion that negations are 'holes' for presuppositions.

(22) John doesn't know that he is incompetent.

The incremental context of $\underline{p}p'$ in $(not \; \underline{p}p')$ is the global context $C$; and since $p$ must be entailed by its local context, we obtain the result that $C$ must entail $p$.

(23) $Sat(C, (not \; \underline{p}p'))$ iff $C \vDash p$

**(iii) (*p and* $\underline{\text{q}}$q′) presupposes (*if p . q*)**

The incremental context of $\underline{q}q'$ in the formula $(p \; and \; \underline{q}q')$ is $C \wedge \mathbf{p}$, as was shown at the beginning of this section. In Heim's notation, this derives the result that $C[p \; and \; \underline{q}q'] = C[p][\underline{q}q']$: the incremental context of $\underline{q}q'$ is the original context $C$, updated with $p$.

(24) $lc(C, \underline{q}q', (p \; and \; \underline{q}q')) = C \wedge \mathbf{p}$

This result predicts that (25) should presuppose that *if John is 64 years old, he cannot be hired*:

(25) John is 64 years old and he knows that he cannot be hired.

The incremental context of $\underline{q}q'$ is C ∧ **p**, which must entail **q**; but this is just to say that *C* must guarantee that *if p, q*.

(26) $Sat(C, (p\ and\ \underline{q}q'))$ iff $C \vDash (if\ p.q)$

The same prediction was made by Stalnaker 1974, Karttunen 1974 and Heim 1983, but it is not uncontroversial: van der Sandt 1993 and Geurts 1999 have argued that in many cases the conditional inference is too weak (this has been dubbed the 'Proviso Problem'). The same issue arises with respect to the presuppositions predicted for conditionals and disjunctions; we come back to this problem in Section 2.3.3.

### (iv) (*if p.*$\underline{\mathbf{q}}\mathbf{q}'$) presupposes *p*

The incremental context of $F$ in the formula $(if\ F.G)$ is $C$. For simplicity, we follow Heim 1983 in treating conditionals as material implications. It then follows that when $c'$ denotes $C$, it is innocuous in $(if\ {}^{c'}F.G)$. Furthermore, in case $c'$ denotes a set $x$ that excludes some world $w$ of $C$, if the sentence turns out to be $(if\ d'.b')$ where $d'$ is true in $w$ while $b'$ is false in $w$, we will have that $w \vDash^{c' \to x} (if\ {}^{c'}d'.b')$ (because $c'$ is false in $w$) but $w \nvDash^{c' \to x} (if\ d'.b')$ (because $d'$ is true and $b'$ is false in $w$). Therefore every transparent value for $c'$ must include all of $C$. $C$ is thus the incremental context of $F$.

(27) $lc(C, \underline{p}p', (if\ \_\_.q)) = C$

The projection of presuppositions out of the antecedent of indicative conditionals is a staple of presuppositional studies — so much so that it is sometimes taken as a defining feature of presuppositions. For instance, (28) clearly presupposes that John used to smoke.

(28) If John stopped smoking, he made a wise decision.

This result is immediately derived since we just showed that the context of $\underline{p}p'$ is $C$ itself.

(29) $Sat(C, (if\ \underline{p}p'.q))$ iff $C \vDash p$

**(v) ($if\ p\ .\ \underline{q}q'$) presupposes ($if\ p\ .\ q$)**

As was illustrated in Section 1.2, the incremental context of $\underline{q}q'$ in the formula $(if\ p.\underline{q}q')$ is also $C \wedge \mathbf{p}$. To reiterate, it is immediate that this restriction is innocuous. And if $c'$ denotes a set $x$ that excludes some $p$-world $w$ of $C$, in case $d'$ is true in $w$ we will have both that $w \vDash^{c' \to x} (if\ p.d')$ ($p$ is true, and so is $d'$) while $w \nvDash^{c' \to x} (if\ p.{}^{c'}d')$ ($p$ is true, but ${}^{c'}d'$ is false); hence $C \nvDash^{c' \to x} (if\ p.{}^{c'}d') \Longleftrightarrow (if\ p.d')$. So the smallest innocuous restriction we can find is $C \wedge \mathbf{p}$.[16]

(30) $lc(C, \underline{q}q', (if\ p.\_) = C \wedge \mathbf{p}$

This result predicts that (31) should have a conditional presupposition that *if John is 64 years old, he cannot be hired*:

(31) If John is 64 years old, he knows that he cannot be hired.

The reasoning is the same as for the conjunction in (iii): the sentence $(if\ p.\underline{q}q')$ is acceptable just in case $q$ is entailed by its local context, namely $C \wedge \mathbf{p}$. So it must be that $C \wedge \mathbf{p} \vDash q$, and hence that $C \vDash (if\ p.q)$.

(32) $Sat(C, (if\ p.\underline{q}q'))$ iff $C \vDash (if\ p.q)$

**(vi) ($p\ or\ \underline{q}q'$) presupposes ($if\ (not\ p)\ .\ q$)**

More interestingly, the incremental context of $p$ in $(p\ or\ \underline{q}q')$ is $C \wedge (\mathbf{not\ p})$ — which derives a result that Beaver 2001 argued for on the basis of presuppositional data. Within the present framework, the argument is quite direct: by propositional logic, $(p\ or\ d')$ is always equivalent to $(p\ or\ ((not\ p)\ and\ d'))$, hence $C \vDash^{c' \to C \wedge (\mathbf{not\ p})} (p\ or\ {}^{c'}d') \Longleftrightarrow (p\ or\ d')$; this establishes that $C \wedge (\mathbf{not\ p})$ is a transparent value for $c'$. On the other hand, if $c'$ denotes a set $x$ that excludes a (not $p$)-world $w$ of $C$, in case $d'$ is true in $w$ we will have that

16 We note for future reference that the same reasoning extends to $(if\ p.(not\ q))$: $C \wedge \mathbf{p}$ is clearly a transparent restriction for $q$, and furthermore it entails all transparent restrictions for $q$ (the argument is the same as for the preceding case, reversing the value that we consider for $d'$):

(i) $lc(C, q, if\ p.(not\ \_)) = C \wedge \mathbf{p}$

(This remark will be used in Appendix, in C.24(c).

$w \vDash^{c' \to x} (p \; or \, d')$ but $w \nvDash^{c' \to x} (p \; or \, {}^{c'} d')$, and thus $C \nvDash^{c' \to x} \; (p \; or \, d') \iff (p \; or \, {}^{c'} d')$; this shows that any transparent value for $c'$ must include *all* of $C \wedge$ (**not p**), which is thus the local context we were looking for.

(33) $lc(C, \underline{q}q', (p \; or \_\_)) = C \wedge$ (**not p**)

This result predicts that (34) should have a conditional presupposition that *if John is not less than 64 years old, he cannot be hired.*

(34) John is less than 64 years old, or he knows that he cannot be hired.

The result follows because in the sentence $(p \; or \, \underline{q}q')$ the presupposition $q$ of the second disjunct must be entailed by its local context, namely $C \wedge$ (**not p**). So we must have that $C \wedge$ (**not p**) $\vDash q$, and hence that $C \vDash (if \; (not \, p).\, q)$.

### 2.3.2 Quantificational Examples

In Stalnaker's pragmatic approach, it was difficult to see how a local context could be of predicative type, because predicate denotations are not the kind of things that can be believed. By contrast, our definition of local contexts straightforwardly applies to the predicative case, and derives Heim's predictions in almost all cases.

#### (vii) (*Every P* . $\underline{Q}$Q′) presupposes (*Every P* . *Q*)

Let us first compute the incremental context of the nuclear scope $Q$ in the quantified statement $(Every \; P.Q)$. For $c'$ to be conjoinable with $Q$ in the formula $(Every \; P.{}^{c'}Q)$, it must have the type of a predicate (= $\langle s, \langle e, t\rangle\rangle$). It turns out that the 'smallest' transparent value of $c'$ (i.e. the one that entails all other transparent values) is just the property **P** restricted to the context set, which we write as ${}^{C}\mathbf{P}$ and define as ${}^{C}\mathbf{P} = \lambda w_s \lambda x_e C(w) = 1 \; and \; P(w)(x) = 1$. To see that ${}^{C}\mathbf{P}$ is indeed the local context of $\underline{Q}Q'$, we argue in two steps.

First, it is clear that such a restriction is transparent, i.e. truth-conditionally innocuous: because natural language quantifiers are conservative, within $C$, $(Every \; P.{}^{c'}D')$ is equivalent to $(Every \; P.D')$ whenever $c'$ denotes ${}^{C}\mathbf{P}$.

Second, if $c'$ denotes a property $x$ and is transparent in $(Every \; P.{}^{c'}D')$, then $x$ is entailed by ${}^{C}\mathbf{P}$. For suppose, for contradiction, that this is not the case. Then there is some world $w$ of $C$ and some individual $d$ in the domain for which ${}^{C}\mathbf{P}(w)(d) = 1$ but $x(w)(d) = 0$. Taking $D' = P$, we will have the result that $w \vDash^{c' \to x} Every \; P.D'$, but $w \nvDash^{c' \to x} (Every \; P.{}^{c'}D')$ (since $d$ makes $P$

true, but it makes ${}^{c'}D'$ false). Thus $C \not\models^{c'\to x} (Every\ P.{}^{c'}D') \iff (Every\ P.D')$, which shows that $c'$ is not transparent after all.

(35) $lc(C, Q, (Every\ P.\_\_)) = {}^{C}\mathbf{P}$

This result predicts that the sentences in (36a) and (36b) have a universal presupposition that *every student has a computer*, and that those in (36c) and (36d) presuppose that *every student used to smoke*:

(36) a. Every student takes good care of his computer.
b. Does every student take good care of his computer?
c. Every student has stopped smoking.
d. Has every student stopped smoking?

The result follows because in the sentence $(Every\ P.\underline{Q}Q')$ the presupposition $\mathbf{Q}$ of the nuclear scope must be entailed by its local context, namely ${}^{C}\mathbf{P}$. But this is just to say that for every world $w$ in $C$, for every individual $x$ in the domain of $w$, if $\mathbf{P}(w)(x) = 1$, $\mathbf{Q}(w)(x) = 1$. In other words, the context set $C$ must guarantee *that every P-individual is a Q-individual.*

(37) $Sat(C, (Every\ P.\underline{Q}Q'))$ iff $C \models (Every\ P.Q)$

**(viii) (*No P . $\underline{\text{Q}}$Q′*) presupposes (*Every P . Q*)**

More surprisingly, the same result extends if we consider the formula $(No\ P.{}^{c'}Q)$: the local context of $\mathbf{Q}$ is just ${}^{C}\mathbf{P}$. This is an important observation because it guarantees that the projective behavior of $(No\ P.\underline{Q}Q')$ is identical to that of $(Every\ P.\underline{Q}Q')$: both sentences presuppose that every $P$-individual is a $Q$-individual.[17] Let us see how the result is derived.

By Conservativity, if $c'$ denotes ${}^{C}\mathbf{P}$ this restriction will be truth-conditionally innocuous.

Now suppose, for contradiction, that $c'$ denotes a property $x$ which is not entailed by ${}^{C}\mathbf{P}$, and thus that for some world $w$ and individual $d$, ${}^{C}\mathbf{P}(w)(d) = 1$ but $x(w)(d) = 0$. Pick a predicate $D'$ which is true of $d$ and nothing else in $w$ (i.e. $\mathbf{D}'(w) = \{d\}$). In such a case, $w \models^{c'\to x} (No\ P.{}^{c'}D')$ because the only member of $\mathbf{D}'(w)$, namely $d$, does not belong to $x(w)$, so

17 This result is also significant because it derives stronger presuppositions than a supervaluationist (or Strong Kleene) analysis would. We come back to this point in Section 3.4.

that the nuclear scope ${}^{c'}D'$ has an empty extension in $w$. On the other hand, $w \not\models^{c' \to x} (No\ P.D')$, because $d$ belongs both to ${}^{C}\mathbf{P}(w)$ and to $\mathbf{D}'(w)$. Thus $C \not\models^{c' \to x} (No\ P.{}^{c'}D') \Longleftrightarrow (No\ P.D')$, which shows that $c'$ is not transparent after all.

(38) $lc(C, Q, (No\ P.\_\_)) = {}^{C}\mathbf{P}$

This analysis predicts that the sentences in (39) should trigger universal presuppositions:

(39) a. None of these ten students takes good care of his computer.
⇒ Each of these ten students has a computer.
b. None of these ten students has stopped smoking.
⇒ Each of these ten students used to smoke.

This result follows from the present theory because the local context of $\underline{Q}Q'$ in *(No P . $\underline{Q}Q'$)* is ${}^{C}\mathbf{P}$, which must thus entail **Q**. But this is exactly the same condition we had in (vii) for the sentence *(Every P . $\underline{Q}Q'$)*, and thus in the present case as well it should be that relative to $C$ every $P$-individual satisfies $Q$.

(40) *Sat*(C, (No P . $\underline{Q}$Q′)) iff C ⊨ (Every P . Q)

The inferences that are triggered in French by equivalents of $(Every\ P.\underline{Q}Q')$ and $(No\ P.\underline{Q}Q')$ were tested by Chemla (2007, 2009) with experimental means: in both cases, almost 90% of his subjects derived a universal inference that *each P-individual is a Q-individual* (in Chemla's experiment, the demonstrative *these ten students* in (39) was designed to make it pragmatically unlikely that the speaker had an additional implicit restriction in mind; a similar structure was used with *each*). In the case of $(Every\ P.\underline{Q}Q')$, Chemla's result could be explained away by assuming that the presupposition of the main predicate is also part of the assertive component, which would make the universal inference unsurprising on most theories. But this strategy doesn't explain why a universal inference that *every P-individual is a Q-individual* is also obtained in $(No\ P.\underline{Q}Q')$; Chemla's inference appears to be due to presupposition projection rather than to an entailment.[18]

18 See Appendix E for discussion of universal inferences arising with non-conservative quantifiers.

### 2.3.3 The Proviso Problem

As mentioned, for sentences of the form $(p\ \textit{and}\ \underline{q}q')$ or $(\textit{if}\ p.\underline{q}q')$ we predict a conditional presupposition that $(\textit{if}\ p.q)$. But as was argued in detail in van der Sandt 1993 and Geurts 1996, 1999, these predictions are often too weak, a difficulty that Geurts called the 'Proviso Problem' (see also Gazdar 1979, Karttunen & Peters 1979). The greatest difficulty, for the present theory as well as for dynamic semantics, is to explain the contrast between (41a), which displays the expected presupposition, and (41b), which typically yields a stronger (unconditional) presupposition.

(41) a. Peter knows that if the problem was easy / difficult, someone solved it. (Geurts 1999)
$\not\Rightarrow$ Someone solved the problem.
b. If the problem was easy / difficult, then it isn't John who solved it. (Geurts 1999)
$\Rightarrow$ Someone solved the problem.

There is now a growing body of work that attempts to explain on pragmatic grounds why conditional presuppositions are sometimes strengthened (see for instance Beaver 2001, Heim 2006, Pérez Carballo 2007 and van Rooij 2007). These solutions could be adapted to the present framework, but it is fair to say that the contrast in (41) has not been fully explained yet (Singh (2007b) does explain it, but his account is in part syntactic).

The DRT approach to presuppositions is designed to address this problem. But it raises difficulties of its own (see Beaver 2001 for discussion). Without going into too much detail, let us mention two. First, the DRT approach fails to explain the universal inferences found in $(\textit{No}\ P.\underline{Q}Q')$: the only readings it predicts are that *no P-individual that satisfies Q also satisfies Q′*, or that *no P individual satisfies Q and Q′* — none of which yields the desired inference.[19] Second, the DRT approach can never derive genuine conditional inferences for $(P\ \textit{and}\ \underline{Q}Q')$ or $(\textit{if}\ P.\underline{Q}Q')$; but conditional presuppositions

19 On the other hand, DRT makes welcome predictions when it comes to the restrictors of generalized quantifiers, which generally fail to give rise to universal presuppositions, contrary to what Heim 1983 or the present approach predict (e.g. *No student who knows that he is incompetent applied* does not give rise to an inference that every student is incompetent). Experimental results on presupposition projection out of restrictors are discussed in Chemla 2008a.

are clearly obtained in some cases, as is illustrated in (42).

(42) If you accept this job, will you let your parents know that you work for a {priest | thug}?
⇒ If you accept this job, you will work for a {priest | thug}.

The debate should be considered open at this point; for better or worse, our approach sides with dynamic semantics on the issue of conditional presuppositions.

## 2.4 General Results

We have shown that in some salient examples our analysis derives the same results as Heim 1983. But what about the general case?

In the propositional case, we obtain full equivalence with the system outlined in Heim 1983, enriched with the asymmetric dynamic disjunction of Beaver 2001. Specifically, it can be shown that for any propositional formula $F$ and for any context set $C$, the local contexts as we have defined them always exist. Furthermore, if we write as $C[F]$ the Heimian update of $C$ with $F$, $C[F] \neq \#$ just in case for each presupposition trigger of the form $\underline{d}d'$ that occurs in $F$, $d$ is entailed by its local context as reconstructed here (using our earlier notation, we write this as $Sat(C,F)$). This result is summarized in (43).

