Abstract  Grammatical theories of Scalar Implicatures make use of an exhaustivity operator \texttt{exh}, which asserts the conjunction of the prejacent with the negation of excludable alternatives. We present a new Grammatical theory of Scalar Implicatures according to which \texttt{exh} is replaced with \texttt{pex}, an operator that contributes its prejacent as asserted content, but the negation of scalar alternatives at a \textit{non-at-issue} level of meaning. We show that by treating this non-at-issue level as a \textit{presupposition}, this theory resolves a number of empirical challenges faced by the old formulation of \texttt{exh} (as well as by standard neo-Gricean theories). The empirical challenges include projection of scalar implicatures from certain embedded environments (‘\textit{some under some}’ sentences, \textit{some} under negative factives), their restricted distribution under negation, and the existence of common ground-mismatching and oddness-inducing implicatures. We argue that these puzzles have a uniform solution given a \texttt{pex}-based Grammatical theory of implicatures and some independently motivated principles concerning presupposition projection, cancellation and accommodation.

* For useful suggestions and discussion we thank Danny Fox, Paul Marty, Jacopo Romoli, Yimei Xiang, Brian Buccola, and the participants of the XPrag.de workshop on Exhaustivity in Questions at Universität Tübingen, June 13-14, 2019, and of the ZAS internal Semantics-Pragmatics workshop in June 2019. We benefited greatly from comments by Judith Degen and five reviewers for \textit{Semantics and Pragmatics}. We also thank David-Etienne Bouchard and Renata Shamsutdinova for their excellent technical help. This project has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (grant agreement No 856421). Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) SPP 1727 XPrag.de, Project SA 925/11-2. All errors are our own.

©2021 Itai Bassi and Guillermo Del Pinal and Uli Sauerland
This is an open-access article distributed under the terms of a Creative Commons Attribution License (https://creativecommons.org/licenses/by/3.0/).
Key Words: scalar implicatures, exhaustification, presuppositions, oddness

1 Presuppositional exhaustification: Basic assumptions and implementation

According to grammatical accounts of Scalar Implicatures (SIs), SIs are triggered by an exhaustification operator, \textbf{exh}, which asserts both its prejacent and the negation of each of its excludable alternatives (Chierchia, Fox & Spector 2012, a.o.). This view is captured in (1) (some of the specifics of (1) are based on the account in Magri 2009 and anticipate the discussion to follow).

(1) For a structure \(\phi\) of propositional type and local context \(c\):

a. \(\llbracket \textbf{exh}(\phi) \rrbracket = \llbracket \phi \rrbracket \land \bigwedge \neg \llbracket \psi \rrbracket : \psi \in \text{Excl}(\phi) \land \llbracket \psi \rrbracket \in R\)

b. \(\text{Excl}(\phi)\) is a subset of the set of formal alternatives of \(\phi\), such that, for each \(\psi \in \text{Excl}(\phi)\), \(\llbracket \psi \rrbracket\) isn’t logically entailed by \(\llbracket \phi \rrbracket\) (or equivalently, such that \(\llbracket \phi \rrbracket\) is logically consistent with \(\neg \llbracket \psi \rrbracket\)).

c. \(R = \) a contextually assigned ‘relevance’ predicate which minimally satisfies the following two conditions: (i) the prejacent, \(\phi\), is relevant, i.e. \(\llbracket \phi \rrbracket \in R\), and (ii) any proposition that is contextually equivalent to the prejacent is also in \(R\) (i.e., if \(\llbracket \phi \rrbracket \cap c \equiv \llbracket \psi \rrbracket \cap c\), then \(\llbracket \psi \rrbracket \in R\)).

The entry in (1a) implies that the prejacent of \textbf{exh} and the generated SIs are both communicated at the at-issue (assertive) level of meaning. In this paper, we present evidence that SIs are generally not introduced at the at-issue level of meaning. Specifically, while we agree with grammatical accounts that SIs are the result of an operator in the Logical Form of sentences, we will argue that SIs behave in a manner akin to non-at-issue, presupposed content. For concreteness, we propose that \textbf{exh} be replaced with \textbf{pex} in (2), an exhaustification operator which asserts the prejacent but presupposes the SIs.\footnote{A different line of research has established that \textbf{exh} also applies to the presuppositional component of meaning to yield a class of presuppositions often called \textit{implicated presuppo-}

\footnote{Following standard practice, we will throughout the paper represent a context \(c\) as a set of worlds (‘context set’) compatible with all the propositions mutually believed by the participants in \(c\). We also sometimes use the term ‘common ground’ to informally describe the context set.}

\footnote{Conditions (i) and (ii) are necessary for the range of data we are concerned with in this paper. Generally (i) and (ii) are not seen as sufficient (Chierchia, Fox & Spector 2012, Fox & Katzir 2011, Roberts 2012, Trinh & Haida 2015). This is compatible with our conclusions in this paper, but the weaker condition stated in the text is sufficient for our purposes.}

\footnote{1 Following standard practice, we will throughout the paper represent a context \(c\) as a set of worlds (‘context set’) compatible with all the propositions mutually believed by the participants in \(c\). We also sometimes use the term ‘common ground’ to informally describe the context set.}

\footnote{Generally (i) and (ii) are not seen as sufficient (Chierchia, Fox & Spector 2012, Fox & Katzir 2011, Roberts 2012, Trinh & Haida 2015). This is compatible with our conclusions in this paper, but the weaker condition stated in the text is sufficient for our purposes.}
Presuppositional exhaustification

(2) For a structure $\phi$ of propositional type and a local context $c$:

a. $\llbracket \text{pex}(\phi) \rrbracket = \{ \text{presupposition}: \wedge \neg \llbracket \psi \rrbracket : \psi \in Excl(\phi) \wedge \llbracket \psi \rrbracket \in R \}
   \text{assertion:} \llbracket \phi \rrbracket$

b. $Excl(\phi)$ is defined as in (1b).

c. $R$ is defined as in (1c).

In terms of its effect on different ingredients of meaning, pex is the mirror image of the overt exhaustifier only, on standard accounts of the latter (Horn 1969): pex presupposes what only asserts and vice versa. We also assume, following Magri (2009), that pex is obligatorily adjoined to any structure of propositional type at least once. On this kind of implementation, the optionality of specific SIs, in certain contexts, is modelled as a by-product of the relevance parameter $R$. In particular, strictly excludable alternatives may sometimes not be included in the exhaustified meaning of an expression because they are not in $R$.

As a simple illustration, a sentence like some students passed will be parsed as in (3a). If its only (excludable) alternative is (3b), and that alternative is also relevant in the context of utterance (i.e., if $\llbracket (3b) \rrbracket \in R$), the output of (3a) would be (3c).

(3) Some students passed.

a. $\text{LF: pex}[\text{Some students passed}]$

b. $Excl(\text{some students passed}) = \text{All students passed}$

c. $\llbracket (3a) \rrbracket = \{ \text{presupposition:} \neg \text{all students passed} \}
   \text{assertion:} \text{some students passed}$

This proposal raises an immediate worry: SIs do not feel like presuppositions qua constraints on the common ground. In our example, some students passed (Heim 1991, Sauerland 2002, Percus 2006, Sauerland 2008, Marty 2017). As far as we can see, implicated presupposition are orthogonal to our present proposal to presuppose the SI itself.

4 We briefly return to the relationship between only and pex in the summary section. It should be noted that our proposal for SIs bears some resemblance to certain proposals for the meaning of cleft sentences (e.g. Velleman et al. 2012, Büring & Križ 2013), on which the exhaustivity inference of clefts is introduced at the level of presuppositions while the assertive level conveys just the meaning of the prejacent.

5 Meyer 2015 derives the distribution of exh from more general principles. As far as we can see, her approach would also work for our data, but for ease of presentation we adopt Magri’s stipulation.

6 We will use the terms ‘assertive’ and ‘at-issue’ content interchangeably.
*passed* can be felicitously asserted when the common ground doesn’t entail the ‘not all’ implicature. Why, then, do we propose that SIs are generally introduced as presuppositional rather than assertive content?