(43) Let $C \subseteq W$ be a context set and let $F$ be a propositional formula. Then:
a. For all expressions $a, b, \underline{d}d'$, if $F = a\ \underline{d}d'\ b$, then $lc(C, \underline{d}d', a\_b) \neq \#$.
b. Furthermore, $Sat(C,F)$ iff $C[F] \neq \#$.

In the quantificational case, things are more complicated. In a nutshell, when all the relevant local contexts exist, (44) also holds, but only when two technical conditions are met:

**Non-Triviality** Quantificational clauses should not be 'trivial' (i.e. replaceable with a tautology or a contradiction).

**Constancy** The domain of individuals should be finite, and in addition restrictors should hold true of a constant number of individuals throughout the context set.[20]

20 While Non-Triviality is quite natural (why would one use a quantificational clause if it is

In case local contexts fail to exist, a modified version of the present theory guarantees full equivalence with Heim's result when Non-Triviality and Constancy are satisfied.

It is worth saying how these results are proven. The key observation is that whenever local contexts exist, the present theory is equivalent to the 'Tranparency theory' (Schlenker 2007, 2008b), an analysis that was initially presented as *anti*-dynamic. It was proven in earlier work (Schlenker 2007) that under the conditions of Non-Triviality and Constancy, the Transparency theory is itself equivalent to Heim's dynamic semantics. Indirectly, then, we obtain an equivalence between the present account and Heim 1983.

The relationship between the present analysis, the Transparency theory and dynamic semantics is discussed in Appendix A. In Appendix B, we consider the case in which local contexts fail to exist, and show that an extension of the present analysis yields full equivalence with the Transparency theory — which indirectly proves an equivalence with dynamic semantics. Finally, the main definitions and formal results are stated in Appendix C.

## 3 Extensions

### 3.1 Enriching the Fragment

Up to this point, we have concentrated on a very simple fragment, which allowed us to obtain general results of equivalence with standard dynamic semantics. But our definition of local contexts can in principle be applied to much richer languages — possibly with small adjustments in the definitions. All we need in order to compute the local context of an expression $E$ (whose type 'ends in $t$') in a sentence $a\ E\ b$ is (i) a well-defined syntax and semantics for $S$, and (ii) a notion of equivalence (relative to the context set) between sentences of the form $a\ d'\ b'$ and $a\ {}^{c'}d'\ b'$. We will now briefly consider extensions of our basic language with questions, belief reports, as well as variable-binding operators (though a technical discussion of the latter is left for Appendix D).

trivial?), I cannot think of any good justification for Constancy. Without the latter, we obtain presuppositions that are sometimes weaker than those predicted by Heim 1983 (see Appendix A, fn. 31, for an example).

### 3.1.1 Adding Questions

For concreteness and technical simplicity, we adopt the structured approach to questions advocated in Krifka 2001.[21] In this framework, *yes* and *no* denote functions that return the positive or negative version of their propositional argument, as indicated in (44a). And the meaning of a yes-no question $(?p)$ at a world $w$ is simply a function that takes **yes** or **no** as an input, and returns the value of $p$ or of *not p*, as shown in (44b). Finally, the meaning of an answer $A$ to a question $Q$ is just the value of $Q$ taking as argument the value of $A$, as illustrated in (44c) and (44d) (in our type-theoretic notation, we write $st$ to abbreviate $\langle s, t\rangle$):

(44) a. $\mathbf{yes}^w = \lambda p_{st}\, p$; $\mathbf{no}^w = \lambda p_{st}\, (not\, p)$
b. $\mathbf{(?p)}^w = \lambda f_{\langle st,st\rangle}\, [f(\lambda w'\, \mathbf{p}^{w'})](w)$,
where $f$ ranges over $\{\lambda p_{st}\, p, \lambda p_{st}\, (not\, p)\}$
c. Meaning of the answer 'yes' at w: $(\mathbf{?p})^w(\mathbf{yes}^w) = \mathbf{p}^w$
d. Meaning of the answer 'no' at w: $(\mathbf{?p})^w(\mathbf{no}^w) = (\mathbf{not\ P})^w$

The effect of this semantics is that the meaning of a question $(?p)$ at a world $w$ can be assimilated to the ordered pair of its answers at $w$: $\langle \mathbf{p}_w, (\mathbf{not\ p})_w\rangle$.

Now a natural criterion of equivalence between two questions $Q$ and $Q'$ relative to a context set $C$ is that at each world of $C$, $Q$ and $Q'$ have the same denotation. Using the familiar syntactic reasoning (whereby the only good final is the right parenthesis), we can now take the local context of $\underline{p}p'$ in the question $(?\underline{p}p')$ to be the strongest proposition $x$ that satisfies (45):

(45) for every clause $d'$, $C \vDash^{c'\to x} (?d') = (?^{c'}d')$

In other words, for every clause $d'$, every world $w$ in $C$ should guarantee that $\lambda f_{\langle st,st\rangle}\, [f(\lambda w'\, \mathbf{d}'^{w'})](w) = \lambda f_{\langle st,st\rangle}\, [f(\lambda w'\, ({}^{c'}\mathbf{d}')^{w', c\to x})](w)$. These functions are identical just in case they output the same value for each of their arguments, of which there are only two: **yes** and **no** (if we view the

21 Krifka's (2001) approach is easy to integrate to our analysis because it implies that if two questions $Q$ and $Q'$ are equivalent, then their possible answers $A_1$ and $A_1'$, $A_2$ and $A_2'$, etc. are equivalent as well — which yields an immediate reduction to the assertive case. In other approaches to questions, the equivalence between $Q$ and $Q'$ would only guarantee that the set of $\{A_1, A_2, \dots\}$ is identical to $\{A_1', A_2', \dots\}$ (where these may be sets of true or possible answers, depending on the approach). The integration of various theories of questions into the present framework is left for future research.

meaning of a Krifka-question as a pair of its answers, two questions are identical just in case they are identical coordinate by coordinate). Thus their positive answers should be equivalent, and their negative answers should be as well:

(46) for every clause $d'$, for every world $w$ in $C$,
a. $(\mathbf{?d}')^w(\mathbf{yes}) = (\mathbf{?}^{\mathbf{c}'}\mathbf{d}')^{w,c'\to x}(\mathbf{yes})$, i.e. $\mathbf{d}'^w = {}^{\mathbf{c}'}\mathbf{d}'^{w,c'\to x}$
b. $(\mathbf{?d}')^w(\mathbf{no}) = (\mathbf{?}^{\mathbf{c}'}\mathbf{d}')^{w,c'\to x}(\mathbf{no})$, i.e. $(\mathbf{not\ d}')^w = (\mathbf{not}\ {}^{\mathbf{c}'}\mathbf{d}')^{w,c'\to x}$

It is immediate that (46b) reduces to (46a), which is satisfied if $c'$ denotes $C$, but not if it excludes any world of $C$ (as before, this result builds on the assumption that every proposition can be expressed in the language). So we derive the result that the local context of $\underline{p}p'$ in $?\underline{p}p'$ is the global context $C$. As a result, the question $?\underline{p}p'$ is acceptable in $C$ just in case $C$ entails $\mathbf{p}$, as desired.

*Wh*-questions can be treated in the same way. Let us briefly sketch the argument. In Krifka's framework, (*who P*) (e.g. *Who smoked ?*) receives the analysis in (47) (for simplicity, $P$ is a simple predicate, and we abstract away from the nominal restriction of the *wh*-word — in this case, the fact that *who* only ranges over human beings):

(47) a. Question: who P?
$(\mathbf{who\ P})^w = \lambda i_e\, \mathbf{P}^w(i)$
b. Term Answer: John
$(\mathbf{who\ P})^w(\mathbf{John}^w) = \mathbf{P}^w(j)$

In general, we can compute the value of the local context of $\underline{P}P'$ in *(who* $\underline{P}P'$*)* as the strongest property $x$ (of type $\langle s, \langle e, t\rangle\rangle$) which satisfies the condition in (48a), which reduces to (48b) (we take the domain of individuals to be constant across worlds):

(48) a. for every clause $d'$, for every world $w$ in $C$,
$(\mathbf{who\ d}')^w = (\mathbf{who}\ {}^{\mathbf{c}'}\mathbf{d}')^{w,c'\to x}$
b. for every clause $d'$, for every world $w$ in $C$, for every individual $i$,
$\mathbf{d}'^w(i) = ({}^{\mathbf{c}'}\mathbf{d}')^{w,c'\to x(i)}(i)$.

It is immediate that the condition is satisfied if the denotation $x$ of $c'$ is the property $\lambda w\ \lambda x\ w \in C$ (i.e. the property which in $C$ is true of all individuals, and which outside of $C$ is false of all individuals). But if for any world $w$ of $C$ and any individual $i$, $x(w)(i) \neq 1$, the condition will be violated in case

$\mathbf{d}'(w)(i) = 1$, since in this case ${}^{c'}\mathbf{d}'(w)(i) = 0$. This analysis predicts that wh-questions should yield universal presuppositions, as in (49):

(49) Among your 10 students, who is aware of being incompetent?
⇒ each of your 10 students is incompetent

While universal projection in wh-questions is assumed by some researchers (e.g. Abrusan (2008)), experimental data do not yet decide the issue.[22]

### 3.1.2 Adding Belief Reports

Heim (1992) predicts that *John believes* $\underline{p}p'$ presupposes that *John believes p*.[23] Her analysis is based on a modal treatment of *believe*, in which *John*

22 In recent work, Chemla (2008a) tested French sentences similar to (i), with different presupposition triggers (change of state verbs, possessive descriptions, *know*, *be unaware*. . . ).

(i) Parmi ces 20 étudiants, qui prend soin de son ordinateur?
*Among these 20 students, who takes good-care of his computer ?*

Interestingly, Chemla (2008a) obtained significantly different results depending on the methodology he adopted.

In a gradient inferential task, in which subjects were asked to estimate the degree (0%–100%) to which they made certain inferences, Chemla (2008a) obtained high endorsement rates for universal inferences. Specifically, he got an average of 76.7% of endorsement of the universal inference; for comparison, universal inferences under *no student* were obtained with an average of 86.6%. This suggests that *wh*-questions do trigger universal-like presuppositions.

In a coherence task, in which the negation of the purported presupposition was explicitly asserted, and the resulting discourse was assessed for coherence, universal inferences were obtained much less strongly with embedding under *who* than under no student: the average coherence in the *no student* case was of 51.9%, and of only 16.1% in the *who* case — which suggests *wh*-questions do not trigger universal presuppositions. Needless to say, the problem is currently open. (Thanks to Emmanuel Chemla for discussion of these results).

23 This prediction is controversial, since conditional presuppositions that are not explicitly justified in the preceding discourse are often strengthened to unconditional ones, as shown in (i):

(i) a. John believed I had started proof-reading at 5pm (modified from Geurts 1999)
⇒ I had not been proof-reading before 5pm.

b. John believed that I had not been proof-reading before 5pm, but he thought that I had finally started proof-reading at 5pm.
⇏ I had not been proof-reading before 5pm.

Proponents of DRT claim that (ia) shows that *John thinks that* $\underline{p}p'$ presupposes $p$ rather than *John believes that p*. Proponents of dynamic semantics, with whom our theory sides, *believes F* is true at world $w$ just in case $F$ is true in all worlds compatible with what John believes in $w$ (we call the set of these worlds $Dox_J(w)$). Since this modal analysis is quantificational in nature, we might expect our theory to predict the same result as for universally quantified sentences such as $(Every\ P.\underline{Q}Q')$ — hence a presupposition that *every world in* $Dox_J(w)$ *satisfies the presupposition p of the embedded clause*, which in turn derives Heim's prediction. This is indeed the case, but some care is needed in the implementation because *believe* is an intensional construction.

In our treatment of $(Every\ P.\underline{Q}Q')$, $\underline{Q}Q'$ had the intensional type $\langle s, \langle e, t\rangle\rangle$. When we computed the value of $c'$ in $(Every\ P.{}^{c'}\underline{Q}Q')$, $c'$ also had the type $\langle s, \langle e, t\rangle\rangle$, which was crucial to allow $c'$ to take different values at different worlds of the context set. We must of course preserve this ability in the case of belief reports: a person's beliefs depend on the world of evaluation, and the value of $c'$ should too. But this possibility will be lost if we posit for belief reports the intensional semantics in (50) (we henceforth adopt the simplified syntax $(believe_J\ F)$ for *John believes that F*; $st$ abbreviates the type $\langle s, t\rangle$):

(50) $believe_J$ is of type $\langle s, \langle st, t\rangle\rangle$ and $F$ is of type $\langle s, t\rangle$
$(\mathbf{believe_J\ F})^w = 1$ iff $\mathbf{believe_J}^w(\lambda w'\mathbf{F}^{w'})$ iff for every world $w' \in Dox_J(w)$, $\mathbf{F}^{w'} = 1$

The problem we face is that the value of the initial world parameter w is lost when we evaluate the embedded clause $F$. As a result, the value of the contextual restriction in ${}^{c'}F$ cannot depend on $w$, contrary to what is needed.

This problem is by no means special to the present case: when one considers a language with indexicals, the semantics in (50) turns out to be inadequate. This is because indexicals depend rigidly on the context of utterance rather than on the world of evaluation, a fact that cannot be captured in this semantics. To solve this problem, Kaplan 1989 introduced the device of double indexing, in which every expression is evaluated with respect to a context of utterance in addition to a world parameter. Once this modification is adopted, our problem can be solved — though only if *believe* is not embedded under further modal operators.

Let us now posit the two-dimensional analysis of attitude reports given in (51) (for simplicity, we take the context of utterance $w^*$ to be a world; each earlier type $\tau$ is now replaced with $\langle s, \tau\rangle$ to take into account of this

suggest that independent mechanisms are responsible for the strengthening (see van Rooij 2007 for a recent discussion of pragmatic mechanisms that might strengthen conditional presuppositions to unconditional ones). I believe that the question is currently open.

dependency on the context of utterance).

(51) *believe*$_J$ is of type $\langle s, \langle s, \langle st, t\rangle\rangle\rangle$ and $F$ is of type $\langle s, \langle s, t\rangle\rangle$
$(\mathbf{believe_J}\ F)^{w^*,w} = 1$
iff $\mathbf{believe_J}^{w^*,w}(\lambda w'\ \mathbf{F}^{w^*,w'})$
iff for every world $w' \subset Dox_J(w)$, $\mathbf{F}^{w^*,w'} = 1$

As in Kaplan's system, when we evaluate the truth of a statement with respect to a context of utterance $w^*$, we take the initial world of evaluation to be $w^*$ as well. Taking into account the syntactic fact that a good final for the string ($believe_J\ \underline{p}p'$ can only be the right parenthesis *)*, we can define the local context of $F$ in ($believe_J\ \underline{p}p'$) to be the strongest element $x$ of type $\langle s, \langle s, t\rangle\rangle$ which satisfies the condition in (52):

(52) for every clause $d'$ of type $\langle s, \langle s, t\rangle\rangle$, for every $w^*$ in $C$,
$(\mathbf{believe_J}\ d')^{w^*,w^*,c'\to x} = (\mathbf{believe_J}\ {}^{c'}\mathbf{d'})^{w^*,w^*,c'\to x}$,
i.e. [for every world $w \in Dox_J(w^*)$, $\mathbf{d}^{w^*,w} = 1$]
iff [for every world $w \in Dox_J(w^*)$, $({}^{\mathbf{c}'}\mathbf{d}')^{w^*,w^*} = 1$].

We can now argue that the desired value is $x = \lambda w^*\ \lambda w\ (w^* \in C$ and $w \in Dox_J(w^*))$. First, it is clear that this restriction will never affect the truth conditions, because for every $w^*$ in $C$, the quantification in (52) is restricted to worlds in $Dox_J(w^*)$. Second, the restriction $x$ *will* lead us astray if for some $w*$ in $C$ and some $w$ in $Dox_J(w^*)$, $x(w^*)(w)$. For suppose that $d'$ is a tautology; it will then be true that $(\mathbf{believe_J}\ \mathbf{d'})^{w^*,w^*,c'\to x} = 1$ but not that $(\mathbf{believe_J}\ {}^{\mathbf{c}'}\mathbf{d}')^{w^*,w^*,c'\to x} = 1$, because $w$ is in $Dox_J(w^*)$ but ${}^{\mathbf{c}'}\mathbf{d}'^{w^*,w^*,c'\to x} = 0$.

The conclusion, then, is that a sentence of the form $believe_J\ \underline{p}p'$ presupposes that every world compatible with what the agent believes should satisfy $p$. This result follows because the local context $x$ of the embedded clause should entail $p$; thus it must be that for all worlds $w^*$, $w$, if $x(w^*)(w) = 1$, $\mathbf{p}^{w^*,w} = 1$, with $x = \lambda w^*\ \lambda w\ (w^* \in C\ and\ w \in Dox_J(w^*))$. In other words, it should be the case that for every world $w^*$ in $C$, for every world $w$ in $Dox_J(w^*)$, $\mathbf{p}^{w^*,w} = 1$. This is the standard result obtained in Heim's framework: *John believes that it stopped raining* is taken to presuppose that *John believes that it rained* (Heim 1992).

### 3.1.3 Adding Variable-binding Operators

The language under consideration can also be enriched by adding individual, time or world variables to it. In essence, the analysis can be extended by taking local contexts to be functions from assignment functions to objects of types that 'end in t'. The technical implementation is sketched in Appendix D. (Within a language with explicit world variables, this extension makes it possible to treat modal operators — including ones that are embedded under other modal operators — without recourse to the indexical analysis described in the previous paragraph.)

### 3.2 Dynamic Implementation

Although our analysis did reconstruct a notion of local context, it also departed from dynamic semantics in ... not being truly dynamic at all![24] Specifically, in our system local contexts are derivative from a classical semantics, together with a specification of the syntax of the language under consideration. Still, one could use our framework to constrain a more conservative version of dynamic semantics, one in which all expressions are intrinsically dynamic.[25] We can thus require that for any unary or binary connective $*$, lexical rules specify that the presupposition of $\underline{F}F'$ in $C[*\underline{F}F']$ really be checked with respect to the incremental context of $\underline{F}F'$; and in case no presupposition failure occurs, the update of C with $(*\underline{F}F')$ is simply the subset of worlds of C that satisfy $(*F')$ (given that in such a case $F$ is entailed by the incremental context, this is the same thing as satisfying $(*\underline{F}F')$).

(53) $C[(*\underline{F}F')] \neq \#$ iff $lc(C, \underline{F}F', *\_) \leq \mathbf{F}$
If $\neq \#$, $C[(*\underline{F}F')] = \{w \in C : w \vDash (*F')$

The same reasoning can be applied to binary connectives:

(54) $C[(\underline{F}F' * \underline{G}G')] = \#$ iff
a. it is not the case that $lc(C, \underline{F}F', (\_ * \underline{G}G')) \leq \mathbf{F}$ or
b. $(lc(C, \underline{F}F', (\_ * \underline{G}G')) \leq \mathbf{F}$ and it is not the case that $lc(C, \underline{G}G', (\underline{F}F' * \_)) \leq \mathbf{G}$.
If $\neq \#$, $C[(\underline{F}F' * \underline{G}G')] = \{w \in C : w \vDash (F' * G')\}$.

For instance, for $* = and$, $C[(\underline{F}F' \; and \; \underline{G}G')] = \#$ iff the local context of $\underline{F}F'$, namely $C$ itself, fails to entail $\mathbf{F}$, or if the local context of $\underline{G}G'$, namely $C \wedge \underline{\mathbf{F}}\mathbf{F}'$,

24 While the theory of Heim 1983 was 'entirely' dynamic, Karttunen 1974 combined a static semantics with fully autonomous rules of context update. Heim improved on such a system by showing how the static semantics could be derived from appropriate rules of update. We have attempted to do the converse, i.e. to show that context update can be made to follow from a static semantics.

25 See LaCasse 2008 and Rothschild 2008b,c for very different solutions.

fails to entail **G**. If $C[(\underline{F}F'\ \mathit{and}\ \underline{G}G')] \neq \#$, it is equal to the set of worlds in $C$ which satisfy $F'$ and $G'$. This is exactly the rule that was posited in Heim 1983. But the result is more general: in the propositional case, our templates derive the rules for connectives found in Heim 1983 (augmented with the asymmetric dynamic disjunction of Beaver 2001). The template in (54) can easily be extended to binary connectives that have a different syntax, such as $(\mathit{if}\ F.G)$ or $(Q\,F.G)$; it is noteworthy that the same template applies to both cases because, in our highly simplified fragment, they share the same syntax:

(55) $C[(*\underline{F}F'.\underline{G}G')] = \#$ iff
   a. it is not the case that $lc(C, \underline{F}F', (*\_\_.\underline{G}G')) \leq \mathbf{F}$ or
   b. $lc(C, \underline{F}F', (*\_\_.\underline{G}G')) \leq \mathbf{F}$ and it is not the case that $lc(C, \underline{G}G', (\underline{F}F' * \_\_)) \leq \mathbf{G}$.
   If $\neq \#$, $C[(*\underline{F}F'.\underline{G}G')] = \{w \in C : w \vDash (F' * G')\}$.

(For reasons that we discuss below, the template for quantifiers derives something close, but not identical, to Heim's treatment of quantifiers; see in particular sections C.22–C.23 of the appendix.)

Although our algorithm can be embedded in a standard dynamic framework to constrain the list of possible dynamic entries, given the present data we take this move to be unnecessary and thus undesirable: the algorithm already delivers what we want without a dynamic semantics, and by Occam's razor the latter should presumably be dispensed with. The situation will of course change if independent arguments (e.g. from anaphora resolution) can be given for the dynamic framework.

### 3.3 Local Triviality

Some recent accounts of presupposition projection do without any notion of local context (Schlenker 2007, 2008a, George 2008a,b, Chemla 2008b). The present account, which is in this respect more conservative, has the advantage of allowing for a general theory of triviality which follows Stalnaker's (1978) initial insights: an expression $E$ is locally trivial if it is entailed by its local context; and it is locally contradictory if its negation is entailed by its local context (see Singh 2007a for a recent discussion). This has some welcome empirical consequences.

The following constructions are deviant, most probably because *he is sick* is in some sense redundant:

(56) a. #John has cancer and [he is sick or desperate]
b. John has cancer and he is desperate.
c. #If John has cancer, he is sick or desperate.
d. If John has cancer, he is desperate.

The analysis is straightforward: in (56a) and (56c) *he is sick* is entailed by its incremental context, whereas the situation is different in (56b) and (56d). Similarly, the contrast in (57) can be accounted for in terms of 'incremental triviality':

(57) a. John is in Paris, and he is staying near the Louvre.
b. #John is staying near the Louvre, and he is in Paris.

What about violations of the constraints against expressions that *contradict* their local context? Cases in which one disjunct entails the other are known to be deviant (Hurford 1974), and they have recently been the object of detailed studies (Singh 2008; Chierchia et al. 2008):[26]

(58) a. #John is staying near the Louvre, or he is in Paris.
b. #John is in Paris, or he is staying near the Louvre.
c. John is staying near the Louvre, or at least he is in Paris.
d. #John is at least in Paris, or he is staying near the Louvre.

As was shown in (33), the incremental context of $q$ in ($p$ *or* $q$) uttered in a context set $C$ is $C \wedge (\textbf{not p})$. It follows that (58b) should be incrementally deviant, since for $p$ = *John is in Paris* it is clear that $C \wedge (\textbf{not p})$ entails the negation of *John is staying near the Louvre.* On the other hand, (58a) should not be deviant, because the second disjunct is not locally contradictory (John could fail to be staying near the Louvre while still being in Paris). It is noteworthy that when *at least* is added, as in (58c) and (58d), the data are as expected. We leave this problem for future research[27] (but see Chierchia et al. 2008 for an in-depth discussion of related issues).

26 Special thanks to Benjamin Spector for helpful conversations on this topic and on this specific hypothesis.

27 Anticipating Section 4, we could posit that (58a) is deviant because the first disjunct is contradictory relative to its *symmetric* local context, which takes into account the entire sentence (it turns out that the symmetric context of the first disjunct in (58a) is identical to the incremental context of the second disjunct in (58a), hence the result). But if the symmetric context is what matters, (57a) should be as deviant as (57b), contrary to fact.

### 3.4 Alternative Accounts

The present theory is by no means alone in seeking a solution to the explanatory problem of dynamic semantics. First, there are alternative attempts to constrain standard dynamic semantics, either by imposing a template on possible lexical entries (LaCasse 2008; Rothschild 2008b,c) or by deriving parts of the framework from the logic of common belief (Unger & van Eijck 2007). Second, there are entirely different, non-dynamic accounts that seek to connect presuppositions and implicatures, albeit within a new framework for both (Chemla 2007, 2008b). Third, George (2008a,b) and Fox (2008)have recently revived and considerably improved a non-dynamic trivalent approach that was explored by Peters (1979) and Beaver & Krahmer (2001); they are, in essence, incremental versions of supervaluations or Strong Kleene systems. A general comparison of the present account with these new approaches is left for future research. But since the trivalent approach has some properties in common with the present account, a brief discussion is called for.

For simplicity, we restrict attention to the incremental trivalent account discussed in Fox (2008), based on supervaluations (in the case we consider, similar results are obtained in simple extensions of the Strong Kleene logic, as discussed in George 2008a). Semantic failure is treated as an uncertainty about the value of an expression: if $\underline{p}p'$ is evaluated at $w$ while $p$ is false at $w$, we just don't know whether the clause is true or false; the same holds if the presuppositional predicate $\underline{P}P'$ is evaluated with respect to a world $w$ and an individual $d$ which make $P$ false. The semantic module outputs the value # in case this uncertainty cannot be resolved — which systematically happens with unembedded atomic propositions whose presupposition is not met. But in complex formulas it may happen that *no matter* how the value of $\underline{p}p'$ or $\underline{P}P'$ is resolved, one can still unambiguously determine the value of the entire sentence. This is for instance the case if $(q\ and\ \underline{p}p')$ is evaluated in a which $q$ and $p$ are both false. $\underline{p}p'$ receives the 'indeterminate' value #, but no matter how the indeterminacy is resolved, it won't affect the value of the entire sentence, since it will be false anyway. The same reasoning can be made with respect to every world in the context set: for any world $w$, the sentence will have a determinate truth value in w just in case either (i) $q$ is false in $w$ (so that it doesn't matter how one resolves the indeterminacy of the second conjunct); or (ii) $q$ is true, and in that case the presupposition $p$ of the second conjunct is satisfied. Since we are solely interested in worlds that are compatible with what the speech act participants take for granted, we derive the familiar prediction that the context set must entail that *if q, p*. The beauty of this proposal is that its underlying intuition is completely general: by treating presupposition failure as an instance of 'uncertainty' between truth and falsity, it provides a general recipe for determining under what conditions the uncertainty in question does or does not matter for the entire sentence.

The present analysis has one point in common with this trivalent approach: in order to determine whether a presupposition is justified, it checks whether the information it contributes is in a sense innocuous. In the present framework, for $q$ to be innocuous in the formula $(p\ and\ \underline{q}q')$, $q$ must be entailed by its local context, which is itself the strongest innocuous restriction one can make on the interpretation of the second conjunct. This means that in every world of the context set $C$, $q$ itself should be an innocuous restriction no matter what the second conjunct turns out to be. In the trivalent analysis, we only demand that *in any world $w$ of $C$ in which the presupposition $q$ is not satisfied*, the failure $\underline{q}q'$ gives rise to should be innocuous no matter how it is resolved. In simple cases (propositional examples involving incremental versions of both theories), the two requirements turn out to be equivalent — which is the reason both theories predict that $(p\ and\ \underline{q}q')$ presupposes that *if p, q*.

In quantified cases, however, the incremental version of the present theory predicts stronger presuppositions than this trivalent analysis (see Schlenker 2008c for an argument). This is best illustrated on the example of the formula $(No\ P.\underline{Q}Q')$. We saw that our reconstruction of local satisfaction predicts a universal presupposition, of the form $(Every\ P.Q)$; in effect, our condition is very strong because we require that $Q$ be an innocuous restriction *no matter what the main predicate turns out to be.* The results are much weaker within the trivalent approach because it takes the value of the main predicate as given. To see this, suppose that all $P$-individuals except d fail to satisfy the presupposition $Q$ of the main predicate, but that $d$ satisfies both $Q$ and $Q'$. No matter how the uncertainty about the value of $\underline{Q}Q'$ with respect to the other individuals is resolved, we can be certain that the entire statement will be false, because $d$ alone suffices to refute it. So this is a case in which our reconstruction of dynamic semantics predicts that the sentence should be a presupposition failure, whereas the trivalent approach predicts that it should be false. We leave a more detailed comparison for another occasion (Schlenker 2008c). But it is clear that trivalent approaches are technically and empirically different from our reconstruction of local satisfaction.

## 4 Symmetric Local Contexts and Symmetric Satisfaction

In standard dynamic semantics, the left-to-right asymmetries observed in projection patterns are hardwired in the lexical entries of the various operators. The present account is different: the asymmetry arises because the local context of an expression *E* in a sentence *S* is computed incrementally, on the basis of information available in *S* before *E*. But it is easy to define the *symmetric local context* of *E*, which is computed on the basis of *all* of *S* (except *E*, of course, whose interpretation the local context is supposed to facilitate). There are well- known arguments in favor of a symmetric account of disjunction, illustrated in (59a) and (59b); but they extend to conditionals, as seen in (59c) and (59d):

(59) a. There is no bathroom or the bathroom is well hidden (after Partee).
b. The bathroom is well hidden or there is a no bathroom.
c. If there is a bathroom, the bathroom is well hidden.
d. If the bathroom is not hidden, there is no bathroom.

(59a) and (59c) are correctly predicted by dynamic semantics and the incremental version of our account to carry no presupposition. By contrast, they predict that (59b) and (59d) should presuppose that there is a bathroom. The issue is complex and would require a longer discussion (see Schlenker 2008a,b); but it is plausible that in these examples the presupposition of the first element is justified on the basis of information that appears at the end of the sentence. In fact, when the entire sentence is taken into account, (59b) becomes informationally indistinguishable from (59a). And similarly for (59d) and (59c): trading on the near-equivalence between *If not F, not G* and *if G, F*, when the entire sentence is taken into account, (59d) becomes informationally similar to (59c) — which makes it unsurprising that they should transmit presuppositions in the same way.

We take these observations to suggest that the local context of an expression *E* in a sentence *S* can to some extent be computed by taking into account *all* of *S* except *E*. This option is presumably costly, since (59c) and (59d) are somewhat less felicitous than (59b) and (59d). Still, it appears that the left-right asymmetry we observed is *just* a bias, which can be overcome with some effort. The general availability of symmetric readings has been defended in Schlenker (2008a,b), Chemla (2008b), and Rothschild (2008b,c,a), but it is currently controversial (see Beaver (2008) and Rothschild (2008c) for opposite assessments). If correct, it can be accounted for by providing a symmetric version of all the notions we introduced earlier, as is done in (60) (with the superscript $s$ added to all the definitions to distinguish them from their incremental counterparts):

(60) a. $tr^s(C, d, a\_b) = \{x : x$ is an object of the type specified by $d$ and for every constituent $d'$ of the same type as $d$, $C \vDash^{c' \to x}\ a\ {}^{c'}d'\ b \Longleftrightarrow a\, d'\, b\}$
b. $lc^s(C, d, a\_b)$ = the bottom element of $tr^s(C, d, a\_b)$, if it exists; # otherwise.
c. $Sat^s(C, \underline{d}d', a\_b)$ iff $lc^s(C, \underline{d}d', a\_b) \leq \mathbf{d}$
d. $Sat^s(C, F)$ iff for all $\underline{e}e'$, $a'$, $b'$, if $F = a'\, \underline{e}e'\, b'$, $Sat^s(C, \underline{e}e', a'\_b')$.

Sample predictions (proven in Appendix C.24 are listed in (61); in particular, (61b) and (61c) account for the data in (59c) and (59d).

(61) a. $lc^s(C, \underline{q}q', (\_\ and\ p)) = lc(C, \underline{q}q', (p\ and\ \_)) = C \wedge \mathbf{p}$
b. $lc^s(C, \underline{q}q', (\_\ or\ p)) = lc(C, \underline{q}q', (p\ or\ \_)) = C \wedge (\mathbf{not\ p})$
c. $lc^s(C, \underline{q}q', if\ \_ .\ p) = lc(C, \underline{q}q', (if\ (not\ p).\ (not\ \_))) = C \wedge (\mathbf{not\ p})$
d. $lc^s(C, \underline{P}P', (Every\ (not\ \_).Q)) = {}^{C}(\mathbf{not\ Q})$
e. $lc^s(C, \underline{P}P', (No\ (not\ \_).Q)) = {}^{C}\mathbf{Q}$

How could these predictions be tested? Let us focus on the predictions of the symmetric analysis of $(if\ \underline{q}q'.\ p)$, given in (61c). It can be shown that the symmetric local context of $\underline{q}q'$ is $C \wedge (\mathbf{not\ p})$. Since we take symmetric satisfaction to be possible but dispreferred, we predict that if $q$ is entailed by $(not\ p)$ but not by the global context, the sentence will have an intermediate acceptability status. In order to obtain acceptability judgments (as opposed to inferences), we can make use of presupposition triggers such as *too*, which has the advantage of making accommodation — and in particular local accommodation — very difficult or impossible (why this is so is another matter, which goes beyond the present paper; see Beaver & Zeevat 2007 for discussion). This means that when the presupposition of *too* is not satisfied, the resulting sentence is deviant. One should thus ask subjects to rate the acceptability of sentences such as those in (62),[28] which are the object of an ongoing experiment (conducted in collaboration with Emmanuel Chemla) in French.

(62) L'évolution du salaire des fonctionnaires va être remise à plat.
*the evolution of-the salary of-the civil-servants will be reset to flat*
'The evolution of state employees' salaries will be reconsidered.'