Despite that natural worry, we will argue that there are good reasons to take seriously the hypothesis that SIs are part of non-at-issue, presuppositional content. Specifically, we will examine cases in which SIs triggered in embedded environments behave as presuppositions. In Section 2, we analyse embedded SIs triggered in ‘*some* under *some*’ sentences, and argue that they project as if they were presuppositions. In Sections 3–4, we analyze SIs in certain downward entailing environments, and show that they behave like presuppositions both with respect to projection and local accommodation. Finally, in Section 5, we argue that certain oddness-inducing SIs studied in Magri’s (2009) work are best understood as cases in which there is an inconsistency between the presuppositional content and the common ground. Importantly, each of the cases we examine presents serious difficulties for standard *exh*-based theories — according to which both the prejacent and its SIs are part of assertive content — as well as for various more traditional neo-Grician theories of SIs. Our goal is to show that the difficulties are resolved once the Grammatical theory is reformulated in terms of *pex*.7

To address the initial worry raised for simple cases of SIs, we argue, in Section 6, that *pex* belongs to a class of triggers — which arguably includes similar overt exhaustifiers such as *only* and *even* — whose presuppositions are automatically globally accommodated whenever they are consistent with the common ground. Global accommodation is the process by which, upon hearing an assertion with presupposition *p*, interlocutors automatically adjusts their beliefs about the common ground so as to entail *p*. In a simple case like (3), our hypothesis is that the ‘not all’ entailment is automatically accommodated whenever it is both relevant and compatible with the common ground.8 Thus, the ‘not all’ implicature need not be strictly entailed by the the common ground prior to a felicitous use of (3a). In addition, in Sec-

---

7 To be clear, we do not think that these are the only cases that motivate a *pex*-based Grammatical theory. For example, in Del Pinal, Bassi & Sauerland 2021, we argue that certain puzzles related to free choice inferences, which present a challenge for standard *exh*, can be nicely resolved by a *pex*-based theory of SIs.

8 In Section 5 we explain why global accommodation is usually not possible when a presupposition is in compatible with the common ground.
Presuppositional exhaustification

In section 6 we will discuss the options for modelling the SIs triggered by \textit{pex} as belonging to a different class of non-at-issue content.\footnote{It is increasingly appreciated that there are different notions, and probably different types, of non-at-issue contents, and that some elements that pass certain diagnostics for presupposition-hood fail on others (Simons et al. 2010, Tonhauser et al. 2013). If our proposal is on the right track, then SIs belongs to a class of non-at-issue content that projects like standard presuppositions, yet behaves with respect to global accommodation like more general kinds of non-at-issue content. See Section 6 for discussion.}

It is important to clarify our argumentative strategy from the outset. There are various modifications and refinements of standard \textit{exh}-based Grammatical theories that, to varying degrees of generality and success, try to deal with the empirical puzzles we present. Our goal is to show that a \textit{pex}-based theory provides the most principled and uniform resolution of those puzzles. In each case, we base our account on how the SIs triggered by \textit{pex} interact with independently motivated principles and constraints on guiding presupposition projection and both local and global accommodation.

2 Some under some

In this section we analyze a pattern of inference that arises from sentences in which a weak scalar item is embedded in the scope of another (hence the name ‘\textit{some} under \textit{some}’). The pattern poses a problem for existing theories of SIs, and to our knowledge has never been satisfactorily addressed. We will show that \textit{pex}, coupled with a certain theory of presupposition projection (‘Strong Kleene’), successfully derives the data, thus providing an argument in favor of \textit{pex}.

2.1 Data

Consider (4), where a weak scalar item (\textit{some}) is embedded under another, and two potential implicatures it gives rise to, labeled ‘Local' and ‘Strong Global’ SIs (terminology adapted from Gotzner & Benz 2018):

(4) Some of the girls found some of their marbles.
   a. \underline{Local SI}: Some of the girls found \textbf{some but not all of their marbles}.
   b. \underline{Strong Global SI}: None of the girls found \textbf{all of their marbles}.

\textbf{Gotzner & Benz (2018)} report experimental evidence that suggests that subjects readily derive the strong global implicature for ‘\textit{some} under \textit{some}’ sen-
tences like (4). Note that the strong global SI, taken together with the prejacent, entails the local SI (i.e., the conjunction of (4) and (4b) entails (4a)). This pattern of inference is predicted by virtually all existing theories of SIs, most straightforwardly by any theory that allows (4) to trigger the alternative *some of the girls found all of her marbles.*

To this we add the observation, following Chierchia 2004, that a ‘*some* under *some*’ sentence can also trigger the local SI side by side with what we call a ‘Weak Global’ SI, which is of the general form ‘*not all... some...*’, as shown in (5).¹⁰

(5) Some of the girls found some of their marbles. And some found all of them.

a. **Local SI:** *Some of the girls found some but not all of their marbles.*

b. **Weak Global SI:** *Not all of the girls found some of their marbles.*

¹¹≡ *Some of the girls didn’t find any of their marbles.*

The Weak Global SI together with the prejacent (the first sentence of (5)) does not entail the Local SI. Note importantly that the second sentence of (5) is not compatible with computing the Strong Global SI (4b) for the first sentence in (5), yet both the inferences in (5a) and (5b) still arise; therefore the pattern in (5a) and (5b) does not depend for its existence on deriving the Strong Global SI.

This means that for a ‘*some* under *some*’ sentence, a theory of SIs needs a way to derive both the local and the weak global inferences, *without* necessarily computing the strong global SI. This is summarized in (6). ‘∃¬∀𝑦φ’ abbreviates ∃𝑦φ ∧ ¬∀𝑦φ.

(6) **Desideratum:**

Given a sentence of the form *Some*₁[... *some*₂... *P*(x, y)], derive just the following implicatures:

10 Chierchia’s example is with embedded disjunction under *some*, which has the same logical profile:

(i) Someone smokes or drinks.  

a. **Local SI:** Someone smokes or drinks but not both.

b. **Weak-Global SI:** Not everyone smokes or drinks

11 The terms ‘Strong’ and ‘Weak’ might be misleading in this context, since (4b) as a whole is not logically stronger than (5b) as a whole. Rather this terminology is meant to reflect just the logical relationship between the quantifiers ‘none’ and ‘not all’.
Presuppositional exhaustification

a. \( \exists x[\ldots \exists \neg \forall y[\ldots P(x, y)]] \) (Local SI)
b. \( \neg \forall x[\ldots \exists y[\ldots P(x, y)]] \quad (\equiv \exists x[\ldots \neg \exists y[\ldots P(x, y)]] \) (Weak-Global SI)

As it turns out, (6) presents a difficulty for all existing theories of SIs known to us, Grammatical and Neo-Gricean alike. Below we focus on the predictions of the grammatical \textsf{exh} theory and show that they are inadequate. Before that, let us illustrate why a simple-minded (neo-)Gricean theory does not derive this pattern.

On Gricean accounts, implicatures are derived only globally, by reasoning over (and eventually negating) alternatives to the whole utterance; it is not possible to compute implicatures locally by targeting sub-constituents. Such accounts could derive the local SI indirectly by computing the strong-global SI, as mentioned earlier in the context of (4). But for the purpose of (5) this isn’t enough, because deriving the global SI should be avoided, as we explained. It is then not clear how to derive the local SI here (the Weak-Global SI is easily derivable using the alternative \textit{all of the girls found some of her marbles}).

2.2 An \textsf{exh} paradox

In a standard \textsf{exh}-based theory, it is possible to derive just the Local SI, and it is possible to derive just the Weak-Global SI. But deriving both is, as far as we can see, not possible without ad-hoc stipulations. The structure in (7) at first glance looks like the correct LF representation to derive (6):

\[
(7) \quad \textsf{exh}_1[\text{some of the girls } \lambda x \textsf{exh}_2[\text{some of the marbles } \lambda y [x \text{ found } y]]]
\]

Structure (7) seems promising because each \textsf{exh} operator is meant to deliver one of the two inferences we’re after: \textsf{exh}_2 is supposed to deliver the Local SI, by targeting the embedded ‘some’, and \textsf{exh}_1 is supposed to deliver the Weak Global SI, by targeting the matrix ‘some’. However, the interpretation (7) actually yields, which is given in (8), is too weak, in particular the part in (8b).

\[
(8) \quad \text{Inferences predicted from the parse in (7)}:
\]

a. \textit{Some of the girls found some but not all of their marbles.} (prejacent of \textsf{exh}_1)
b. Not all of the girls found some but not all of their marbles.

(output of exh₁)

While (8a) is equivalent to the desired local SI, (8b) is weaker than the desired Weak Global SI: it only guarantees that some girls found either none or all of their marbles, but what is needed is to guarantee that some girls found none of the marbles.

To get the desired Weak Global SI we would want to replace ‘some but not all’ in (8b) with a plain ‘some’. The culprit for that some but not all is of course the presence of exh₂ downstream; but we cannot just choose to remove exh₂ from the structure, because its presence is required to generate the local SI in (8a).