28 *Aussi* associates with focus, which can cause undesired ambiguities. To circumvent the problem, we inserted *aussi* right after a strong pronoun (e.g. *eux aussi*, literally 'them too'), which yielded unambiguous sentences.

a. Si les infirmières sont augmentées, les salaires des enseignants seront eux aussi { A. revalorisés / B. bloqués }.
*if the nurses are getting-a-raise, the salaries of-the teachers will-be them too* { A. *revalued* / B. *blocked* }.
'If the nurses get a raise, the teachers' salaries too will be { A. increased / B. frozen }'

b. Si les infirmières sont augmentées, les salaires des enseignants seront { A. revalorisés / B. bloqués }.
*if the nurses are getting-a-raise, the salaries of-the teachers will-be* { A. *revalued* / B. *blocked* }.
'If the nurses get a raise, the teachers' salaries will be { A. increased / B. frozen }.'

c. Si les salaires des enseignants ne sont pas eux aussi { A. revalorisés / B. bloqués }, les infirmières ne seront pas augmentées.
*If the salaries of-the teachers NE are not them too* { A. *revalued* / B. *blocked* }, *the nurses NE will-be not getting-a-raise*
'If the teachers' salaries are not too { A. increased / B. frozen } too, the nurses won't get a raise.'

d. Si les salaires des enseignants ne sont pas { A. revalorisés / B. bloqués }, les infirmières ne seront pas augmentées.
*If the salaries of-the teachers NE are not* { A. *revalued* / B. *blocked* }, *the nurses NE will-be not getting-a-raise*
'If the teachers' salaries are not { A. increased / B. frozen }, the nurses won't get a raise.'

(62a)A displays the canonical order *if* $p$, $\underline{q}q'$ where $p$ entails $q$: the presupposition of the consequent is satisfied by the antecedent. (62a)B should be deviant because the presupposition of the consequent is not entailed by the antecedent, and is in fact contradictory with it. (62b) offers non-presuppositional controls. Finally, (62c) and (62d) are analogous to (62a) and (62b), except that *if F, G* is replaced with *if not G, not F* — which makes it possible to test the predictions of the symmetric analysis. If presuppositions are preferably satisfied incrementally, but can at some cost be satisfied symmetrically, we predict that (62a)A should be acceptable, that (62a)B and (62b)B should unacceptable, and — crucially — that (62c)A should have an intermediate status (see Schlenker 2008b, Chemla 2008a, and Chemla & Schlenker 2009 for some experimental results that confirm the hypothesis).

# 5 Perspectives

## 5.1 Presupposition and Accommodation

Now that we have seen how our reconstruction of local contexts can derive some classic results, it is worth stepping back to ask what it means in the present framework to presuppose something, and how accommodation can be incorporated into the analysis.

The local context of an expression $E$ can be seen as the *semantic contribution* made by those words that precede $E$ given the shared assumptions of the speech act participants. As a result, any part of $E$ which is entailed by its local context will be redundant. We can thus view the presupposition $d$ of an expression $\underline{d}d'$ as a part of the meaning that should not make any contribution to the conversation — in other words, one that should be trivial in its local context. In uttering $S$, a speaker presupposes that $p$ just in case $p$ must be part of the common ground if the presupposition of every expression $\underline{d}d'$ that occurs in $S$ is to be entailed by its local context. The dynamic tradition viewed the local context of an expression $E$ as a *belief state* obtained by the speech act participants before $E$ was taken into account, and it similarly required that presuppositions be trivial in their local contexts. With respect to unembedded clauses, the two approaches are similar: relative to a context set $C$, the presupposition $d$ of a clause $\underline{d}d'$ is trivial just in case it is entailed by $C$. With respect to embedded triggers, Stalnaker's dynamic approach had to rely on some 'intermediate belief states' — which led to the conceptual and technical problems discussed in Section 1.1. By contrast, the present approach has no difficulty assessing the contribution of the string that precedes $E$ when $E$ is embedded; and it is just as unproblematic to check that the presupposition of $E$ (if any) is redundant in its local context.[29]

Turning to the topic of accommodation, we should distinguish between two issues. *Global accommodation* is the process by which cooperative speech act participants are willing (within reason) to revise their belief states to guarantee that some linguistic constraints are satisfied in the conversational exchange; thus if I tell you that *My sister is pregnant*, you will be willing to add to your beliefs the assumption that I have a sister, even if this fact was not

29 The intuition that the presupposition of an expression $E$ must make no semantic contribution given the words that precede $E$ is common to the present approach and to the Transparency theory developed in Schlenker 2007, 2008a. This explains why equivalence results can be proven between the two theories despite the fact that they are conceptually dissimilar. See Appendix A for a more detailed discussion.

common knowledge before I uttered the sentence. As noted in Lewis 1979, the existence of such a process is virtually necessary given the cooperative nature of communication. *Local accommodation*, by contrast, is a mechanism that does not follow from conceptual considerations in Heim's approach,[30] though it is empirically motivated by examples like (63), which are marked but nonetheless possible:

(63) The king of France isn't bald because there is no king of France!

In this case, global accommodation won't help because it would produce a global context that entails that France is a monarchy — an undesirable outcome, in particular in view of the end of the sentence. Heim 1983 suggested that in this case one can apply, as a last resort, a rule of local accommodation: the global context is left untouched, but the local context is strengthened so as to entail the presupposition of the beginning of the first clause. If we write as $\underline{p}p'$ *the king of France is bald* (with $p$ = France is a monarchy), the update process in a context set $C$ would normally proceed as in (64a), which yields a failure if $C$ doesn't entail that France is a monarchy. By contrast, (64b) is the update process with local accommodation, which guarantees that the presupposition of $\underline{p}p'$ is satisfied in its local context without thereby implying that $C$ must entail that France is a monarchy. As is seen in (64c), this has the same effect as treating $\underline{p}p'$ as if $p$ were part of its assertive component.

(64) a. **Update without local accommodation**
$C[\textit{not}\, \underline{p}p'] = C - C[\underline{p}p']$, unless $C[\underline{p}p'] = \#$
b. **Update with local accommodation**
$C[\textit{not}\, \underline{p}p'] = C - C'[\underline{p}p']$, with $C' = \{w \in C : p \text{ is true in } w\}$
c. $C - C'[\underline{p}p'] = C - C[p][\underline{p}p'] = C - C[(p\ \textit{and}\ p')]$

Building on the bivalence of our fragment, we can imitate the effects of Heim's local accommodation by stipulating that, in case global accommodation fails, one may lift the requirement that the presupposition $p$ of an expression $\underline{p}p'$ must be entailed by its local context. Since the semantic component of our analysis already treats $\underline{p}p'$ as bivalent, it has the same meaning as $(p\ \textit{and}\ p')$ and we immediately obtain the result in (64c).

30 It should be pointed out that the DRT approach of van der Sandt (1993) and Geurts (1999) offers a different picture because it takes accommodation (whether local, intermediate or global) to be the very foundation of the theory of presupposition computation. In that framework, local accommodation is just as natural as global accommodation.

Some researchers (van der Sandt 1993; Geurts 1999) have argued that an additional process of 'intermediate accommodation' is sometimes available; it functions like local accommodation, but it targets a context which is neither the local nor the global one to guarantee that the presupposition of an expression is satisfied. The existence of such a process is controversial (Beaver 2001), though it is very simple to implement in the DRT framework. In any event, whatever stipulations can implement it within Heim's dynamic semantics can be adapted to the present framework: we can stipulate that, under certain specified conditions, one can interpret a sentence $a[_{\alpha} \ldots \underline{d}d' \ldots]b$ (where the constituent $\alpha$ has a type that 'ends in $t$') as if it were $a^{c^+}[_{\alpha} \ldots \underline{d}d' \ldots]b$, where $c^+$ is a strengthened context, possibly at an intermediate site, which guarantees that the presupposition of $d$ is entailed by its local context. In this case the presence of $c^+$ may well affect the truth conditions rather than just the felicity conditions of the sentence. Of course, in the case of local accommodation this process is redundant with the mechanism we sketched in the preceding paragraph, whereby a presupposition is treated as if it were part of the asserted meaning. Thus the leaner theory is the one that only postulates local accommodation; whether it will suffice depends on the eventual outcome of the debate on intermediate accommodation.

## 5.2 Concluding Remarks

To conclude, we hope to have shown that a modified notion of 'local context' can be defined which improves on dynamic semantics in two respects. First, and foremost, the resulting theory of local satisfaction is fully predictive: as soon as the classical semantics and the syntax of any operator are specified, its projection behavior is automatically predicted as well. Second, it shows that the main results of the dynamic approach to presuppositions can be reconstructed within a theory that eschews Context Change Potentials, and is in fact classical. Third, our analysis is potentially more fine-grained than dynamic semantics because it can treat the left-to-right asymmetry found in projection patterns as a processing bias rather than as a hard-wired property of operators. This predicts gradient judgments of acceptability, and might account for the existence — and relative difficulty — of a variety of new 'symmetric readings'.

In addition to the issue of accommodation, three main questions remain for future research. First, can the limited availability of symmetric readings be demonstrated experimentally? Second, how does our approach compare to other recent solutions to the projection problem which also address the Explanatory Challenge introduced in Section 1.1 — in particular, the trivalent accounts of George (2008a,b) and Fox (2008), the dynamic approaches of Rothschild (2008b,c) and LaCasse (2008), and the implicature-based analysis of Chemla (2008b)? Finally, how do recent experimental results (e.g. Chemla 2009, 2008a) bear on the present theory? Initial data confirm some of our results (e.g. the existence of univeral projection under the determiner *no*), but raise important new challenges: contrary to the predictions of the present theory, presupposition projection out of restrictors and out of the scope of numerical quantifiers (*more than five*, *less than five*, *exactly five*) does not appear to give rise to universal inferences; some of the competing accounts (especially those of George, Fox and Chemla) might in this respect be at an advantage. The debate promises to be lively.

## A Equivalence with the Transparency Theory and with Dynamic Semantics

In this appendix, we show that our reconstruction of dynamic semantics is equivalent to the Transparency theory, an analysis that was initially presented as *anti*-dynamic. The incremental version of the Transparency theory was itself shown in earlier work to be equivalent to Heim's dynamic semantics under relatively broad conditions (Schlenker 2007); when these are satisfied, we thus have an indirect proof that the incremental version of the present proposal is equivalent to Heim's dynamic semantics. Throughout this section, we assume that local contexts exist, which is not always the case; we revisit this question in Appendix B, where we show that a natural extension of our proposal yields full equivalence with the Transparency theory even when local contexts fail to exist.

### A.1 The Transparency Theory

The Transparency theory purports to do without any notion of local context, and to explicate presupposition projection in purely pragmatic terms, on the basis of two Gricean principles of manner. Starting from a sentence *S* and a specification of its classical semantics (with distinguished presupposition triggers), the reasoning is as follows.

A presupposition is viewed as a distinguished entailment, one that 'wants' to be articulated as a separate conjunct. All things being equal, then, one should say *It is raining and John knows it* rather than *John knows that it is raining*. The constraint that demands that presuppositions be articulated separately is called *Be Articulate*; it can be seen as a Gricean maxim of manner, since it imposes a condition on the way in which certain meanings should be expressed.

(65) **Be Articulate**
Say $a\ (d\ and\ \underline{d}d')\ b$ rather than $a\ \underline{d}d'\ b$.

A second principle of manner, *Be Brief,* limits the effects of *Be Articulate*. The intuition is that in any syntactic environment $a\_b$, one should *not* say $a\ (d\ and\ blah)\ b$ in case the words *d and* are certain to be eliminable without truth-conditional loss. *Be Brief* was taken to come in an incremental and in a symmetric version.

In the incremental version, *d and* is considered idle in case *no matter what follows*, these words are certain to be eliminable given what is already assumed in the conversation. For instance, if it is already assumed that John is in Paris, it will be idle to start any sentence with *John is in Paris and....* Similarly, no matter what is assumed, a sentence that starts with *If John is staying near the Louvre,* ***he is in Paris and ...*** will contain a redundancy, because the words in bold are certain to be eliminable without truth conditional loss.

In the symmetric version of *Be Brief*, the entire syntactic environment of a conjunction *... F and G ...* is taken into account when deciding whether the words *F and* are redundant. All the cases excluded by the incremental version are excluded by the symmetric version, but additional cases are ruled out by the symmetric version. For instance, *John is in Paris and he is happy, if he is staying near the Louvre* is prohibited by the symmetric but not by the incremental version; for no matter what the second conjunct *blah* turns out to be, one can be certain that *John is in Paris and blah, if John is staying near the Louvre* is equivalent to *Blah, if John is staying near the Louvre.*

(66) **Be Brief** (Slightly generalized from Schlenker 2008b)
Let $C$ be a context set, and let $d$ be an occurrence of an expression whose type 'ends in $t$' in a sentence $a\,(d\ \mathit{and}\ d')\,b$.

a. **Incremental version**
$d$ is 'incrementally transparent' — and violates the incremental version of *Be Brief* — just in case for any expression $g$ of the same type as $d$, for any good final $b'$, $C \vDash\ a\,(d\ \mathit{and}\ g)\,b' \Longleftrightarrow a\,g\,b'$.

b. **Symmetric version**
$d$ is 'symmetrically transparent' — and violates the symmetric version of *Be Brief* — just in case for any expression $g$ of the same type as $d$, $C \vDash\ a\,(d\ \mathit{and}\ g)\,b \Longleftrightarrow a\,g\,b$

With these principles in place, a theory of presupposition projection was developed by positing that *Be Brief* cannot be violated, while *Be Articulate* can be. This may be encoded by postulating (for instance in an optimality-theoretic framework) that *Be Brief* is more highly ranked than *Be Articulate*:

(67) *Be Brief* >> *Be Articulate*

Together, these principles predict that in any syntactic environment a presupposition trigger $\underline{d}d'$ must be expressed as $(d\ \mathit{and}\ \underline{d}d')$, unless $d$ is (incrementally or symmetrically) transparent. To give an example, $\underline{p}p'$ presupposes that $p$, because it is precisely in case $C \vDash p$ that we can be sure that for any $g$, $C \vDash\ (p\ \mathit{and}\ g) \Longleftrightarrow g$. Similarly, $(\mathit{if}\ \ p\,.\,\underline{p}p')$ does not presuppose anything, because no matter what $C$ and $g$ are, $C \vDash\ (\mathit{if}\ \ p\,.\,(p\ \mathit{and}\ g)) \Longleftrightarrow (\mathit{if}\ \ p\,.\,g)$. The same reasoning could in principle apply to the sentence $\underline{p}p'\ \mathit{if}\ p$, but only if we apply the symmetric rather than the incremental version of *Be Brief*. Furthermore, we can posit that *both* versions of *Be Brief* are in fact at work, but that an articulated sentence is more deviant — and hence its unarticulated counterpart *more acceptable* — if it is ruled out by the incremental version of *Be Brief*. This immediately derives the preference for sentences of the form $(p\ \mathit{and}\ \underline{q}q')$ over $(\underline{q}q'\ \mathit{and}\ p)$ in case $p$ entails $q$.

Taken together, *Be Brief* and *Be Articulate* imply that a presupposition trigger $\underline{d}d'$ in a syntactic environment $a\_b$ satisfies the incremental or the symmetric version of the Transparency theory just in case its competitor $a\,(d\ \mathit{and}\ \underline{d}d')\,b$ is ruled out by the relevant version of *Be Brief*. To indicate that $\underline{d}d'$ is acceptable according to the incremental or symmetric version of the Transparency theory, we write $\mathit{Transp}^i(C, \underline{d}d', a\_b)$ or

$Transp^s(C, \underline{d}d', a\_b)$.

(68) a. $Transp^i(C, \underline{d}d', a\_b)$ iff for any expression $g$ of the same type as $d$, for any good final $b'$, $C \vDash a\ (d\ and\ g)\ b' \Longleftrightarrow a\ g\ b'$.
b. $Transp^s(C, \underline{d}d', a\_b)$ iff for any expression $g$ of the same type as $d$, $C \vDash\ a\ (d\ and\ g)\ b \Longleftrightarrow a\ g\ b$.

We can then say that a formula $f$ is acceptable according to the Transparency theory just in case every occurrence of any presupposition trigger $\underline{d}d'$ is acceptable; and here too the notion comes in two versions, though both are defined in the same way relative to the relevant version of *Transp*.

(69) For any $v \in \{i, s\}$, $Transp^v(C, F)$ iff for every expression $\underline{d}d'$, for all strings $a$, $b$, if $F = a\ \underline{d}d'\ b$, then $Transp^v(C, \underline{d}d', a\_b)$.

In Schlenker 2007, it was shown that for the very fragment we have assumed in the present article, with expressions of the form $(not\ F)$, $(F\ and\ G)$, $(F\ or\ G)$, $(if\ F. G)$, $(Q\ F. G)$, the incremental version of the Transparency theory derives almost all the results of Heim 1983. We will now extend these results to our reconstruction of dynamic semantics by showing that the latter is itself equivalent to the Transparency theory; near-equivalence with Heim's dynamic semantics will immediately follow.

### A.2 Equivalence with the Transparency theory

Our reconstruction of dynamic semantics does things in two steps:

(70) It starts by defining the local context of an expression $\underline{d}d'$ in an environment $a\_b$ as the strongest $c'$ for which $c'$ is (incrementally or symmetrically) transparent in $a\ {}^{c'}g\ b$ relative to the context set $C$.

(71) It then requires that the value of $c'$ should entail **d**.

The Transparency theory does essentially the same thing, but in a single step: given a sentence $a\ \underline{d}d'\ b$, it simply asks whether $\underline{d}$ is (incrementally or symmetrically) transparent no matter what the assertive component $d'$ turns out to be. Because the theory is based on a competition between $a\ \underline{d}d'\ b$ and its 'articulated' competitor $a\ (d\ and\ \underline{d}d')\ b$, the relevant notion of 'transparency' involves a full conjunction (i.e. we ask whether $d\ and$ could be eliminated without truth-conditional loss), but the end result is still that the presupposition must be transparent.

It can be shown that whenever the local context $\underline{d}d'$ exists, $\underline{d}d'$ satisfies Transparency (in its incremental or symmetric version) just in case $d$ is entailed by its (incremental or symmentric) local context. Using again the superscript $i$ for incremental notions and $s$ for their symmetric counterparts, we obtain the following result:

(72) **Equivalence with Transparency — Special Case**
For any $v \in \{i, s\}$, for every formula that has the form
$a\,\underline{d}d'\,b$, if $lc^v(C, \underline{d}d', a\_b) \neq \#$,
then $Transp^v(C, \underline{d}d', a\_b)$ iff $Sat^v(C, \underline{d}d', a\_b)$.

The argument is straightforward; we only sketch it for the incremental version (the argument is analogous for the symmetric version, taking $b' = b$).

First, suppose that $Transp^i(C, \underline{d}d', a\_b)$. Then for every $g$ of the same type as $d$ and for every good final $b'$, $C \vDash\ a\,(d\ and\ g)\,b' \iff a\,g\,b'$. Using our superscript notation, this also means that $C \vDash\ a\,{}^d g\,b' \iff a\,g\,b'$, and thus that **d** is a transparent restriction for $g$. Since $lc^i(C, \underline{d}d', a\_b)$ is the bottom element of the set of transparent restrictions, it immediately follows that $lc^i(C, \underline{d}d', a\_b) \leq \mathbf{d}$.

Second, suppose that $lc^i(C, \underline{d}d', a\_b) \leq \mathbf{d}$. Then for every $g$ of the same type as $d$, for every good final $b'$:

(73) a. $C \vDash^{c' \mapsto lc^i(C, \underline{d}d', a\_b)}\ a\,{}^{c'}g\,b' \iff a\,g\,b'$
b. $C \vDash^{c' \mapsto lc^i(C, \underline{d}d', a\_b)}\ a\,{}^{c'}(d\ and\ g)\,b' \iff a\,(d\ and\ g)\,b'$

Since $lc^i(C, \underline{d}d', a\_b) \leq \mathbf{d}$, replacing $g$ with $(d\ and\ g)$ in $a\,{}^{c'}g\,b'$ won't affect the truth conditions:

(74) $C \vDash^{c' \mapsto lc^i(C, \underline{d}d', a\_b)}\ a\,{}^{c'}g\,b' \iff a\,{}^{c'}(d\ and\ g)\,b'$

Putting (73a–b) and (74) together, we conclude that:

(75) $C \vDash^{c' \mapsto lc^i(C, \underline{d}d', a\_b)}\ a\,{}^{c'}(d\ and\ g)\,b' \iff a\,g\,b'$

Since $c'$ does not occur in this formula, the value assigned to $c'$ is irrelevant and we obtain the result that $C \vDash a\,(d\ and\ g)\,b' \iff a\,g\,b'$, which shows that $\underline{d}d'$ satisfies Incremental Transparency.

More generally, it follows that an entire formula $F$ satisfies Transparency (in its incremental or symmetric version) just in case *each* presupposition is entailed by its (incremental or symmetric) local context:

(76) **Consequence**
For any $a$ and any $v \in \{i, s\}$, for any formula $F$, for every expression $\underline{d}d'$ and for all strings $a, b$, if $F = a\ \underline{d}d'\ b$ and if $lc^v(C, \underline{d}d', a\_b) \neq \#$, then: $Transp^v(C, F)$ iff $Sat^v(C, F)$

## A.3 Equivalence with Standard Dynamic Semantics

It was shown in Schlenker 2007 that in the propositional case the incremental version of the Transparency theory is fully equivalent to Heim's dynamic semantics (augmented with the asymmetric dynamic disjunction of Beaver 2001). In the quantificational case, the equivalence holds only if two additional assumptions are made:[31]

(77) **Non-Triviality**
Quantificational clauses should not be 'trivial' (i.e. replaceable with a tautology or a contradiction).

(78) **Constancy**
The domain is finite, and in addition restrictors should hold true of a constant number of individuals throughout the context set.

These assumptions are stated precisely in Appendix C.9 and in Schlenker 2007. Let us just recapitulate the main conclusion:

(79) Under the assumptions of Non-Triviality and Constancy,
a. $C[F] \neq \#$ iff $Transp^i(C, F)$.
b. If $C[F] \neq \#$, $C[F] = \{w \in C : w \vDash F\}$

We just showed that whenever local contexts exist, our reconstruction of dynamic semantics is equivalent to the Transparency theory. Furthermore,

31 Non-Triviality without Constancy does not suffice to predict universal presuppositions for all generalized quantifiers. Consider for instance the sentence (*less than 2 students.* $\underline{Q}Q'$) in a context set $C$, with $C = \{w, w'\}$, **student**$(w) = \{s\}$, **student**$(w') = \{s, s'\}$, $\mathbf{Q}(w) = \emptyset$ and $\mathbf{Q}(w') = \{s, s'\}$, and $\mathbf{Q}'(w) = \mathbf{Q}'(w') = \{s, s'\}$. We note that $w \vDash$ (*less than 2 students.* $\underline{Q}Q'$) while $w' \nvDash$ (*less than 2 students.* $\underline{Q}Q'$), so Non-Triviality is satisfied (because this quantificational statement is equivalent neither to a tautology nor to a contradiction). Still, we have that for every $d'$, $C \vDash$ (*less than 2 students.*($Q$ *and* $d'$)) $\iff$ (*less than 2 students.* $d'$), despite the fact that $w \nvDash$ (*every student.* $d$). The point is that the one and only student in $w$, namely $s$, does not satisfy the presupposition $Q$, but since the quantificational statement evaluated at $w$ is trivially true, this does not affect the principle of Transparency.

Constancy entails that in each world the domain of individuals is finite, which by results proven in Appendix C.16 guarantees that local contexts always exist (see also Appendix B for discussion). So we obtain in this way a relatively general equivalence between the present system and standard dynamic semantics.

(80) **Equivalence with Standard Dynamic Semantics**
Let $C$ be a context set and $F$ be a formula which satisfies Non-Triviality and Constancy. Then for every presupposition trigger $\underline{d}d'$, for all strings $a, b$, if $F = a\,\underline{d}d'\,b$, then $lc^i(C, \underline{d}d', a\_b) \neq \#$. Furthermore, $Sat^i(C, F)$ iff $C[F] \neq \#$.

### A.4 Differences between the Transparency theory and the present analysis

Despite these results of equivalence, there are several differences between the Transparency theory and the present theory. The most important one is conceptual: the Transparency theory has no notion of local contexts, and is entirely based on Gricean maxims of manner; by contrast, the present analysis follows Stalnaker, Karttunen and Heim in positing that a presupposition must be satisfied in its local context. As a result, some of the conceptual arguments leveled against the Transparency theory do not hold against the present analysis. Specifically, critics of the Transparency theory objected that (a) *Be Articulate* lacks independent motivation, and that (b) it sometimes makes incorrect predictions in case either the articulated competitor of the form $(d\ and\ \underline{d}d')$ is ungrammatical, or is too complicated to be expressed.[32]

Since *Be Articulate* plays no role in the present theory, neither of those potential problems arises here. On the other hand, empirical criticisms (for instance with respect to the predictions of the symmetric versions of either analysis) have the same bite for both theories.

Still, there are other differences between the Transparency theory and our reconstruction of local contexts.

i. As was pointed out in Section 3.3, our analysis of local contexts can offer a general theory of 'local triviality.' By contrast, the versions of

32 See the commentaries by Beaver (2008), Chemla (2008c), Fox (2008), Krahmer (2008), Rothschild (2008c), Sauerland (2008), and van der Sandt (2008) in *Theoretical Linguistics*, which also includes a reply (Schlenker 2008b).

*Be Brief* that are assumed in the Transparency theory are by no means general: they explain under what conditions the first conjunct of an expression *F and G* is redundant, but they are hopelessly silent about innumerable cases of redundancy; for instance, they are powerless to explain why *if F, F* or *F or F* are felt to be redundant, since these examples do not even include a conjunction.[33]

ii. The symmetric version of the Transparency theory is difficult to motivate, whereas the symmetric version of our reconstruction of local contexts falls out more naturally. As was mentioned above, incremental Transparency can be motivated on the basis of a processing metaphor: the beginning of a conjunction, *F and* , is incrementally transparent just in case one can determine *as soon as one has heard it* that it is certain to be eliminable without truth-conditional loss. But the symmetric version is much less natural: one must somehow pretend that one has heard the beginning of the sentence $a$, the end of the sentence $b$, and the beginning of the conjunction $d$, but crucially not the end of the conjunction [*and*] $d'$.[34] The symmetric theory of local contexts is arguably more natural: it simply computes the local context of an expression $E$ by taking into account the entire environment in which $E$ occurs.

iii. Finally, it should be mentioned that in case local contexts fail to exist, the Transparency theory is simpler than our reconstruction of local contexts. The latter must be extended to deal with these cases - which is the topic of Appendix B.

33 The Transparency theory also fails to explain why *F and G* is deviant when *G* follows from *F*, as in (57) in the text, copied below as (i):

(i) a. John is in Paris, and he is staying near the Louvre.
b. #John is staying near the Louvre, and he is in Paris.

34 This odd wrinkle is arguably essential to make the right predictions. Consider for instance the sentence *It is John who won.* The negation and the question tests suggest that its presupposition is that *exactly one person won*; and the assertive component has to be that *John won.* But in most cases, if John won, nobody else did, so the assertive component (quasi-)entails the presupposition. If one *did* take into account the second conjunct when determining whether the first one is redundant, one would have to predict that *Exactly one person won and it is John who did* must be ruled out by the symmetric version of Be Brief. Since the articulated competitor is ruled out, the sentence *It is John who won* should in general be acceptable without a presupposition! This appears to be incorrect.

## B Existence of Local Contexts

In this Appendix, we characterize the cases in which local contexts exist, and extend the theory to cases in which they don't. In a nutshell, as long as the semantics is extensional and the domain of individuals in each possible world is of finite size, we can guarantee that local contexts exist. But the result fails to hold when infinite domains are considered; in such cases the satisfaction theory must be redefined in a slightly more complicated way.

### B.1 When local contexts exist

In the propositional case, it can be shown that local contexts (both incremental and symmetric) always exist; a simple proof is given in Appendix C.16(i). In the quantificational case, local contexts may fail to exist, for reasons we will turn to shortly. But there are still broad conditions under which their existence is guaranteed. The details of the proof are laid out in Appendix C.16(ii), but the main observation is quite simple. In all cases, the set of transparent restrictions is closed under finite (generalized) conjunction: if $x$ and $x'$ are two transparent restrictions, then so is $x \wedge x'$. When the set of transparent restrictions is finite, we can ensure that it has a bottom element. For instance, if the set contains the context denotations $x_1$, $x_2$ and $x_3$, we start by taking the intersection of $x_1$ and $x_2$, which entails both and must be in the set; then we take its intersection with $x_3$ — the result is again in the set, and it entails $x_1$, $x_2$, and $x_3$, so it is the bottom element we were looking for. The procedure can be applied whenever the set of transparent restrictions is finite. This condition happens to be met whenever all the relevant domains of individuals are themselves finite. So we can derive a general condition that guarantees that local contexts do exist.

### B.2 When local contexts don't exist

Interestingly, there are cases in which local contexts don't exist. From the preceding remarks, we can already infer that the relevant examples must involve infinite domains of individuals. We start from the formula ($\textit{infinitely-many}\, P.\, {}^{c'}Q$), and consider the set of transparent values for $c'$ (in this case there is no difference between the incremental and the symmetric version of the analysis). We assume that there are infinitely many elements in $\mathbf{P}(w)$, the value of $P$ at a certain world $w$ of $C$. Now we note that for $c'$ to be transparent in $(\textit{infinitely-many}\ P.\ {}^{c'}Q)$, its value $x$ must be such that $x(w)$ contains infinitely many elements. For if not, $(\textit{infinitely-many}\ P.\ {}^{c'}Q)$ would be false at $w$ but $(\textit{infinitely-many}\ P.\ P)$ would be true — and $c'$ wouldn't be transparent after all. Next, we show that for any transparent value $x$ for $c'$, we can find a 'smaller' value $x'$ which is also transparent. We define $x'$ as identical to $x$, except that we take one arbitrary element out of $x(w)$. It is clear that $x'$, which itself contains infinitely many elements, is transparent — for the simple reason that the truth of the statement *infinitely many Ps are Qs* is utterly insensitive to whatever happens to any given finite set of elements. Since $x$ was arbitrary, we have shown that the set of transparent restrictions does not have a bottom element (see Appendix C.23 for a more thorough treatment).

What can be done in this case? The problem arose because we have an infinite series of increasingly stronger transparent values for $c'$, with no bottom element. One solution is to redefine the notion of satisfaction in a way that does not depend on the existence of a bottom element. This can be done by introducing a notion *Sat′* which is defined directly in terms of the set $T$ of transparent restrictions: the presupposition is satisfied in this new sense just in case *there exists a member of T such that every element of T that entails it also entails the presupposition.*[35] As before, this notion comes in an incremental and in a symmetric version, which we distinguish using the superscripts $i$ and $s$ respectively.

(81) For every $v \in \{i,s\}$, $Sat'^{v}(C,\underline{d}d',a\_b)$ iff for some $x \in tr^{v}(C,\underline{d}d',a\_b)$, for every $x'$, if $[x' \leq x\ \ and\ \ x' \in tr^{v}(C,\underline{d}d',a\_b)]$, then $C \vDash^{c' \mapsto x'} c' \leq d$.

Of course when the set $T$ of transparent restrictions has a bottom element $c*$, a presupposition is satisfied in the new sense just in case it is in the old sense: if $c*$ entails the presupposition, taking $x = c*$, we immediately see that the condition in (81) is met. Conversely, if the condition in (81) is met, then the bottom element $c*$ must entail the presupposition, which is thus satisfied in the old sense. However when local contexts fail to exist, we

35 It can be noted that the problem we face and the solution we explore have counterparts in David Lewis's study of *Counterfactuals* (1973). Lewis defined a non-monotonic semantics for conditionals whose main intuition was that *if F, G* is true in world $w$ just in case the closest $F$-worlds from $w$ are also $G$-worlds. But Lewis argued that sometimes there is an infinite series of increasingly 'closer' $F$-worlds to $w$; for such cases the truth conditions of conditionals had to be adapted. In essence, *if F, G* was deemed true just in case for some world $x$, every $F$-world which is closer than it (to the world of evaluation) is a $G$-world.

obtain new predictions, which turn out to be fully equivalent to those of the Transparency theory:

(82) **Equivalence with Transparency — General Case**
The revised definition of satisfaction yields full equivalence with the Transparency theory. Specifically, for any $v \in \{i, s\}$, for every formula that has the form $a\ \underline{d}d'\ b$, $Sat'^{v}(C, \underline{d}d', a\_b)$ iff $Transp^{v}(C, \underline{d}d', a\_b)$.

A simple proof is given in Appendix C.21. This equivalence need not be a good thing, because in the somewhat arcane cases in which local contexts don't exist the Transparency theory (and our revised theory of satisfaction) make predictions that are arguably too weak (see Appendix C.23). It might thus prove fruitful in future research to explore alternative extensions of our primitive notion of satisfaction to derive slightly stronger predictions.

## C Definitions and Formal Results

In this appendix, we define one base language (called $L$) and compare three accounts of presupposition projection in that language: dynamic semantics, the Transparency theory, and our reconstruction of local satisfaction.[36]

### C.1 Syntax of $L$

- Generalized quantifiers: $Q ::= Q_i$
- Predicates: $P ::= P_i \mid P_i P_k$
- Propositions: $\mathbf{p} ::= \mathbf{p_i} \mid \mathbf{p_i p_k}$
- Formulas: $\mathbf{F} ::= \mathbf{p} \mid (\mathit{not}\,\mathbf{F}) \mid (\mathbf{F}\ \mathit{and}\,\mathbf{F}) \mid (\mathbf{F}\ \mathit{or}\,\mathbf{F}) \mid (\mathit{if}\,\mathbf{F}.\,\mathbf{F}) \mid (Q_i\ P.\,P)$

To state some of our principles, the official object language is enriched with:

- Predicate conjunction: if $P$ and $P'$ are predicates, so is $(P\ \mathit{and}\,P')$.
- Restrictions of predicative and propositional types:

36 The Transparency theory is an essential intermediary because it was shown in earlier work to be partly equivalent to dynamic semantics (Schlenker 2007); since our reconstruction of local satisfaction is itself technically close to the Transparency theory, as shown in Appendix A, we obtain the main equivalence results through the Transparency theory.