One might suggest that exh can optionally be deleted/disappear from alternatives. Then the (relevant) alternative to the higher exh could have the embedded exh deleted from it, as shown in (9b):

(9)  a. Prejacent:
    some girls λₓ exh₂ [some of their marbles λᵧ [x found y]]

b. Alternative:
    all girls λₓ exh₂ [some of their marbles λᵧ [x found y]]

That would work, but it is unclear why exh₂ could disappear from alternatives. Meyer (2013) independently suggested that exh cannot be deleted for the purposes of alternative generation. Moreover, the overt exhaustifier only, whose semantics is closely related to exh, does not have this option either; (10) does not allow the relevant inference, which would be possible if only could be deleted in an analogous way to (9b). (10) in fact strongly conveys that all girls found some of their marbles.

(10) Some girls only found SOME of their marbles.

a.  ↝ Not all girls found some-but-not-all of their marbles.

b.  ↣ Not all girls found some of their marbles.

Chierchia (2004) seemed to have suggested a remedy along similar lines to (9), though couched in a different framework for computing SIs. In his system, SIs are produced not with the help of a syntactic exh operator but by the hard-coded semantic compositional rules. Chierchia’s (2004: p. 62) suggestion was that SIs are “removed in DE(=Downward Entailing) contexts”. While this is the guiding intuition behind our account too, Chierchia’s proposal doesn’t contain enough detail to cash it out.
Presuppositional exhaustification

Still, one might hold on to the intuition that \textbf{exh} is active in the prejacent but not in the alternative. One way of trying to justify the deletion of \textbf{exh} in (9b) is to hold that it follows as a consequence of applying a principle of Economy to the step-wise computation of (7).\textsuperscript{13} Fox & Spector 2018 proposed an Economy condition which in effect bans the insertion of \textbf{exh} in DE environments (in normal circumstances). With Economy, the basic idea would be that the SI associated with matrix \textbf{exh} in (7) is computed first, delivering (11a). We then compute the SIs associated with each of the embedded tokens of \textbf{exh}, but only if each is independently licensed by Economy. Economy licenses the computation of an SI for the first conjunct of (11a), since \textbf{exh} occurs in an UE-environment, but not for the second one, since there \textbf{exh} occurs in a DE-environment. We can represent this result as in (11b), which supports the target reading in (11c).

(11) a. \[\text{[some of the girls } \lambda x \text{ exh}2 \text{ [some of the marbles } \lambda y [x \text{ found } y]]} \]
   \[\land \neg \text{[all of the girls } \lambda x \text{ exh}2 \text{ [some of the marbles } \lambda y [x \text{ found } y]]} \]

b. \[\text{[some of the girls } \lambda x \text{ exh}2 \text{ [some of the marbles } \lambda y [x \text{ found } y]]} \]
   \[\land \neg \text{[all of the girls } \lambda x \text{ exh}2 \text{ [some of the marbles } \lambda y [x \text{ found } y]]} \]

c. \[\text{[some of the girls found some-but-not-all of their marbles]} \]
   \[\land \neg \text{[all of the girls found some of their marbles]} \]

Economy, however, is a principle which guides the selection of parses for expressions; it is not a principle that allows one to selectively ignore the contribution of the syntactic pieces of one and the same parse at different levels of meaning produced by that parse. Therefore, we do not find this sketch of explanation based on Economy a real theoretical option. Within a grammatical theory of SIs, in which \textbf{exh} is optional, Economy could only affect the selection of whether to parse an expression like (12) from options like (12a) or (12b) (a.o., not relevant for deriving our target reading). Once (12a) is selected, \textbf{exh} is a syntactic item, which, not being a focused item for \textbf{exh}, should be uniformly present across alternatives, just as any other overt or covert non-focused items.

(12) Some of the girls found some of their marbles.

\textsuperscript{13} This line of reasoning was employed by Elliott & Marty 2019 to account for related data.
This then is the conundrum for the standard \textit{exh}-based theories in trying to account for (6): the embedded \textit{exh} is required for one task (local SI), but its contribution interferes with achieving the other task (Weak Global SI).

2.3 A \textit{pex} solution

We now show that \textit{pex} derives our desideratum. The key point is that, given independently-proposed mechanisms about how presupposition projection works, a separation between the presuppositional and at-issue content of \textit{pex} allows the presence of an embedded \textit{pex} to not necessarily affect the contribution of material higher up in the structure.

Let us look at the relevant LF in (13), where \textit{pex} replaces \textit{exh}:

\begin{equation}
\begin{array}{l}
\text{pex}_1 [\text{some girls } \lambda_x \text{ pex}_2 [\text{some of the marbles } \lambda_y [x \text{ found } y]]] \\
\end{array}
\end{equation}

To see what is predicted for (13), we need to know how quantifiers handle presuppositions in their scope, because (13) features a presupposition trigger — \textit{pex}_2 — in the scope of the matrix quantifier (and it also features the same \textit{pex}_2 at the level of the alternative that \textit{pex}_1 negates; see below). Our analysis will henceforth rest on a specific — yet independently argued for — framework for presupposition projection: the Strong Kleene trivalent logic (George 2008, Fox 2013, Winter 2019 a.o.). We show now that given this logic for presuppositions, the embedded \textit{pex} will be semantically ‘transparent’ when it comes to evaluating the contribution of the higher \textit{pex}, avoiding the conundrum faced by \textit{exh}-based theories.

In Strong Kleene, the method for computing the presuppositions of a complex expression $\phi$ consists of first specifying its bivalent truth conditions (Truth conditions and Falsity conditions). These are the conditions under which, roughly speaking, $\phi$ is guaranteed to have a bivalent truth value, even if some of its sub-constituents do not have a bivalent truth value. The conditions under which it is not possible to determine such a bivalent value for $\phi$ are the conditions under which $\phi$ is assigned the third truth value, #. The predicted presupposition of $\phi$ is the disjunction of its truth and falsity conditions (i.e., the conditions under which it is not #). See George 2008 and
Presuppositional exhaustification

Fox 2013 for a more thorough discussion and technical implementation; this general description is sufficient for our purposes.

We also need to reformulate our lexical entry for \textit{pex} from (2a) to respect the switch to a trivalent framework. The most natural way of doing so in line with our core proposal is given in (14):

\begin{align}
[\textsf{pex}(\phi)] = \begin{cases} 
1, & \text{if } [\phi] = 1 \land \bigwedge(\lbrack \psi \rbrack = 0) : \psi \in \text{Excl}(\phi) \land \lbrack \psi \rbrack \in R \\
0, & \text{if } [\phi] = 0 \\
#, & \text{otherwise}
\end{cases}
\end{align}

This entry says that the conditions under which \textsf{pex}(\phi) is true are the expected ones (the same as \textsf{exh}), but the conditions under which \textsf{pex}(\phi) is false are identical to the conditions under which \phi is false. As long as we are restricting our attention to alternatives of \phi that are strictly stronger than \phi, which is what we do throughout the paper, the predicted presupposition here (i.e., the disjunction of truth and falsity) derives our core proposal, namely the presupposition is that all the exclusable relevant alternatives are false.\footnote{In the more general case, i.e. when merely non-weaker alternatives are also considered as exclusable (and relevant), then the predicted presupposition is merely disjunctive: either the prejacent is false, or all exclusable (relevant) alternatives are false. Equivalently: if the prejacent is true, all exclusable (relevant) alternatives are false.}

\footnote{One could imagine an alternative entry where the exclusion of alternatives is duplicated in the falsity conditions (the addition is underlined):}

\begin{align}
[i] \quad [\textsf{pex}(\phi)] = \begin{cases} 
1, & \text{if } [\phi] = 1 \land \bigwedge(\lbrack \psi \rbrack = 0) : \psi \in \text{Excl}(\phi) \land \lbrack \psi \rbrack \in R \\
0, & \text{if } [\phi] = 0 \land \bigwedge(\lbrack \psi \rbrack = 0) : \psi \in \text{Excl}(\phi) \land \lbrack \psi \rbrack \in R \\
#, & \text{otherwise}
\end{cases}
\end{align}

As a simple illustration:

(15) She found some of her marbles.

\begin{itemize}
\item[(a)] \quad [\textsf{pex}(15)] = \begin{cases} 
1, & \text{if she found some but not all her marbles} \\
0, & \text{if she found none of her marbles} \\
#, & \text{otherwise}
\end{cases}
\item[(b)] \quad \text{Predicted presupposition:} \\
\quad \text{Either she found none of her marbles, or some but not all her}
\end{itemize}

\footnote{This would produce the same presupposition as (14) if only strictly stronger alternatives are considered, and a non-disjunctive presupposition in the more general case (cf. footnote 14). As the discussion below (esp. surrounding (22)) will hopefully make clear, however, the entry in (14) is a crucial ingredient in deriving our desideratum for \textit{some under some}; (i) would not give us the right result. The contribution of \textsf{pex} to the falsity conditions of its prejacent needs to be vacuous.}
marbles.
≡ She didn’t find all her marbles.