– if $c'$ is a predicative context variable and if $P$ is a predicate, ${}^{c'}P$ is a predicative expression;

– if $c'$ is a propositional context variable and if $F$ is a formula, ${}^{c'}F$ is a formula.

The 'propositional fragment' of $L$ is the language defined by the expressions in bold.

We start by defining a classical semantics, which we call $I$. On a technical level, we assume that:

- each propositional letter is assigned by $I$ a function of type $\langle s, t \rangle$
- each predicate letter is assigned by $I$ a function of type $\langle s, \langle e, t \rangle \rangle$
- each generalized quantifier $Q_i$ corresponds to a 'tree of numbers' $f_i$, which associates a truth value with each pair of the form $(a, b)$ with $a$ = the number of elements that satisfy the restrictor but not the nuclear scope and $b$ = the number of elements that satisfy both the restrictor and the nuclear scope.

Logical constants are given a syncategorematic semantics.

Instead of writing $I(F)(w) = 1$, we sometimes use the notation $w \vDash F$. We will also abbreviate $I(F)(w)$ as $\mathbf{F}^w$. When we use an extended language to state some of our principles, we will sometimes need assignment functions, and we will write $w \vDash^s F$, $\mathbf{F}^{w,s}$ to indicate relativization of the relevant notions to the assignment function $s$.

When certain elements are optional, we place angle brackets $\langle \cdots \rangle$ around them and around the corresponding part of the semantic rules.

**C.2 Classical Semantics** (Called $I$ in what follows)

$$w \vDash p \text{ iff } \mathbf{p}^w = 1$$
$$w \vDash \underline{p}p' \text{ iff } \mathbf{p}^w = \mathbf{p}'^w = 1$$
$$w \vDash (not\, F) \text{ iff } w \nvDash F$$
$$w \vDash (F\ and\ G) \text{ iff } w \vDash F \text{ and } w \vDash G$$
$$w \vDash (F\ or\ G) \text{ iff } w \vDash F \text{ or } w \vDash G$$
$$w \vDash (if\ F.\ G) \text{ iff } w \nvDash F \text{ or } w \vDash G$$
$$w \vDash (Q_i \langle \underline{P} \rangle P'\ .\ \langle \underline{Q} \rangle Q') \text{ iff } f_i(a_w, b_w) = 1, \text{ with}$$
$$a_w = \{d \in D\colon \langle \mathbf{P}^w(d) = 1 \text{ and}\rangle\ \mathbf{P}'^w(d) = 1 \text{ and } \langle \mathbf{Q}^w(d) = 0 \text{ or}\rangle\ \mathbf{Q}'^w(d) = 0\}$$
$$b_w = \{d \in D\colon \langle \mathbf{P}^w(d) = 1 \text{ and}\rangle\ \mathbf{P}'^w(d) = 1 \text{ and } \langle \mathbf{W}^w(d) = 1 \text{ and}\rangle\ \mathbf{Q}'^w(d) = 1\}$$

For reasons of simplicity, we will assume throughout that $L$ is *extremely* expressive: any proposition or property can be expressed by an atomic expression:

**C.3 Expressivity** Every proposition and every property is denoted by some atomic expression of $L$.

We repeat from the text our definitions of generalized entailment and generalized conjunction; they are intended for much richer type-theoretic languages, but are applicable in the present framework.

**C.4 Generalized Entailment**

i. If $x$ and $x'$ are two objects of a type $\tau$ that 'ends in t,' and can take at most $n$ arguments, $x \leq x'$ just in case whenever $y_i, \ldots y_n$ are objects of the appropriate type, if $x(y_1)\ldots(y_n) = 1$, then $x'(y_1)\ldots(y_n) = 1$.

ii. If $E$ and $E'$ are two expressions of a type $\tau$ that 'ends in t,'

$$w \vDash^s (E \leq E') \text{ iff } \mathbf{E}^{w,s} \leq \mathbf{E}'^{w,s}$$

## C.5 Generalized Conjunction

i. If $x$ and $x'$ are two objects of a type $\tau$ that 'ends in t,' and can take at most $n$ arguments, of types $\tau_1, \ldots, \tau_n$ respectively, then

$$x \wedge x' = \lambda y_{1\tau_1} \ldots \lambda y_{n\tau_n}[x(y_1)\ldots(y_n) = x'(y_1)\ldots(y_n) = 1]$$

ii. If $E$ and $E'$ are two expressions of a type $\tau$ that 'ends in t,'

$$({}^{\mathbf{E}'}\mathbf{E})^{w,s} = (\mathbf{E}' \textit{ and } \mathbf{E})^{w,s} = \mathbf{E}'^{w,s} \wedge \mathbf{E}^{w,s}$$

We define on the basis of C.2 a dynamic semantics which corresponds to Heim's analysis (Heim 1983), except that (i) it applies to all generalized quantifiers, (ii) it does not include variables, (iii) it applies to disjunction, which Heim does not discuss (here we follow Beaver 2001).

## C.6 Dynamic Semantics

$$
\begin{aligned}
C[p] &= \{w \in C\colon \mathbf{p}^w = 1\}\\
C[\underline{p}p'] &= \#\text{ iff for some } w \in C, \mathbf{p}^w = 0\\
&\quad \text{if } \neq \#, C[\underline{p}p'] = \{w \in C\colon \mathbf{p}'^w = 1\}\\
C[(\textit{not}\, F)] &= \#\text{ iff } C[F] = \#\\
&\quad \text{if } \neq \#, C[(\textit{not}\, F)] = C - C[F]\\
C[(F \textit{ and } G)] &= \#\text{ iff } C[F] = \#\text{ or } (C[F] \neq \#\text{ and } C[F][G] = \#)\\
&\quad \text{if } \neq \#, C[(F \textit{ and } G)] = C[F][G]\\
C[(F \textit{ or } G)] &= \#\text{ iff } C[F] = \#\text{ or } (C[F] \neq \#\text{ and } C[not\ F][G] = \#)\\
&\quad \text{if } \neq \#, C[(F \textit{ or } G)] = C[F] \cup C[not\ F][G]\\
C[(\textit{if}\, F.\, G)] &= \#\text{ iff } C[F] = \#\text{ or } (C[F] \neq \#\text{ and } C[F][G] = \#)\\
&\quad \text{if } \neq \#, C[(\textit{if}\, F.\, G)] = C - C[F][not\ G]\\
C[(Q_i\ \langle \underline{P}\rangle P'.\ \langle \underline{R}\rangle R')] &= \#\text{ iff } \langle\text{for some } w \in C,\ d \in D, \mathbf{P}^w(d) = 0\rangle \text{ or}\\
&\qquad \langle\text{for some } w \in C,\ d \in D, \langle \mathbf{P}^w(d) = 1 \text{ and}\rangle\\
&\qquad \mathbf{P}'^w(d) = 1 \text{ and } \mathbf{R}^w(d) = 0\rangle\\
&\quad \text{if } \neq \#, C[(Q_i\ \langle \underline{P}\rangle P'.\ \langle \underline{R}\rangle R')] =\\
&\qquad \{w \in C\colon f_i(a_w, b_w) = 1\}
\end{aligned}
$$

with

$$
\begin{aligned}
a_w &= \{d \in D\colon \langle \mathbf{P}^w(d) = 1 \text{ and}\rangle\ \mathbf{P}'^w(d) = 1 \text{ and } \langle \mathbf{Q}^w(d) = 0 \text{ or}\rangle\ \mathbf{Q}'^w(d) = 0\}\\
b_w &= \{d \in D\colon \langle \mathbf{P}^w(d) = 1 \text{ and}\rangle\ \mathbf{P}'^w(d) = 1 \text{ and } \langle \mathbf{W}^w(d) = 1 \text{ and}\rangle\ \mathbf{Q}'^w(d) = 1\}
\end{aligned}
$$

The Transparency theory is based on the classical semantics in C.2; it comes in an incremental version and in a symmetric version.

## C.7 Transparency: Principles

i. **Be Articulate**
In any syntactic environment, express the meaning of an expression $\underline{d}d'$ as $(d\ and\ \underline{d}d')$.

ii. **Be Brief — Incremental Version**
Given a context set $C$, a predicative or propositional occurrence of $d$ is infelicitous in a sentence that begins with $\alpha\ (d\ and$ if for any expression $\gamma$ of the same type as $d$ and for any good final $\beta$,

$$C \vDash\ [\alpha\,(d\ and\ \gamma)\,\beta] \Longleftrightarrow [\alpha\,\gamma\,\beta]$$

iii. **Be Brief — Symmetric Version**
Given a context set C, a predicative or propositional occurrence of $d$ is (somewhat) infelicitous in a sentence of the form $\alpha\ (d\ and\ d')\ \beta$ if for any expression $\gamma$ of the same type as $d$,

$$C \vDash\ [\alpha\ (d\ and\ \gamma)\ \beta] \Longleftrightarrow [\alpha\,\gamma\,\beta]$$

iv. **Ordering of Principles**
Be Brief (in either version) $\gg$ Be Articulate.

## C.8 Transparency: Derived Notions

i. **Incremental Transparency**
= **Be Articulate** + Incremental Version of **Be Brief**
Let $C$ be a context set and $F$ be a formula. *$F$ satisfies Incremental Transparency relative to $C$* (abbreviation: $Transp^i(C,F)$) just in case for any presuppositional expression $\underline{d}d'$, for any strings $\alpha$ and $\beta$, if $F = \alpha\,\underline{d}d'\,\beta$, then for any constituent $\gamma$ of the same type as $d$ and for any good final $\beta'$,

$$C \vDash\ [\alpha\,(d\ and\ \gamma)\,\beta'] \Longleftrightarrow [\alpha\,\gamma\,\beta']$$

ii. **Symmetric Transparency**
= **Be Articulate** + Symmetric Version of **Be Brief**
Let $C$ be a context set and $F$ be a formula. *$F$ satisfies Symmetric Transparency relative to $C$* (abbreviation: $Transp^s(C, F)$) just in case for any presuppositional expression $\underline{d}d'$, for any strings $\alpha$ and $\beta$, if $F = \alpha\,\underline{d}d'\,\beta$, then for any constituent $\gamma$ of the same type as $d$,

$$C \vDash\ [\alpha\,(d\ and\ \gamma)\,\beta] \Longleftrightarrow [\alpha\,\gamma\,\beta]$$

In a broad range of cases, the incremental version of Transparency theory is equivalent to standard dynamic semantics.

## C.9 Incremental Transparency vs. Standard Dynamic Semantics (from Schlenker 2007)

i. **Non-Triviality**
Let $C \subseteq W$ be a context set and let $F$ be a formula. $\langle C, F\rangle$ satisfies **Non-Triviality** just in case for any initial string of $F$ of the form $\alpha\,A$, where $A$ is a quantificational clause (i.e. a formula of the form $(Q_i\,G\ .\ H)$), there is a good final $\beta$ such that:

$$C \nvDash\ [\alpha\,A\,\beta] \Longleftrightarrow [\alpha\,T\,\beta]$$

$$C \nvDash\ [\alpha\,A\,\beta] \Longleftrightarrow [\alpha\,F\,\beta]$$

where $T$ is a tautology and $F$ is a contradiction.

ii. **Constancy**
Let $C$ be a context set and $F$ be a formula. $\langle C, F\rangle$ satisfies **Constancy** just in case

a. the (unique) domain of individuals is of constant finite size over $C$, and

b. the extension of each restrictor that appears in $F$ is of constant size over $C$.

iii. **Theorem 1**
Consider the propositional fragment of $L$. Let $C \subseteq W$ be a context set and let $F$ be a formula. Then

a. $Transp^i(C, F)$ iff $C[F] \neq \#$

b. If $C[F] \neq \#$, $C[F] = \{w \in C\colon\ w \vDash F\}$

iv. **Theorem 2**
[Here we state only a *consequence* of Theorem 2 from Schlenker 2007.]

Let $C \subseteq W$ be a context set and let $F$ be a formula of $L$. Suppose that $\langle C, F\rangle$ satisfies Non-Triviality and Constancy. Then

a. $Transp^i(C, F)$ iff $C[F] \neq \#$

b. If $C[F] \neq \#$, $C[F] = \{w \in C\colon\ w \vDash F\}$

We not turn to the definition of transparent restrictions, of local contexts and of local satisfaction.

**C.10 Transparent Restrictions** Let $C \subseteq W$ be a context set and let $a\,d\,b$ be a formula, where $d$ has a type that 'ends in t'; let $c'$ be a variable of the same type as $d$.

i. $tr^i(C, d, a\_b) = \{x\colon\ x$ is an object of the type specified by $d$ and for every constituent $d'$ of the same type as $d$, for every good final $b$, $C \vDash^{c'\to x} [a\ {}^{c'}d'\ b'] \iff [a\,d'\,b']\}$

ii. $tr^s(C, d, a\_b) = \{x\colon\ x$ is an object of the type specified by $d$ and for every constituent $d'$ of the same type as $d$, $C \vDash^{c'\to x} [a\ {}^{c'}d'\ b] \iff [a\,d'\,b]\}$

**C.11 Local Contexts**

i. $lc^i(C, d, a\_b)$ = the bottom element of $tr^i(C, d, a\_b)$ if such an element exists; # otherwise.

ii. $lc^s(C, d, a\_b)$ = the bottom element of $tr^s(C, d, a\_b)$ if such an element exists; # otherwise.

C.12–C.16 are concerned with the existence of local contexts.

**C.12 Lemma 1. Propositional Fragment** For any $C \subseteq W$, for any formula $E$, if $a\,E\,b$ is a formula of the propositional fragment, $lc^i(C, E, a\_b) \neq \#$ and $lc^s(C, E, a\_b) \neq \#$.

*Proof.* We define $LC^i := \lambda w.1$ iff for some formula $g$, for some good final $b'$, $w \nvDash^{f\to F} [a\ {}^{f}g\ b'] \iff [a\,g\,b']$, where $F$ is a contradiction.

*Step 1:* $LC^i \in tr^i(C, E, a\_b)$

For every $w \in C$, either $LC^i(w) = 1$, in which case the contextual restriction $LC^i$ is innocuous at $w$ and $w \vDash^{c' \to LC^i} [a\, {}^{c'}g\, b'] \iff [a\, g\, b']$; or $LC^i(w) = 0$, which means that for every formula $g$, for every good final $b'$, $w \vDash^{f \to F} [a\, {}^{f}g\, b'] \iff [a\, g\, b']$, and thus $w \vDash^{c' \to LC^i} [a\, {}^{c'}g\, b] \iff [a\, g\, b']$.

*Step 2:* For every $x \in tr^i(C, F, a\_b)$, $LC^i$ entails $x$.

Suppose, for contradiction, that $LC^i(w) = 1$ and $x(w) = 0$. Since $x \in tr^i(C, F, a\_b)$, for every formula $g$ and good final $b'$, $w \vDash^{c' \to x} [a\, {}^{c'}g\, b] \iff [a\, g\, b']$, and since $x(w) = 0$ and the logic is extensional, $w \vDash^{f \to F} [a\, {}^{f}g\, b'] \iff [a\, g\, b']$. But by the definition of $LC^i$ this means that $LC^i(w) = 0$, contrary to hypothesis. The proof is similar for $lc^s(C, F, a\_b) \neq \#$. □

**C.13 Lemma 2. Closure under Finite Conjunction** For any $C \subseteq W$, for any formula $a\, d\, b$, $tr^i(C, d, a\_b)$ and $tr^s(C, d, a\_b)$ are closed under finite conjunction.

*Proof.* Assume that $x', x'' \in tr^i(C, d, a\_b)$. We have in particular that for any admissible $d'$ and for any good final $b'$,

By the semantics of ${}^{e}F$,

(83) $$C \vDash^{e \to x' \wedge x'',\ c' \to x',\ c'' \to x''} [a\, {}^{e}d'\, b'] \iff D[a\, {}^{c'}(c''\ and\, d')\, b']$$

Because $x' \in tr^i(C, d, a\_b)$,

(84) $$C \vDash^{e \to x' \wedge x'',\ c' \to x',\ c'' \to x''} [a\, {}^{c'}(c''\ and\, d')\, b'] \iff [a\, (c''\ and\, d')\, b']$$

By the semantics of ${}^{c''}F$,

(85) $$C \vDash^{e \to x' \wedge x'',\ c' \to x',\ c'' \to x''} [a\, (c''\ and\, d')\, b'] \iff [a\, {}^{c''}d'\, b']$$

Because $x'' \in tr^i(C, d, a\_b)$,

(86) $$C \vDash^{e \to x' \wedge x'',\ c' \to x',\ c'' \to x''} [a\, {}^{c''}d'\, b'] \iff [a'\ d'\, b']$$

By (83–86),

(87) $$C \vDash^{e \to x' \wedge x'',\ c' \to x',\ c'' \to x''} [a\, {}^{e}d'\, b'] \iff [a'\ d'\, b']$$

By (87) since $c'$ and $c''$ don't appear,

(88) $$C \vDash^{e \to x' \wedge x''} [a\, {}^{e}d'\, b'] \iff [a'\ d'\, b']$$

(89) $$(x' \wedge x'') \in tr^i(C, d, a\_b)$$

The proof is similar for $tr^s(C, d, a\_b)$. □

**C.14 Lemma 3: Finite Sets** For every $v \in \{i, g\}$, if $tr^v(C, d, a\_b)$ is finite

(90) $$lc^v(C, d, a\_b) \neq \#$$

*Proof.* Immediate from C.13. □

**C.15 Lemma 4: Pointwise Construction of Local Contexts** (This lemma crucialy relies on the extensionality of the fragment.) For every $v \in \{i, g\}$, if for every $w \in C$, $lc^v(\{w\}, d, a\_b) \neq \#$, then $lc^v(C, d, a\_b) \neq \#$.