Armed with these assumptions, we now go back to our LF in (13). The task is to specify its truth and falsity conditions. According to (14), this LF is true if the prejacent of the matrix exhaustifier \( \text{pex}_1 \) is true and the alternative to it is false; false if the prejacent is false; and \# otherwise. For convenience, the prejacent and alternative for \( \text{pex}_1 \) are shown in (16), so the interpretation of the whole LF is succinctly represented in (17).

(16) The prejacent and (relevant) alternative of \( \text{pex}_1 \) in (13):
   a. \textit{prejacent}:
      \[
      \text{some girls} \lambda_x \text{pex}_2 [\text{some of the marbles} \lambda_y [x \text{ found } y]]
      \]
   b. \textit{alternative}:
      \[
      \text{all girls} \lambda_x \text{pex}_2 [\text{some of the marbles} \lambda_y [x \text{ found } y]]
      \]

   (17) \[
   \boxed{\mathbf{13}} = \begin{cases} 
   1, & \text{if } \boxed{16a} = 1 \land \boxed{16b} = 0 \\
   0, & \text{if } \boxed{16a} = 0 \\
   \#, & \text{otherwise}
   \end{cases}
   
   To unpack (17), we need to know the truth and falsity conditions of (16a), and the falsity conditions of (16b). Starting with (16a), this is an existential statement; Strong Kleene’s logic produces the following 3-valued semantics for such statements:

(18) \textbf{Strong Kleene semantics for existential quantifiers}. An expression of the form \([\text{Some } \xi_x : \psi_x] \) (\( \xi \) is the restrictor and \( \psi \) the scope of the quantifier) is \textbf{True} if (i) holds, \textbf{False} if (ii) holds, and \# if neither (i) nor (ii) hold.

   \[
   \begin{align*}
   & (i) \ \exists x \in [\xi] : [\psi](x) = 1 & \text{(Truth conditions)} \\
   & (ii) \ \forall x \in [\xi] : [\psi](x) = 0 & \text{(Falsity conditions)}
   \end{align*}
   
   In prose, in order to guarantee that \([\text{Some } \xi_x : \psi_x] \) is true, one has to find merely one individual in the domain of quantification which satisfies the scope; if one is found, it is irrelevant whether the other individuals satisfy the presuppositions of the scope or not. But in order to guarantee that \([\text{Some } \xi_x : \psi_x] \) is false, one has to consider all individuals in the domain and show that they falsify the scope (thereby satisfying the presuppositions).

   Applying (18) to (16a), we get that for (16a) to be True, it is necessary that some girls find some-but-not-all of their marbles, and for it to be False, it is
Presuppositional exhaustification

necessary that all girls don’t find any marbles. \( \psi \) in our case is of the form ‘\( \text{pex}_2(\mathbf{x} \text{ found some of } \mathbf{x}’\text{’s marbles}) \)’, and its 3-valued semantics was given in (15a).

\[
(19) \quad \lfloor (16a) \rfloor = \begin{cases} 
1, & \text{if some girls found some-but-not-all marbles} \\
0, & \text{if all girls didn’t find any marbles} \\
\#, & \text{otherwise}
\end{cases}
\]

This allows us to update the meaning for the whole LF in (17) as follows:

\[
(20) \quad (17) = \lfloor (13) \rfloor = \begin{cases} 
1, & \text{if some girls found some-but-not-all marbles} \\
\land \lfloor (16b) \rfloor = 0 \\
0, & \text{if no girls found any marbles} \\
\#, & \text{otherwise}
\end{cases}
\]

Turning to (16b), this is a universal statement; Strong Kleene’s system produces the following 3-valued semantics for universal statements:

\[
(21) \quad \textbf{Strong Kleene semantics for Universal quantifiers.} \text{ An expression of the form } [\text{All } \xi_{\mathbf{x}} : \psi_{\mathbf{x}}] \text{ (} \xi \text{ is the restrictor and } \psi \text{ the scope of the quantifier) is True if (i) holds, False if (ii) holds, and } \# \text{ if neither (i) nor (ii) hold.}
\]

\[
(i) \quad \forall \mathbf{x} \in \lfloor \xi \rfloor : \lfloor \psi \rfloor(\mathbf{x}) = 1 \quad \text{(Truth conditions)}
\]

\[
(ii) \quad \exists \mathbf{x} \in \lfloor \xi \rfloor : \lfloor \psi \rfloor(\mathbf{x}) = 0 \quad \text{(Falsity conditions)}
\]

In order to guarantee that [All \( \xi_{\mathbf{x}} : \psi_{\mathbf{x}} \)] is false, one has to find merely one individual in the domain of quantification which falsifies the scope; if one is found then it is irrelevant whether the others satisfy the scope’s presuppositions or not. This is the mirror image of (18).

Applying (21) to (16b), we get that for (16b) to be False, it is necessary that some girls don’t find any of their marbles. This result obtains crucially due to the fact that \( \text{pex}_2 \) does not contribute anything to the Falsity condition of its prejacent. In other words, \( \text{pex}_2 \) is vacuous here because ‘\( \text{pex}_2(\mathbf{x} \text{ found some of } \mathbf{x}’\text{’s marbles}) \)’ (our \( \psi \)) is false just in case the existential statement ‘\( \mathbf{x} \text{ found some of } \mathbf{x}’\text{’s marbles} \)’ is false, namely iff \( \mathbf{x} \) found none of \( \mathbf{x}’ \text{’s marbles} \). It is this fact that is responsible for deriving the Weak-Global SI that was problematic for the \( \text{exh} \) theory (see below).\(^{16}\)

\(^{16}\) As the savvy reader may notice, the line of reasoning in the text also predicts that \( \text{pex} \) under negation will not contribute anything to the truth conditions of the negated sentence. We
We have now reached the final truth and falsity conditions for the whole LF, which are as follows:

\[
(22) \quad \llbracket (13) \rrbracket = \begin{cases} 
1, & \text{if some girls found some-but-not-all of their marbles} \\
& \quad \wedge \text{some girls didn't find any of their marbles} \\
0, & \text{if no girls found any of their marbles} \\
#, & \text{otherwise}
\end{cases}
\]

The description of the truth conditions in (22) are exactly the two inferences we were after: the first line of it is the local SI, and the second line is the Weak-Global SI. So if we just concentrate on the inferences coming from the truth conditions of the LF, we have derived our desideratum, repeated in (23):

\[
(23) \quad \textbf{Desideratum (repeated from (6))}:
\]

Given a sentence of the form \( \text{Some}_x[\ldots \text{some}_y[\ldots P(x, y)]\ldots] \), derive just the following implicatures:

\begin{enumerate}
\item \( \exists x[\ldots \exists y \wedge \neg \forall y[\ldots P(x, y)]] \) (Local SI)
\item \( \neg \forall x[\ldots \exists y[\ldots P(x, y)]] \) \( \equiv \ \exists x[\ldots \neg \exists y[\ldots P(x, y)]] \) (Weak-Global SI)
\end{enumerate}

In a sense we are done, then. But the Strong Kleene system we relied on to obtain this result makes predictions not only about the truth and falsity conditions of LFs, but also about their presuppositions. What is the predicted presupposition here? Recall that the semantic presupposition of an LF according to this theory is the disjunction of its truth conditions and falsity conditions. The disjunctive presupposition predicted for (22) is thus that either no girls found any of their marbles (F conditions), or some girl found some but not all of her marbles and some girl didn’t find any of her marbles (T conditions). We can equivalently state this as a material implication as follows:

\[
(24) \quad \text{Presupposition predicted for (22)}:
\]

If some girls found some of their marbles, then some girl found some but not all of her marbles and some girl didn’t find any of her marbles.

argue in section 3.1 below that this is exactly what explains why SIs are not normally detected under negation.
Presuppositional exhaustification

With the help of some logic, (24) turns out to be equivalent to the statement in (25). Notice incidentally that the result entails the Weak-Global SI, as (25) makes transparent.

(25) Presupposition predicted for (22):
Some girl didn’t find any of her marbles, and if some girl found some then some girl found just some.
\[ \exists x \neg \exists y P_{x,y} \land (\exists x \exists y P_{x,y} \rightarrow \exists x \exists \forall y P_{x,y}) \]

Standardly, the presuppositions of an LF must be entailed by the Common Ground before evaluating the LF’s assertive component (truth and falsity). This sounds problematic, because (25) is not felt to necessarily be entailed by the common ground prior to evaluating our sentence (associated with the pattern of inference we are concerned with). But as we alluded to in Section 1, we will argue in Section 6 that the presuppositions associated with pex must be globally accommodated whenever possible, i.e. whenever those presuppositions are not in contradiction with the common ground. For the current case, (25) is quietly and immediately accommodated by a hearer of the sentence in any context with which it is compatible, so it would not be perceived as a pre-condition on the use of the some-under-some sentence in question.

3 pex and the restricted distribution of SIs under negation

This section discusses the distribution of SIs under negation. We will point out the advantages of pex in explaining both why, normally, SIs are perceived as not being computed under negation (Section 3.1), and why they sometimes are computed in those positions, by unifying the latter cases with the phenomenon of ‘presupposition cancellation’ via local accommodation (Section 3.2).

3.1 The vacuity of pex under negation

SIs are not normally computed in the scope of negation. The sentences in (26), if read with neutral intonation (default accenting), do not allow the indicated readings which can be described as enriching a weak scalar item (or, some) with a SI in the scope of negation. This is corroborated by the infelicity of the continuation sentences in parentheses, which would only be compatible with the enriched meaning.
(26)  a. Alex didn’t talk to Mary or Sue. (#She talked to both)  
\begin{center} \textit{\textit{\sim} it is not true that Alex talked to just one of them} \end{center}  
b. I don’t think someone cheated. (#I think they all did)  
\begin{center} \textit{\textit{\sim} it is not true that (I think that) some-but-not-all cheated} \end{center}

Existing grammatical theories have to say something special to block this, since if \textbf{exh} was freely available in the syntax there is no \textit{a priori} reason why in (26) it couldn’t be embedded under negation, resulting in the problematic parses in (27):

(27)  a. not [\textbf{exh} [Alex talked to Mary or Sue]] (not > or-but-not-both)  
b. not (believe) [\textbf{exh} [someone cheated]] (not > some-but-not-all)

In order not to overgenerate this, Fox & Spector (2018) propose an economy condition which generally bans the presence of \textbf{exh} under negation.\footnote{Their condition can be obviated in certain conditions, cf. our discussion surrounding (29). Fox \& Spector (2018) build their economy condition on the observation that computing a SI under negation weakens the overall interpretation of the sentence, and weaker meanings are generally dispreferred to stronger ones. Enguehard \& Chemla (2019) propose a similar constraint on the insertion of \textbf{exh}, though they motivate their principle on different grounds than Fox \& Spector 2018.}

Given the \textbf{pex} theory, however, the economy condition on the distribution of exhaustification is not needed. Due to the revised semantics of \textbf{pex}, LFs analogous to (27) (i.e. with an embedded occurrence of \textbf{pex}) do not give rise to embedded SIs.

To see this, consider the LF of (27b) under the \textbf{pex} theory, in (28a).\footnote{We continue to assume that matrix \textbf{pex} is also present, but in this case it will be vacuous so it isn’t represented. We also abstract away from the contribution of ‘believe’ in (27b) and drop it from the representation for the purpose of simplicity.} The embedded \textbf{pex} will derive the presupposition that not all cheated, (28b). Since it is a presupposition, it will project through negation to the matrix level. The presupposition is entailed by the assertive component of the whole sentence, which is just a negated existential — the basic semantics of the sentence.

(28)  a. [not [\textbf{pex} [someone cheated]]]  
b. Projected presupposition: \textit{not all cheated}  
c. At-issue content of (28a): \textit{no one cheated}

Thus, in these simple cases a structure with \textbf{pex} embedded under (one) negation does not yield embedded SIs. What it does yield, if exhaustification is active due to Relevance, is a presupposition that projects. As mentioned
Presuppositional exhaustification

earlier, we will propose in Section 6 that the presuppositions associated with \textit{pex} must be globally accommodated whenever possible. Given this, it is expected that (28b) is not detected \textit{qua} presupposition. Because it is already entailed by the at-issue content, the presupposition is true when the at-issue content is.\textsuperscript{19} So a hearer with LF (28a) in mind who is guided by the requirement to immediately accommodate \textit{pex}'s presupposition should not be led to conclude that the Common Ground should already entail (28b).\textsuperscript{20}

### 3.2 \textit{pex} and presupposition cancellation

This \textit{pex}-based approach allows us to account for a related fact about SIs, namely that under special conditions they can appear under negation. Specifically, when the target sentences are pronounced with the ‘contradiction contour’ (Horn 1989)— H* pitch on negation, L+H* on the scalar item and LH% as the boundary tone—we get the reading that was missing in (26) (see e.g. Meyer 2016 for discussion).

\begin{enumerate}[a.]
  \item Alex didn’t\textsubscript{H*} talk to Mary OR\textsubscript{L+H*} Sue\textsubscript{LH%}… she talked to both!
  \item I don’t\textsubscript{H*} think SOME\textsubscript{L+H*} one cheated on the exam\textsubscript{LH%}… everyone did!
\end{enumerate}

This pattern bears a striking resemblance to the phenomenon known as \textbf{presupposition cancellation}. In presupposition cancellation, presuppo-

\textsuperscript{19} See Section 4 and Ahn, Saha & Sauerland 2020 for cases in which the presuppositions triggered by \textit{pex} in other kinds of DE environments are more easily detectable, and in particular, are not entailed by the assertive part.

\textsuperscript{20} Peter Lasersohn (p.c.) and a reviewer asked about cases similar to those in (27) but with expressions that do not have scalar terms, such as (ia). Given obligatory \textit{pex}, and paralleling our account of scalar terms in (28), we are committed to a parse as in (ib).

\begin{enumerate}[i.]
  \item Mary did not sing.
  \item not \textbf{[pex [Mary sang]]}
\end{enumerate}

Assuming that the (relevant) alternatives to \textit{Mary sang} are \{\textit{Bill sang, Sue sang}\}, the predicted presupposition here according to our theory is the material conditional \textit{if Mary sang, Bill and Sue didn’t} (see footnote 14). We again respond to this seemingly problematic conclusion using our running theme that the presuppositions of \textit{pex} will be immediately (globally) accommodated upon processing the LF. The truth conditions here too entail the presupposition; if Mary didn’t sing, it follows that the material conditional \textit{If Mary sang, Bill and Sue didn’t} is true.
sitional content in the scope of negation does not project through it, but appears to be at-issue. Some illustrative examples are in (30):

(30)  

a. Mary isn’t$_H^*$ late to the meeting AGAIN$_L^* + H^* L^H%$... she has never been late before!
   
   (negation denies the presup. of [again $\phi$], i.e. “$\phi$ happened before”)

b. Mary didn’t$_H^*$ STOP$_L^* + H^* L^H%$ smoking$_L^H%$... she never used to!
   
   (negation denies the presup. of stop, namely “used to”)

c. Mary can’t$_H^*$ climb THE$_L^* + H^* L^H%$ tree in the garden$_L^H%$... because there are two of them!
   
   (negation denies the uniqueness presup. of the)

d. Chris didn’t$_H^*$ MANAGE$_L^* + H^* L^H%$ to solve the problem$_L^H%$... it was quite easy for him
   
   (Horn 1989)
   
   (negation denies the ‘it-was-hard’ presup. of manage)

(29) and (30) have the same prosodic signature, and moreover they both require the continuation sentence to be felicitous with this prosody. This similarity suggests that the two phenomena — SI under negation and presupposition cancellation — should be accounted for by the same mechanism. Crucially, our approach based on pex — according to which SIs are presuppositions — allows for such a unified account.

Assume following Heim 1983 that presupposition cancellation is possible because, under certain conditions, presuppositions can be “locally accommodated” under negation. Let us further model local accommodation as is standard within trivalent logic, using the $\mathcal{A}$ operator (Beaver & Krahmer 2001). $\mathcal{A}$ turns a trivalent proposition into a bivalent one by collapsing undefinedness with falsity:

(31)  

Let $\phi$ be a sentence with a presupposition $p$ and assertion $p’$
   
   (notation: $\phi = \overline{pp’}$). Then,

\[
\llbracket \mathcal{A}(\phi) \rrbracket = \begin{cases} 
1, & \text{if } \llbracket \phi \rrbracket = 1 \\
0, & \text{if } \llbracket \phi \rrbracket = \# \text{ or } 0
\end{cases} 
\text{(i.e. if } p \land p’ \text{ or } \neg p \text{ or } \neg p’ \text{)}
\]

When (31) is embedded under negation, the whole sentence is predicted to be true if the presupposition is false; this derives presupposition cancellation:  

---

21 (32) assumes standard semantics for negation which flips truth- and falsity-conditions.
Presuppositional exhaustification

\[
\text{\textit{not}}(\mathcal{A}(\phi)) = \begin{cases} 
1, & \text{if } [\phi] = \# \text{ or } 0 \quad \text{(i.e. if } \neg p \text{ or } \neg p') \\
0, & \text{if } [\phi] = 1 \quad \text{(i.e. if } p \land p')
\end{cases}
\]

On our \texttt{pex}-based proposal, SIs under negation are derived with the same mechanism: these are cases where an \texttt{A} operator takes scope in between negation and \texttt{pex}. We illustrate the analysis in (33).