*Proof.* The idea is to construct the bottom element point-wise, i.e. world by world. We define $LC^v$ as $\lambda w_s \,.\, lc^v(\{w\}, d, a\_b)$ if $w \in C$, $z$ otherwise, where $z$ is the null object of type $t$ if $d$ is propositional, and where $z$ is the null object of type $\langle e, t\rangle$ if $d$ is predicative.

It is immediate that $LC^v \in tr^v(C, d, a\_b)$ (because of the extensionality of the fragment).

Suppose, for contradiction, that for some $x \in tr^v(C, d, a\_b)$, $x$ is not entailed by $LC^v$. Then there is some world $w$ such that $x(w)$ is not entailed by $LC^v(w)$. It couldn't be that $w \notin C$, since in that case $LC^v(w) = z$, which entails everything. So $w \in C$. Clearly, since $x \in tr^v(C, d, a\_b)$, $x(w) \in tr^v(\{w\}, d, a\_b)$. But by assumption $LC^v(w)$ is the bottom element of $tr^v(\{w\}, d, a\_b)$, so it entails $x(w)$, *contra* hypothesis. □

**C.16 Existence Theorem: Existence of Local Contexts** Let $C \subseteq W$ be a context set and let $a\,E\,b$ be any formula.

i. If $a\,\underline{d}d'b$ belongs to the propositional fragment, then for every $v \in \{i, s\}$, $lc^v(C, E, a\_b) \neq \#$.

ii. If for every $w \in C$, the domain of individuals in $w$ is of finite size, then for every $v \in \{i, s\}$, $lc^v(C, E, a\_b) \neq \#$.

*Proof.* (i) is just Lemma 1 (C.12). (ii) follows from Lemma 3 (C.14) and Lemma 4 (C.15) together with the following observation: if the domain of individuals in $w$, $D_w$, is finite, then for any intensional type $\tau$ there are only finitely many functions of type $\tau$ with $D_s = \{w\}$.[37] It follows that $tr^v(\{w\}, E, a\_b)$

37 The proof is by induction on primitive types relative to $w$. Clearly, $D_w$ and $\{0, 1\}$ are finite. Furthermore, if E is finite, so are $D_w \to E$ and $\{0, 1\} \to E$.

is finite. By Lemma 3 (C.14), we construct $lc^v(\{w\}, E, a\_b)$ for every $w \in C$. By Lemma 4 (C.15), this makes it possible to construct $lc^v(C, E, a\_b)$. □

We now define our reconstruction of local satisfaction, in case local contexts exist, but also in the more general case in which they need not exist.

**C.17 Definition of Local Satisfaction — Special Case** (when local contexts exist) Let $C \subseteq W$ be a context set.

i. For every $v \in \{i, s\}$, for all expressions $\underline{d}d'$, $a$, $b$, if $lc^v(C, \underline{d}d', a\_b) \neq \#$, $Sat^v(C, \underline{d}d', a\_b)$ just in case $lc^v(C, \underline{d}d', a\_b) \leq \mathbf{d}$.

ii. For every $v \in \{i, s\}$, for every formula $F$, if for all expressions $a$, $b$, $\underline{e}e'$ such that $F = a\,\underline{e}e'\,b$, $lc^v(C, \underline{d}d', a\_b) \neq \#$, $Sat^v(C, F)$ just in case for every expression $\underline{e}e'$, for all strings $a$, $b$, if $F = a\,\underline{e}e'\,b$, then $Sat^v(C, \underline{e}e', a\_b)$.

**C.18 Definition of Local Satisfaction — General Case** (local contexts need not exist) Let $C \subseteq W$ be a context set.

i. For every $v \in \{i, s\}$, for all expressions $\underline{d}d'$, $a$, $b$, $Sat'^v(C, \underline{d}d', a\_b)$ iff for some $x \in tr^v(C, \underline{d}d', a\_b)$, for every $x'$, if $[x' \leq x$ and $x' \in tr^v(C, \underline{d}d', a\_b)]$, then $C \vDash^{c' \to x'} c' \leq d$.

ii. For every $v \in \{i, s\}$, for every formula $F$, $Sat'^v(C, F)$ just in case for every expression $\underline{e}e'$, for all strings $a$, $b$ if $F = a\,\underline{e}e'\,b$, $Sat'^v(C, \underline{e}e', a\_b)$.

**C.19 Lemma 5: when local contexts exist, the definitions in C.18 and C.17 are equivalent** For every $v \in i, s$, if $lc^v(C, \underline{d}d', a\_b) \neq \#$, $Sat^v(C, \underline{d}d', a\_b)$ iff $Sat'^v(C, \underline{d}d', a\_b)$.

*Proof.* First, if $Sat^v(C, \underline{d}d', a\_b)$, then by taking $x = ls^v(C, \underline{d}d', a\_b)$, we can find an $x \in tr^v(C, \underline{d}d', a\_b)$ such that, for every $x'$, if $[x' \leq x$ and $x' \in tr^v(C, \underline{d}d', a\_b)]$, then $C \vDash^{c' \to x'} c' \leq d$.

Second, if $Sat'^v(C, \underline{d}d', a\_b)$, there is some $x$ such that, for every $x'$, if $[x' \leq x$ and $x' \in tr^v(C, \underline{d}d', a\_b)]$, then $C \vDash^{c' \to x'} c' \leq d$.

Since $lc^v(C, \underline{d}d', a\_b)$ is the bottom element of $tr^v(C, \underline{d}d', a\_b)$, $lc^v(C, \underline{d}d', a\_b) \leq x$, and therefore $C \vDash^{c' \to lc^v(C, \underline{d}d', a\_b)} c' \leq d$. By the definitions of $tr^v(C, \underline{d}d', a\_b)$ and $lc^v(C, \underline{d}d', a\_b)$, it must be the case that

$lc^{v}(C, \underline{d}d', a\_b) \leq C$. Therefore, $lc^{v}(C, \underline{d}d', a\_b) \leq \mathbf{d}$. In other words, $Sat^{v}(C, \underline{d}d', a\_b)$. □

Before we go further, it is worth pointing out that the incremental version of local satisfaction systematically predicts presuppositions that are at least as strong as those predicted by the symmetric version (the same conclusion holds of the incremental vs. symmetric version of all the theories under study in this appendix).

**C.20 Incremental Satisfaction predicts stronger presuppositions than Symmetric Satisfaction** For any context set $C$, for all expressions $\underline{d}d'$ and for all strings $a$, $b$,

i. $tr^{i}(C, \underline{d}d', a\_b) \subseteq tr^{s}(C, \underline{d}d', a\_b)$

Furthermore, if $lc^{s}(C, d, a\_b) \neq \#$ and $lc^{i}(C, d, a\_b) \neq \#$,

ii. $lc^{s}(C, d, a\_b) \leq lc^{i}(C, d, a\_b)$

iii. if $Sat^{i}(C, d, a\_b)$, then $Sat^{s}(C, d, a\_b)$.

*Proof.* Immediate. □

We now turn to a comparison between Incremental Satisfaction, Incremental Transparency and Dynamic Semantics

**C.21 Theorem. Equivalence with Transparency** Let $C \subseteq W$ be a context set. Then:

i. For any $v \in \{i, s\}$, for every formula that has the form $a\,\underline{d}d'\,b$, $Sat'^{v}(C, \underline{d}d', a\_b)$ iff $Transp^{v}(C, \underline{d}d', a\_b)$.

ii. Therefore, for any $v \in \{i, s\}$, for every formula $F$, $Sat'^{v}(C, F)$ iff $Transp^{v}(C, F)$.

*Proof of (i).* We start with the incremental version.

⇒ Suppose that
$Sat'^{i}(C, \underline{d}d', a\_b)$. Then for some $x \in tr^{i}(C, \underline{d}d', a\_b)$,

$$C \vDash^{c' \to x} c' \leq d$$

For every expression $d''$ of the same type as $d$, for every good final $b'$,

$$C \vDash^{c' \to x} a\,(d\; and\, d'')\, b' \Longleftrightarrow a^{\,c'}(d\; and\, d'')\, b'$$
$$C \vDash^{c' \to x} a^{\,c'}(d\; and\, d'')\, b' \Longleftrightarrow a^{\,c'} d''\, b'$$
$$C \vDash^{c' \to x} a^{\,c'} d''\, b' \Longleftrightarrow a\, d''\, b$$

Hence,

$$C \vDash a\,(d\; and\, d'')\, b' \Longleftrightarrow a\, d''\, b'$$

$\Leftarrow$ Suppose that $Transp^i(C, \underline{d}d', a\_b)$. Clearly, $\mathbf{d} \in tr^i(C, \underline{d}d', a\_b)$. Furthermore, for every $x'$,

$$[x' \in tr^i(C, \underline{d}d', a\_b) \text{ and } x' \le \mathbf{d}] \implies C \vDash^{c' \to x'} c' \le d$$

So $Sat'^i(C, \underline{d}d', a\_b)$.

The argument is similar for the symmetric version of *Sat* and *Transp*. □

*Proof of (ii).* Immediate given (i). □

**C.22 Theorem. Equivalence with Standard Dynamic Semantics** Let $C \subseteq W$ be a context set and $F$ be a formula which satisfies Non-Triviality and Constancy. Then:

i. for all expressions $a$, $b$, $\underline{d}d'$, if $F = a\,\underline{d}d'\,b$, $lc^i(C, \underline{d}d', a\_b) \neq \#$. Furthermore,

ii. $Sat^i(C, F)$ iff $Sat'^i(C, F)$ iff $C[F] \neq \#$.

*Proof of (i).* Immediate from C.16(ii) and the fact that Constancy implies that for each $w \in C$, the set of individuals in $w$ is of finite size. □

*Proof of (ii).* The first equivalence follows from (i) and the Lemma in C.19. The second equivalence follows from (i), C.9(iii) and C.21(ii). □

Finally, we consider an example in which local contexts do not exist, which makes it necessary to resort to the alternative definition of satisfaction, $Sat'$.

**C.23 Infinitely Many** Consider the formula $F = (Infinitely\text{-}many\, P.\, \underline{Q}Q')$. We assume that there are infinitely many elements in $\mathbf{P}^w$.

i. $\underline{Q}Q'$ has no local context in the context set $\{w\}$.

ii. $Sat'^i(C, F)$ (or equivalently $Transp^i(C, F)$) need not entail that $C \vDash (Every\, P.\, Q)$.

*Proof of (i).* First, we note that if $x$ is a transparent value for $c'$, $x(w)$ must itself contain infinitely many elements. For if not, $(Infinitely\text{-}many\, P.\, {}^{c'}P)$ would be false at $w$ but $(Infinitely\text{-}many\, P.\, P)$ would be true — and $c'$ wouldn't be transparent. Second, we show that for any transparent value $x$ for $c'$, we can find a 'smaller' value $x'$ which is also transparent. Since $x(w)$ must contain infinitely many elements, we just take one arbitrary element out of $x(w)$, obtaining an $x'(w)$ distinct from $x(w)$ with $x'(w) \leq x(w)$ (and also $x \leq x'$). And it is clear that $x'$, which itself contains infinitely many elements, is transparent (because the truth of the statement *infinitely many Ps are Q* is insensitive to whatever happens to any given finite set of elements). Since $x$ was arbitrary, we have shown that the set of transparent context denotations simply does not have a bottom element. □

*Proof of (ii).* Assume that in $w$, $Q$ holds true of all $P$-individuals except a (non-zero) finite number. We have that for any predicative $D$, $w \vDash (Infinitely\text{-}many\, P.\, (Q\ and\ D)) \iff (Infinitely\text{-}many\, P.\, D)$, so $Transp^i(C, F)$, and therefore (by C.21) $Sat^i(C, F)$. Still, $w \nvDash (Every\, P.Q\,)$. □

### C.24 Examples of Symmetric Local Contexts[38] [listed in (61) in the text; since predicate negation is not part of our 'official' fragment, we tacitly adopt an extended fragment in this paragraph]

38 As noted in a different context by (Rothschild 2008c) and (Beaver 2008), as well as in an earlier version of the present paper (Feb. 8, 2008), the symmetric version of our analysis (as well as of the Transparency theory) will run into problems when several presupposition triggers occur in the same sentence. Thus in the example in (i), it is predicted that no presupposition failure obtains, despite the fact that both $\underline{p}p'$ and $\underline{q}q'$ trigger a failure on their own. (Beaver discusses the case in which $q = p$, which leads to the same result.)

(i) a. $(\underline{p}p'\ and\ \underline{q}q')$

b. $C = \{w_1, w_2\}$, $w_1 \nvDash p$, $w_2 \nvDash p$ and $w_2 \vDash q$

To address this issue, we can define for each string $s$ a string $s^*$ obtained by deleting all underlined material. We then define a slightly modified notion of transparency: $tr^{*s}(C, \underline{d}d', a\_b) = tr^s(C, \underline{d}d', a^*\_b^*)$, and we keep the rest of the theory, replacing $tr^s$ with $tr^{*s}$. In (i), the effect is to compute the local context of $\underline{p}p'$ relative to $({}^{c'}\underline{p}p'\ and\ q')$, and to compute the local context of $\underline{q}q'$ relative to $(p'\ and\ {}^{c'}\underline{q}q')$. In effect, we thus restrict the

a. $lc^s(C, \underline{q}q', (\_\_ \textit{and}\ p)) = lc^i(C, \underline{q}q', (p\ \textit{and}\ \_\_)) = C \wedge \mathbf{p}$

In other words, the symmetric local context of $\underline{q}q'$ in $(\underline{q}q'\ \textit{and}\ p)$, as well as in $(p\ \textit{and}\ \underline{q}q')$, is $C \wedge \mathbf{p}$.

*Proof.* In the incremental case, syntactic considerations guarantee that a formula of the form $p\ \textit{and}\ d'\ b'$, where $p$ and $d'$ are constituents, must end with a right parenthesis, so that $b' = )$. Hence $c'$ is symmetrically transparent in $({}^{c'}\underline{q}q'\ \textit{and}\ p)$ just in case it is incrementally transparent in $(p\ \textit{and}\ {}^{c'}\underline{q}q')$ — whence the result. □

b. $lc^s(C, \underline{q}q', (\_\_ \textit{or}\ p)) = lc^i(C, \underline{q}q', (p\ \textit{or}\ \_\_)) = C \wedge (\textbf{not p})$

*Proof.* Same as in (a). □

c. $lc^s(C, \underline{q}q', (\textit{if}\ \_\_\,.\ p)) = lc^i(C, \underline{q}q', (\textit{if}\ (\textit{not}\ p).\ (\textit{not}\ \_\_))) = C \wedge (\textbf{not p})$

In other words, the symmetric local context of $\underline{q}q'$ in $(\textit{if}\ \underline{q}q'\,.\ p)$, as well as in $(\textit{if}\ (\textit{not}\ p).\ (\textit{not}\ \underline{q}q'))$, is $C \wedge (\textit{not}\ \mathbf{p})$

*Proof.* $S$ is *symmetrically* transparent for $c'$ in $(\textit{if}\ {}^{c'}\underline{q}q'\,.\ p)$ just in case

i. for every propositional constituent $d'$, $C \vDash^{c' \to S} (\textit{if}\ {}^{c'}d'.\ p) \iff (\textit{if}\ d'.\ p)$.

$S$ is *incrementally* transparent for $c'$ in $(\textit{if}\ (\textit{not}\ p).\ (\textit{not}\ {}^{c'}\underline{q}q'))$ just in case

ii. for every propositional constituent $d'$, for every good final $b'$,
$C \vDash^{c' \to S} [(\textit{if}\ (\textit{not}\ p).\ (\textit{not}\ {}^{c'}d'\ b'] \iff [(\textit{if}\ (\textit{not}\ p).\ (\textit{not}\ d'\ b']$.

With a bit of syntactic reasoning (based on the formal fragment in (10), it can be argued successively that $d'$ must be immediately followed by $[)]$, and that $(\textit{not}\ {}^{c'}d')$ must itself by followed by $[)]$, so that the condition in (ii) is in effect that:

ii′. for every propositional constituent $d'$, $C \vDash^{c' \to S} [(\textit{if}\ (\textit{not}\ p).\ (\textit{not}\ {}^{c'}d'))] \iff [(\textit{if}\ (\textit{not}\ p).\ (\textit{not}\ d'))]$.

informational basis of context computation to the ‘assertive’ component of the rest of the sentence.