(33) I don’t think \textit{SOME} in \textit{L}+\textit{H}∗… they all did!
   a. \textit{not} \textit{A} [\textit{pex} \{someone cheated\}]
   b. \textit{\sim} \textit{It is not true that some-but-not-all cheated.}

Indeed, in our theory, deriving an embedded SI under negation requires application of \texttt{A} immediately below negation;\textsuperscript{22} as we showed in (28), without \texttt{A} the information that the relevant alternative is locally exhaustified will project through negation rather than enter the at-issue/assertive content that negation targets. This helps explain the restricted distribution of embedded SIs: local accommodation under negation is known to be possible only as a last-resort mechanism, to rescue an otherwise inconsistent discourse (Gazdar 1979, Heim 1983). On the hypothesis that local accommodation involves the insertion of \texttt{A} under negation, this means that inserting \texttt{A} under negation is possible only to rescue an inconsistent discourse. Note that the continuation sentences in (29) threaten the consistency of the whole discourse, so inserting \texttt{A} is possible, and the embedded SI reading arises, which accounts for why the continuation sentence is necessary for the embedded SI reading to arise. In addition, we hypothesize that embedding \texttt{A} under negation requires the contradiction contour (perhaps just the \textit{L}+\textit{H}∗ part of it), accounting for the contour’s obligatoriness when cancelling presupposed SIs, as illustrated in (29), and more standard presuppositions, as illustrated in (30).\textsuperscript{23}

\textsuperscript{22} Note that such a configuration is equivalent to embedding the bivalent \texttt{exh} under negation without \texttt{A}: \texttt{[not(A(pex(\psi)))]} = \texttt{[not(exh(\psi)))].}

\textsuperscript{23} The correlation between local accommodation and the contradiction contour is arguably not exception-free. Cancelling the existence presupposition of ‘the’ (as opposed to its uniqueness presupposition, cf. (30c)) doesn’t seem to require or even allow a \textit{L}+\textit{H}∗ accent on \textit{the}:

(i) Mary didn’t \textit{H}∗ meet the king of France… because there is no king of France.

We are not sure why the existence presupposition of \textit{the} might be different. It is worth mentioning that Coppock & Beaver (2015) argue that the existence inference is not hard-coded as a presupposition of \textit{the}, as opposed to the uniqueness inference.
4 SIs under (negative) factives

In Section 3, we discussed simple expressions with negation over (exhaustified) weak scalar terms and argued that pex predicts their intuitive truth-conditions both when the embedded SIs project and also in special cases in which they are 'canceled' via local accommodation. In this section, we continue to explore the behavior of SIs in downward entailing (DE) environments, focusing now on the slightly more complex cases of SIs under negative factive predicates. What is interesting about these cases is that—unlike in the case of simple negation without local accommodation—the SIs which project from these DE environments have a noticeable effect on the overall entailments of the target expressions. As we will see, these cases present a serious challenge to standard exh-based theories yet can be easily handled with pex.

4.1 Gajewski & Sharvit’s (2012) puzzle

The basic puzzle, introduced by Gajewski & Sharvit 2012 (henceforth G&S), can be illustrated by sentences like (34), where the negative factive (un)aware, a presuppositional trigger, has a complement with the scalar item some.

(34) John is unaware that some of the students smoke.

On its default reading, (34) entails that John is unaware that there are any smoking students. In addition, it also entails that, in the actual world, some but not all students smoke. Let us characterize this state of affairs by calling the latter inference a ‘projective’ inference, and the former the ‘embedded’ inference.

(35) a. Projective inference: Some but not all of the students smoke.

\[ \exists x \neg \forall x [\text{student}(x) \land \text{smoke}(x)] \]

b. Embedded inference: John does not hold the belief that there are smoking students.

\[ \neg \text{BEL}_{\text{john}} : \exists x [\text{student}(x) \land \text{smoke}(x)] \]

The key observation, then, is that there is a scalar enrichment at the level of the projective content, which is lacking at the level of the embedded content.

G&S, followed by Spector & Sudo 2017 (S&S), claim that (35a) is a presupposition of (34) while (35b) is the assertive content. We can for now remain neutral concerning the exact pragmatic status of the scalar enrichment in (35a), but we will get back to this shortly. What matters, at this point, is that
Presuppositional exhaustification

an adequate account of (34) should predict both the (35a) and (35b) inferences.

This pattern of inferences is a problem for naïve exh theory, as discussed in detail in Gajewski & Sharvit 2012 and Spector & Sudo 2017. Embedding exh under unaware, as in (36a), contributes a scalar implicature. This generates the appropriate ‘projective’ inference, as captured in (36b), assuming the ‘some but not all’ output of the embedded exh projects from under the factive predicate unaware. However, it yields a content that is too weak per the ‘embedded’ inference, as captured in (36c), where bel\_\text{\_}x,\_w stands for the set of worlds compatible with x’s beliefs in w.

(36) Wrong inference on exh theory

a. John is unaware that [exh [some of the students smoke]]

b. \(\leadsto \) Some-but-not-all students smoke.

c. \(\leadsto \) John does not hold the belief that some-but-not-all students smoke.

\(\neg \forall w \in \text{BEL}_{\text{\_} john,\_w} : \exists \forall x [\text{student}_w(x) \land \text{smoke}_w(x)]\)

(36c) says, in other words, that John’s belief state does not entail that some-but-not-all smoke. This is perfectly compatible with John’s belief state entailing that all smoke. But that is too weak, as the sentence (in its default, unmarked reading) intuitively excludes the possibility that John believes that all students smoke. That is, (34) is infelicitous in a context where John believes that all students smoke, as (37) shows.

(37) Context: John believes that all students smoke. In fact only some do.

#John is unaware that some of the students smoke.

Other possible LFs don’t seem to help. For example, removing the embedded exh from (36a) would remove the problematic SI from the scope of unaware and thus correctly capture the embedded inference. However, we would then predict a projective inference without an SI, which is too weak, namely, the inference that some or all students smoke.

Gajewski & Sharvit 2012, Spector & Sudo 2017 and Marty & Romoli 2021 all offer accounts of SIs under (negative) factives, such as (34), that modify and supplement standard Grammatical theories so as to endure that exh is sensitive to and affects, in certain specific ways, any presuppositions triggered in its prejacent and/or its alternatives. Our pex-based account, however, is unique in that it treats covert exhaustification as itself a presupposition trig-
Our goal here is neither to evaluate nor to compare those theories with our own—a rather complex task that would require discussion of various kinds of more complex SIs under negative factives. Instead, our goal in what follows is to show that our pex-based Grammatical theory predicts the basic facts in a rather straightforward way, without the need for any additional and potentially ad hoc stipulations.25

4.2 pex and projection of SIs under negative factives

To deal with basic cases of SIs under (negative) factives, such as (34), we propose an LF with embedded pex as in (38), where we ignore for simplicity an additional matrix pex which can be associated with, say, John, since those additional potential enrichments are not relevant for our purposes.

(38) John is unaware that [pex [some of the students smoke]]

We assume a fairly ordinary semantics for the factive unaware, which can be thought of as an amalgamation of negation and a positive factive aware/know. Just like its positive counterpart, unaware presupposes its complement. But it asserts the negation of what its positive counterpart asserts.

(39) \([x \text{ is unaware that } p]^{w} :\)
   a. Presupposes: \([p]^{w} = 1.\)
   b. Asserts: \(\exists w' \in \text{bel}_{x,w} : [p]^{w'} = 0\)
      (It is compatible with x’s beliefs in w that p is false.)

Cast in Strong Kleene semantics, we assume that this is the result of the trivalent entry for unaware in (40), where the presupposition—the semantic value of the complement of unaware—is a conjunct in both truth- and falsity-conditions.

---

24 This is part of the reason why those accounts, unlike our own, derive the target readings by adjoining a an exahustificaiton operator at matrix level rather than embedded over the complement of the factive verb.

25 In Del Pinal, Bassi & Sauerland 2021 we discuss a wider range of cases of SIs under (negative) factives, including free choice inferences, and include a detailed comparative discussion of Gajewski & Sharvit 2012, Spector & Sudo 2017 and Marty & Romoli 2021 and our pex-based Grammatical account.
Presuppositional exhaustification

\[
[\text{x is unaware that } p]_w = \begin{cases} 
1, & \text{if } [p]_w = 1 \land \exists w' \in \text{BEL}_{x,w} : [p]_{w'} = 0 \\
0, & \text{if } [p]_w = 1 \land \forall w' \in \text{BEL}_{x,w} : [p]_{w'} = 1 \\
\#, & \text{otherwise}
\end{cases}
\]

Applied to our LF in (38), where the complement of unaware is a structure appended with \text{pex}, we get (41):

\[
[(38)]_w = \begin{cases} 
1, & \text{if } [\text{pex}(\text{some}...) ]_w = 1 \\
& \land \exists w' \in \text{BEL}_{x,w} : [\text{pex}(\text{some}...) ]_{w'} = 0 \\
0, & \text{if } [\text{pex}(\text{some}...) ]_w = 1 \\
& \land \forall w' \in \text{BEL}_{x,w} : [\text{pex}(\text{some}...) ]_{w'} = 1 \\
\#, & \text{otherwise}
\end{cases}
\]

The part to focus on is the second conjunct in the truth conditions. Part of our proposal, recall, is that \text{pex} does not contribute anything to falsity conditions. This means that $[\text{pex}(\text{some smoke})]_{w'} = 0$ iff $[\text{some smoke}]_{w'} = 0$ iff none smoke$_{w'}$. The semantics then delivers that the LF is true if (i) some-but-not-all smoke, and (ii) it is compatible with John’s beliefs that no student smokes. This is shown in (42).

\[
[(38)]_w = \begin{cases} 
1, & \text{if some-but-not-all smoke}_{w} \\
& \land \exists w' \in \text{BEL}_{x,w} : \text{none smoke}_{w'} \\
0, & \text{if some-but-not-all smoke}_{w} \\
& \land \forall w' \in \text{BEL}_{x,w} : \text{some-but-not-all smoke}_{w'} \\
\#, & \text{otherwise}
\end{cases}
\]

This captures, via the LF in (38), the target reading of (34). The first conjunct in the truth conditions in (42) captures the ‘projective’ inference in (35a) (that some but not all students smoke in the evaluation world), and the second conjunct captures the ‘embedded’ inference in (35b) (that it is compatible with what John believes that no students smoke).\footnote{Our \text{pex}-based system makes predictions about a host of other embedded Downward Entailing environments that we haven’t examined here. We leave a detailed discussion of those predictions for another occasion, in particular SIs in the antecedent of conditionals and in the restrictor position of universals.}

We can now return to the earlier question regarding the status of the projective inference of (34) (that some but not all students smoke in the evaluation world). Note that, in (42), the projective inference is derived as a presupposition, because the disjunction of truth- and falsity-conditions entails it. This is a consequence of the way that the framework encodes the...
factivity of *unaware* (cf. (40)). G&S, and S&S following them, also treat this inference as a presupposition. But does this mean that the information that some-but-not-all smoke must be entailed by the common ground prior to evaluating an assertion of (34)? Or are these cases compatible with our hypothesis, mentioned earlier and elaborated in Section 6, that, in general, SIs that are compatible but not strictly entailed by the common ground are automatically accommodated?

The facts seem to fit our hypothesis: sentences like (34) can be used when the projective content is merely compatible with but not entailed by the common ground. In fact, S&S note (in footnote 8) that in a context that is compatible with the proposition that some or all of the students smoke—i.e. a context where it isn’t common ground that not all smoke—(34) sounds felicitous.

(43) **Context:** We know some of the students are smokers, but are wondering if all are. We are thinking of asking John, but then are told:

John is unaware that some of the students smoke.

(Spector & Sudo 2017: fn. 8)

Now, one might suggest instead that in this context the *not all* implicature is not computed—after all, implicatures are generally optional. This seems to be the analysis advocated by S&S, but it runs into a problem. As also noticed by S&S, when the *not all* inference is not compatible with the context, infelicity does ensue:

(44) We know all students smoke. Yet John is wondering if any student smokes.

#John is unaware that some of the students smoke.

(Spector & Sudo 2017: ex. 7)

In (44), the information that all students smoke is established in the common ground. If (43) is felicitous only because we do not compute the ‘not all’ SI, it isn’t entirely clear why that same strategy isn’t available to rescue (44).

From our perspective, however, the contrast between (43) and (44) is unsurprising. Although the projective inference is a presupposition of (34), it is globally accommodated when it is consistent with the common ground. In (43), there is no direct conflict between the projective presupposition and the common ground, so accommodation takes place. In (44), in contrast, the presupposition directly contradicts the common ground, so accommodation is
Presuppositional exhaustification

not possible; it is an independent generalization about presuppositions that they cannot be accommodated when they contradict the common ground.\(^{27}\)

The generalization just mentioned is going to take a center stage in the next two sections, which are devoted to interactions between SIs, conceived of as a species of non-at-issue content, and the common ground. We will discuss in detail and defend the hypothesis that pex belongs to a class of presuppositions that are (i) automatically globally accommodated when compatible with the common ground, yet (ii) generate infelicity when they are incompatible with the common ground.

5 Common ground-mismatching SIs

The last puzzle for theories of SIs that we will discuss involves odd assertions of expressions with scalar terms that seem to trigger SIs which are inconsistent with the common ground. Standard examples are sentences like (45a)-(45b), which are odd given a common ground with normal information such as that Italians come from the same country and height is a relatively stable property.

(45) \(C\): standard adult background information
   a. #Some Italians come from a warm country.
   b. #Sue is sometimes tall.

Magri 2009 argues that the oddness of the sentences in (45) is due to a clash between their SIs, which are obligatorily triggered by covert \textit{exh}, and the common ground. We review Magri’s basic account in Section 5.1. We accept the key components of Magri’s account. However, after considering a broader set of oddness cases in Sections 5.2 and 5.3, we argue that Magri’s theory should be formulated in terms of \textit{pex} rather than \textit{exh}.\(^{28}\)

\(^{27}\) Still, why can’t interlocutors simple drop the ‘not all’ SI from the projective content and rescue from infelicity the assertion of (34) in contexts like (44)? This constraint follows from our core Grammatical theory. First, the embedded \textit{some} clause has to be parsed with \textit{pex}, so that the factive presupposition we are evaluating against the common ground is \textit{pex[\textit{some smoke}].} Second, given the common ground in (44), the prejacent ‘some’ and its ‘all’ alternatives are contextually equivalent, which means that the latter is necessarily relevant and hence obligatorily excluded.

\(^{28}\) The \textit{pex} based theory of oddness we develop in this section shares various components—including the use of obligatory exhaustification with \textit{pex}—with the theory defended in Del Pinal 2021. Still, the discussions are intended to be complementary, in the sense that, aside from the basic cases in (45), they cover distinct additional oddness patterns. One important
5.1 A review of Magri’s theory

Magri’s (2009) account refines a fairly natural analysis of the oddness of (45a) and (45b). A sentence like (45a) has a scalar alternative *all Italians come from a warm country*. This alternative is obligatorily negated, with the result that exhaustification of (45) entails that not all Italians come from the same country — an inference which conflicts with the common ground belief that Italians come from the same country. The same line of explanation applies to (45b): its scalar alternative, *Sue is always tall*, is obligatorily negated, leading to the inference that Sue is sometimes but not always tall, a piece of information that conflicts with standard information about the relative stability of human height.

As Magri points out, the observation that SIs can clash with the common ground and generate oddness is problematic for standard neo-Gricean theories of SIs. Neo-Griceans conceive of SIs as computed by listeners to ensure that speakers come out as cooperative and rational conversational partners. So why would interlocutors compute an SI that is inconsistent with the common ground and results in infelicity? Why not stick, in cases like (45a)–(45b), with the literal interpretations which not only do not clash with the common ground but can when added to it convey useful information such as that Italy is warm and that Mary is tall?²⁹

Magri argues that the Grammatical account of SIs is best suited to incorporate the notion of common ground-mismatching SIs. Working with standard *exh* (see (1)), his Grammatical account rests on two stipulations about the distribution and computations of *exh*:³⁰

(I) *exh* is obligatorily attached to proposition type structures.

²⁹ Our goal here is not directly to undermine neo-Gricean or in general pragmatic approaches to Magri-style oddness patterns. For a critical discussion of such pragmatic approaches (and indeed approaches which try to explain Magri-style oddness without appealing to common ground-mismatching implicatures), see Del Pinal 2021. The latter also includes a discussion of specific cases of embedded Magri-style oddness, important for discriminating between pragmatic and grammatical theories of oddness, but less so for discriminating between *exh* and *pex*-based accounts.