But since we have treated conditionals as material implications, (i) is equivalent to (ii′) (just take the contraposition of each side of the biconditional). It follows that the transparent values of $c'$ are the same in both cases, and that the corresponding local contexts are also identical. The desired result follows once one observes (as was done in the general case in fn. 16) that $lc^i(C, \underline{q}q', (\mathit{if}\ (\mathit{not}\ p)\ .\ (\mathit{not}\ \_))) = C \wedge (\mathbf{not\ p})$. □

d. $lc^s(C, \underline{P}P', (\mathit{Every}\ (\mathit{not}\ \_).\ Q)) = lc^i(C, \underline{P}P', (\mathit{Every}\ (\mathit{not}\ Q).\ \_) = {}^C(\mathbf{not\ Q})$

In other words, the symmetric local context of $\underline{P}P'$ in $(\mathit{Every}\ (\mathit{not}\ \underline{P}P').\ Q)$, as well as in $(\mathit{Every}\ (\mathit{not}\ Q).\ \underline{P}P'))$, is ${}^C(\mathbf{not\ Q})$ (i.e. the property of being a not-$Q$ individual restricted to $C$).

*Proof.* $S$ is symmetrically transparent for $c'$ in $(\mathit{Every}\ (\mathit{not}\ {}^{c'}\underline{P}P').\ Q)$ just in case

i. for every predicative constituent $d'$, $C \vDash^{c' \to S} (\mathit{Every}\ (\mathit{not}\ {}^{c'}\underline{P}P').\ Q) \iff (\mathit{Every}\ (\mathit{not}\ \underline{P}P').\ Q)$.

By the equivalence (for all $F$) between $(\mathit{Every}\ F.\ Q)$ and $(\mathit{Every}\ (\mathit{not}\ Q).\ (\mathit{not}\ F))$, taking $F = (\mathit{not}\ \underline{P}P')$, (i) is equivalent to

ii. for every predicative constituent $d'$, $C \vDash^{c' \to S} (\mathit{Every}\ (\mathit{not}\ Q).\ {}^{c'}\underline{P}P') \iff (\mathit{Every}\ (\mathit{not}\ Q).\ \underline{P}P')$.

But (ii) is the condition that must be met by $S$ if $c'$ is to be incrementally transparent in $(\mathit{Every}\ (\mathit{not}\ Q).\ {}^{c'}\underline{P}P')$. By the computations of Section 2.3.2 (in (35)), the desired result follows. □

e. $lc^s(C, \underline{P}P', (\mathit{No}\ (\mathit{not}\ \_).\ Q)) = {}^C\mathbf{Q}$

In other words, the symmetric local context of $\underline{P}P'$ in $(\mathit{No}\ (\mathit{not}\ \underline{P}P').\ Q)$ is ${}^C\mathbf{Q}$ (i.e. the property of being a $Q$-individual restricted to $C$).

*Proof.* The proof is similar to that in (d), using this time the observation that $(\mathit{No}\ F.\ Q)$ is equivalent to $(\mathit{No}\ Q.\ F)$ and taking $F = (\mathit{not}\ \underline{P}P')$. □

## D Adding Variable-binding Operators

The problem we encountered with belief operators in Section 3.1.2 re-emerges in languages which have variable-binding operators (by contrast, the system we discussed in the rest of this article contained quantifiers, but no variables at all). We will henceforth assume that we have a fully extensional language which includes individual and world variables, and we will sketch a possible extension of the analysis by discussing an example.

In *Every student believes that he is starting to make progress,* we want the local context of the embedded predicate to be dependent on a variable ($x_1$) bound by *every student*, and on a world variable ($w_2$) bound by the attitude operator — as well as by the free world variable ($w_0$) which denotes the world of utterance:[39]

(91) $[$*every student*-$w_0]\lambda x_1[w_0\ [x_1$ *believes*
$\lambda w_2[w_2\ [$*he*$_1\ [^{c'}$*is starting to make progress*$]]]]]$

Following Heim 1991, we take *student* to have a world argument, and thus we write *student*-$w_0$ for the predicate true of the students at $w_0$. In (91), *believes* takes three arguments: a world variable $w_0$, an individual variable $x_1$, and a propositional argument which has the form of a λ-expression with abstraction of a world variable (note that the world argument of a verb phrase appears as its *last* (i.e. highest) argument, right above the canonical position of the subject).

In such a framework, the meaning of any expression can be seen as a function from assignment functions to objects of a specified type; in essence, assignment functions play a role analogous to that of possible worlds in the variable-free system discussed in the main text. A simple way to allow $c'$ to depend on all the necessary variables is to make its value dependent on an entire assignment function as well.[40] We assume that the world of utterance is denoted by the distinguished variable $w_0$, and that the context set is identified to a set of assignment functions. Thus the context set encodes what is taken for granted by the speech act participants about the values of variables, including the distinguished variable $w_0$ which denotes [what they take to be] the world of utterance. In this modified framework, local

39 See Cresswell (1990) and Heim (1991) for arguments that the full power of such a language is semantically and syntactically justified; time variables, or for that matter variables of any type, could easily be added as well.

40 It is interesting to note that exactly the same technical solution was proposed to handle implicit domain restrictions on quantifiers by Heim (1991).

contexts are functions from assignment functions to objects of type $\langle s, t\rangle$, or $\langle s, \langle e, t\rangle\rangle$, as the case may be (in the earlier system, they were functions from possible worlds to objects of type $\langle s, t\rangle$ or $\langle s, \langle e, t\rangle\rangle$). All we need to apply our analysis to this case is an ordering on the possible values of $c'$; we extend our earlier definition point-wise:

(92) If $x$ and $x'$ are function from assignment functions to objects of type $\tau$, where $\tau$ 'ends in t,' then $x \le x'$ iff for every assignment function $s$, $x(s) \le x'(s)$.

The rest of the analysis is as before. When we apply the new definition to (91), the local context of the embedded predicate is defined as the strongest $x$ which satisfies (93) (for clarity, we continue to separate the value assigned to $c'$ from the value of other contextual parameters — in this case the assignment function $s$; we also omit some brackets from the Logical Forms):

(93) For every predicative expression $d'$, for every $s$ in $C$, $([\textbf{every student-w}_\textbf{0}]\lambda \textbf{x}_\textbf{1}\ \textbf{w}_\textbf{0}\ \textbf{x}_\textbf{1}\ \textbf{believes}\ \lambda \textbf{w}_\textbf{2}[\textbf{w}_\textbf{2}\ \textbf{he}_\textbf{1}\ {}^{\textbf{c}'}\textbf{d}'])^{\textbf{s},\textbf{c}'\to \textbf{x}} =$ $([\textbf{every student-w}_\textbf{0}]\lambda \textbf{x}_\textbf{1}\ \textbf{w}_\textbf{0}\ \textbf{x}_\textbf{1}\ \textbf{believes}\ \lambda \textbf{w}_\textbf{2}[\textbf{w}_\textbf{2}\ \textbf{he}_\textbf{1}\ \textbf{d}'])^{\textbf{s},\textbf{c}'\to \textbf{x}}$

We will now show that the value $x$ of the local context is the function from assignment functions to objects of type $\langle s, \langle e, t\rangle\rangle$ defined by (94):

(94) For every assignment function $s'$, $x(s') = \lambda w_s \lambda d_c 1$ iff for some $s$ in $C$, $s' \approx_{x_1, w_2} s$ and $w = s'(w_2)$ and $d = s'(x_1)$ and $d$ is a student in $s'(w_0)(= s(w_0))$ and $w \in Dox_d(s'(w_0))$, where $s' \approx_{x_1, w_2} s$ indicates that $s'$ is identical to $s$ except possibly for the values it assigns to $x_1$ and $w_2$.

The argument is in two steps.

First, we check that the condition in (93) is satisfied by $x$ as defined in (94). By construction, $x(s')$ denotes a property that holds true of $s'(x_1)$ for any assignment function $s'$ that could matter for the evaluation of the truth conditions of the leftmost formula in (93), hence the result.

Second, we check that any $x'$ that satisfies the condition in (93) is entailed by $x$. In other words, if for some (appropriate) $s'$, $w$ and $d$, $x(s')(w)(d) = 1$ but $x'(s')(w)(d) = 0$, $x'$ then fails to satisfy the condition in (93). So let us suppose that $x'$ satisfies (95), and let us show that $x'$ fails to satisfy (93).

(95) For some assignment function $s'$, for some world $w$, for some individual $d$, for some $s$ in $C$, $s' \approx_{x_1, w_2} s$ and $w = s'(w_2)$ and $d = s'(x_1)$ and $d$ is a student in $s'(w_0)(= s(w_0))$ and $w \in Dox_d(s'(w_0))$ and $x'(s')(w)(d) = 0$.

We take $d'$ to be a tautologous property. This immediately ensures that the right-hand side of (93) is satisfied. Still, the left-hand side isn't satisfied: $s'$ suffices to refute it, because it is an $x_1, w_2$-variant of $s$ which assigns to $x_1$ an individual $d$ who is a student in $s'(w_0)$ and to $w_2$ a world $w$ which is compatible with $d$'s beliefs in $s'(w_0)$, and yet $d$ fails to have property $({}^{c'}\mathbf{d}')^{s',c'\to x'}$ in $w$ because $\mathbf{c}'^{s',c'\to x'} = x'(s')$ is false of $d$ in $w$.

To complete the discussion of our example, we check that we obtain the desired result when we add the requirement that $x$ should entail the presupposition of the embedded predicate in (91). If we abbreviate *has started to make progress* as $\underline{p}p'$, with a presupposition $p$ (= *didn't make progress before*), the requirement is thus that $x$ should entail $\mathbf{p}$; given our point-wise definition of entailment, this means that:

(96) for every assignment function $s'$, $x(s') \leq \mathbf{p}(s')$

$x(s')$ is itself a function of type $\langle s, \langle e, t\rangle\rangle$, whose value is given in (94); as for $\mathbf{p}(s')$, it is a constant function of type $\langle s, \langle e, t\rangle\rangle$. So the condition that must be fulfilled is that:

(97) for every assignment function $s'$, for every world $w$, for every individual $d$, $x(s')(w)(d) \leq \mathbf{p}(s')(w)(d)$

The condition is thus that whenever $x(s')(w)(d) = 1$, $\mathbf{p}(s')(w)(d) = 1$. It is satisfied just in case:

(98) for every $s$ in $C$, for every $d$ that is a student in $s(w_0)$, for every world $w \in Dox_d(s(w_0))$, $\mathbf{p}(s)(w)(d) = 1$

(Since $\mathbf{p}$ is a constant function over the domain of assignment functions, this can be paraphrased more intuitively as: if $d$ is a student in a world $w'$ compatible with what the speech act participants take for granted, if $w$ is a world compatible with what $d$ believes in $w'$, then $d$ satisfies $p$ in $w$.)

It is clear that (98) entails (97), because $x'(s')(w)(d)$ can only take the value 1 if for some $s \in C$, $s' \approx_{x_1,w_2} s$ and $d$ is a student in $s(w_0)$ and $w \in Dox_d(s(w_0))$. Conversely, (97) entails (98): for every $s \in C$, for every $d$ that is a student in $s(w_0)$ and for every world $w \in Dox_d(s(w_0))$, consider $s' = s[x_1 \to d][w_2 \to w]$. It is clear that $x(s')(w)(d) = 1$, hence $\mathbf{p}(s')(w)(d) = 1$ and thus $\mathbf{p}(s)(w)(d) = 1$ — which proves (98).

## E Universal Inferences from Non-Conservative Quantifiers

In our discussion of *every* and *no*, the argument crucially relied on the property of conservativity. What about non-conservative operators, such as *only*? We predict that they will exhibit a very different pattern of projection. Consider the sentence in (99) (with focus on *bad*):

(99) In my class, only $\text{bad}_F$ students have stopped complaining.

My impression is that (99) gives rise to an inference that *all* students in my class used to complain (some speakers believe that a slightly weaker inference is obtained). Let us see how this result can be derived. For simplicity, we abstract away from the presupposition triggered by *only* (treating it as part of the assertion), and we posit the truth conditions in (100), which are motivated by the non-presuppositional example in (101).

(100) (*only* $\textit{bad}_F$ *students . Q*) is true at a world $w$ just in case in w: (a) *at least one bad student satisfies Q,* and (b) *every student that satisfies Q is a bad student.*

(101) a. Only $\text{bad}_F$ students skipped class yesterday.
b. (a) is true if and only if at least one bad student skipped class yesterday, and all students who skipped class yesterday are bad students.

We make the Assumption in (102) and derive the result in (103).

(102) *Assumption*: throughout C, the extension of *bad student* is non-empty.

(103) The local context of $Q$ in (*only* $\textit{bad}_F$ *students . Q*) is ${}^{C}\textbf{student}$, the property of being a student restricted to the context set $C$ (thus ${}^{C}\textbf{student} = \lambda w \lambda x (w \in C \text{ and } x \text{ is a student in } w)$).

It follows from (103) that (99) triggers a presupposition that every student (in my class) used to complain, since the property ${}^{C}\textbf{student}$ should entail the presupposition **complain.** We prove (103) in two steps. First, it is clear that if $c'$ denotes ${}^{C}\mathbf{student}$, the desired equivalence will hold: for any $d'$ of predicative type,

(104) $C \vDash^{c' \to {}^{C}\mathbf{student}} (\textit{only bad}_F \textit{ students . } {}^{c'}d') \Longleftrightarrow (\textit{only bad}_F \textit{ students . } d')$

Second, let us assume that $c'$ denotes a property $x$ and that for some world $w$ of $C$ and for some individual $i$, $\mathbf{student}(w)(i) = 1$ but $x(w)(i) = 0$. We wish to show that for some $d'$, $C \nvDash^{c' \to x} (\textit{only bad}_F \textit{ students . } {}^{c'}d') \Longleftrightarrow (\textit{only bad}_F \textit{ students . } d')$; this will prove that if the denotation x of $c'$ is not entailed by ${}^{C}\mathbf{student}$, $c'$ is not a transparent restriction — in other words, ${}^{C}\mathbf{student}$ entails every transparent restriction for $d'$.

We argue by cases: first, we consider the case in which $i$ is bad student (*Case 1*); second, we consider the case in which $i$ is a student who is not a bad student (*Case 2*). In each case, we find a value of $d'$ for which $C \nvDash^{c' \to x} (\textit{only bad}_F \textit{ students . } {}^{c'}d') \Longleftrightarrow (\textit{only bad}_F \textit{ students . } d')$.

**Case 1** $i$ is a bad student. We take $\mathbf{d}(w) = \{i\}$. It is clear that $w \vDash^{c' \to x} (\textit{only bad}_F \textit{ students . } d')$, because $i$ is a bad student that satisfies $d'$, and every student that satisfies $d'$ is identical to $i$, and is thus a bad student. However, $w \nvDash^{c' \to x} (\textit{only bad}_F \textit{ students . } {}^{c'}d')$, because the extension of ${}^{c'}d'$ is empty. So $w \nvDash^{c' \to x} (\textit{only bad}_F \textit{ students . } {}^{c'}d') \Longleftrightarrow (\textit{only bad}_F \textit{ students . } d')$.

**Case 2** $i$ is a student who is not a bad student. By the Assumption in (102), there exists at least one individual who is a bad student.

**Case 2a** $x(w)$ includes at least one individual $i'$ who is a bad student. We take $\mathbf{d}'(w) = \{i, i'\}$ — and thus ${}^{c'}\mathbf{d'}(w) = \{i'\}$ (since by assumption $x(w)(i) = 0$). Clearly, $w \nvDash^{c' \to x} (\textit{only bad}_F \textit{ students . } d')$, because $i$ satisfies $d'$ but $i$ is not a bad student. Still, $w \vDash^{c' \to x} (\textit{only bad}_F \textit{ students . } {}^{c'}d')$: $i'$ is a bad student, and satisfies ${}^{c'}d'$; and since ${}^{c'}\mathbf{d'}(w) = \{i'\}$, every element that satisfies ${}^{c'}d'$ is a bad student. So $w \nvDash^{c' \to x} (\textit{only bad}_F \textit{ students . } {}^{c'}d') \Longleftrightarrow (\textit{only bad}_F \textit{ students . } d')$.

**Case 2b** $x(w)$ includes no individual who is a bad student. We take $d'$ = *bad student*. Clearly, $w \vDash^{c' \to x} (\textit{only bad}_F \textit{ students . } d')$, since by the Assumption in (102) there exists at least one individual who is a bad student. Still, $w \nvDash^{c' \to x} (\textit{only bad}_F \textit{ students . } {}^{c'}d')$ because by assumption $x(w)$ includes no individual who is a bad student. Thus in this case too $w \nvDash^{c' \to x} (\textit{only bad}_F \textit{ students . } {}^{c'}d') \Longleftrightarrow (\textit{only bad}_F \textit{ students . } d')$.

We make similar predictions for the sentence *Only students$_F$ have stopped complaining*, with the additional complexity that the alternatives to students must be made clear, and the domain restriction must be made explicit.

(105) In my Department, only students$_F$ have stopped complaining.

If the Department is composed of students and professors, we predict a presupposition that all students and all professors used to complain. We leave a more precise assessment of this prediction for future research.

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