³⁰ We had already mentioned stipulation (I) in Section 1. Stipulation (II) was already incorporated the definition of *exh* in (1). Specifically, it was stipulated there that *R*, the Relevance parameter that restricts the propositions among *Excl*(φ) that *exh* negates, always includes
Presuppositional exhaustification

(II) For any context $c$, alternatives to the prejacent $\phi$ which are logically non-weaker but contextually equivalent to $\phi$ in $c$ must be negated in the course of computing $\text{exh}(\phi)$ in $c$ (i.e. such alternatives cannot be dismissed as irrelevant for the purposes of $\text{exh}$).\footnote{For the following, a restriction to logically stronger alternatives, such as the one adopted in \cite{Magri2009}, would also be feasible. But given the empirical support for the weaker restriction we discussed above, we adopt the hypothesis that exhaustification negates relevant, non-weaker alternatives, as formulated in (1), (2), and here.}

In addition, Magri proposes an oddness filter that works together with (I) and (II) to determine the set of utterances that are marked as odd:

(III) **Oddness filter.**
If $\text{exh}(\phi)$ is inconsistent with the common ground in context $c$, then $\text{exh}(\phi)$ is odd in $c$.

The basic idea, then, is that sentences like (45a) and (45b) are obligatorily parsed as (46a) and (46b) respectively. These LFs, in turn, generate the enriched readings in (47a) and (47b), which clash with the common ground $C$. Applying the oddness filter (III), we then correctly predict that (45a) and (45b) are odd in $C$.

(46)  
   a. $\text{exh}$ [Some Italians come from a warm country]
   b. $\text{exh}$ [Sue is sometimes tall]

(47)  
   a. some Italians come from a warm country $\land \neg$all Italians come from a warm country
   b. Sue is sometimes tall $\land \neg$Sue is always tall

Why are stipulations (I)–(III) needed? If we reject the obligatoriness of $\text{exh}$ (I), we could select parses without $\text{exh}$, and thus prevent expressions like (45a) and (45b) from triggering the problematic enrichments. If we reject the hypothesis that $\text{exh}$ is sensitive to logical and not contextual entailment (II), the application of $\text{exh}$ for (46a) and (46b) would be vacuous (in a normal common ground): in each case, the prejacent is contextually equivalent to its logically stronger alternative, so the latter would not be negated and no clash with the common ground would ensue. Finally, if we dropped the oddness filter (III), we could say that when a speaker asserts a content which clashes with the common ground, listeners should, in general (i.e., when the utterance

\footnote{For the following, a restriction to logically stronger alternatives, such as the one adopted in \cite{Magri2009}, would also be feasible. But given the empirical support for the weaker restriction we discussed above, we adopt the hypothesis that exhaustification negates relevant, non-weaker alternatives, as formulated in (1), (2), and here.}
is not obviously metaphorical, figurative, etc.), revise their beliefs about what is in the common ground prior to assessing the truth value of the utterance. The utterance might still be judged false, but this need not generate oddness due to a clash with the common ground.\(^{32}\)

5.2 An overgeneration problem

Although promising, Magri’s account of oddness faces a problem when we consider the effect on felicity judgments of different kinds of mismatches with the common ground. It predicts that any expression which mismatches with the common ground will be filtered out as odd, but this is not borne out. Consider first the following contrast:

(48) \(C\) : normal adult background information

   a. #Some tigers are carnivorous.
   b. Only SOME tigers are carnivorous.
   c. Some but not all tigers are carnivorous.

While (48a) has the signature of Magri-style oddness, (48b) and (48c) feel markedly different: intuitively, it seems more appropriate to classify them as downright false rather than as odd. Yet Magri’s theory, as currently formulated, predicts that (48a)–(48c) should all be equally odd in \(C\). This is because they are all covertly exhaustified, following stipulation (I), and the resulting interpretations are all inconsistent with the common ground. Hence

\(^{32}\) Interestingly, Degen, Tessler & Goodman (2015) show that standard RSA models make incorrect predictions – relative to those provided by ordinary speakers – about interpretation of Magri-style sentences. Specifically, standard RSA models overweight prior expectations about the world (i.e., overweight certain components of the common ground), and as a result do not generate scalar enrichments that conflict with those prior expectations. This result reinforces Magri’s conjecture that simple Gricean or pragmatic models predict that listeners shouldn’t compute common ground-mismatching SIs, but should instead combine the literal readings of expressions like ‘Sue is sometimes tall’ with prior common ground beliefs/expectations (height is stable) to get (unattested) enrichments such as that ‘Sue is (stably/simply) tall. We should note, however, that Degen, Tessler & Goodman (2015) also show that the predictions of RSA models, for these Magri-style oddness cases, can be greatly improved if even ‘stable’ beliefs in the common ground are modeled as more akin to negotiable hypotheses between interlocutors. Interestingly, and as will become clear below, our final account of oddness also builds on the insight that a more refined account of the conditions on which stable beliefs in the common ground can be revised or dropped. It would be interesting to compare, in future work, the more refined RSA models presented in Degen, Tessler & Goodman 2015 with our pex-based theory of oddness.
the oddness filter in (III) predicts them to all pattern together and be classified as odd.  

One might initially take this data to suggest that, for some reason, common ground-mismatching inferences triggered by covert operators are judged odd, as in (48a) assuming it is parsed with exh, whereas those triggered by overt operators or materials are judged to be (obviously) false but not odd, as in (48b) and (48c). This is not the correct generalization, however, as shown by the contrasting pattern in (49):

(49) \( C : \) normal adult background information   
  a. Some tigers are herbivorous. 
  b. #Only SOME tigers are herbivorous. 
  c. Some but not all tigers are herbivorous.

In (49) the sentence with the covert exhaustification sounds merely false, while the one with only sounds odd— the inverse of (48). The key difference from (48) is that the prejacent covert exh in (49) (some tigers are herbivorous) is held in the common ground to be false rather than true. Note that this difference does not affect the (c) examples— (49c) is judged not odd (but false), just like (48c).

Can these patterns be explained by holding that, despite the intended description of the common ground, readers tend to revise the common ground differently across the target cases? This suggestion is promising. Consider an assertion of \( \phi \) by \( S \) which is inconsistent with the common ground \( C \) as represented by listener \( L \). Since hypotheses about what is in the common ground are in general defeasible, \( L \) could take \( S \)’s assertions of \( \phi \) as evidence that \( C \) is not actually the common ground. Indeed, this is plausibly part of what is happening in the non-odd cases in (48)– (49). From this perspective, the puzzle is precisely to explain why, although relatively to Magri’s account the expressions in (48) have (nearly) the same truth-conditional meaning, and so do the ones in (49), only some of those expressions succeed as calls to revise the common ground. That this is so can be illustrated by observing the

---

33 We assume that redundancy, relative to the common ground, doesn’t categorically lead to oddness. An assertion whose content is strictly entailed by background information can be used as reminder, to make specific entities, properties or contents salient, and to make sure that interlocutors are actually aligned with respect to particular components of the common ground. The kind of context we are imagining is one in which uttering tigers are carnivorous is clearly not as odd as uttering some tigers are carnivorous.
following contrasts in the kinds of continuations that our target expressions support:

(50)  

\[-\]

\[
C: \text{normal adult background information}
\]

a. A: Some tigers are carnivorous. #I know you think that is an insane thing to say, but I read that in a recent issue of *Science*.

b. Only some/some but not all tigers are carnivorous. I know you think that is an insane thing to say, but I read that in recent issue of *Science*.

This also sheds light on why oddness filters such as Magri’s (III)—and precursors such as the principle that assertions should be neither trivial nor collapse the common ground—are not sufficiently fine-grained, given Magri’s theory of exhaustification, to explain contrasts like those above. If mismatches between assertions and the hypothesized common ground can be used as evidence to revise the common ground, why can we successfully do that for (48b) and (48c) but not for (48a), or for (49a) and (49c) but not for (49b)?

In our view, the patterns in (48)–(49) reveal that oddness-inducing inferences are sensitive to different ingredients of meaning. Specifically, we claim that oddness arises when the common ground-conflicting information is contributed at a presuppositional level. In contrast, when common ground-conflicting information is communicated fully at the assertive/at-issue level, interlocutors can successfully use it to revise the common ground, and can then judge the information as merely false (or proceed to change their own beliefs, depending on the degree of speaker authority, among other factors). Sentences (48b), (48c), (49a) and (49c) are not odd because they convey the common ground-conflicting information (i.e. not all tigers are carnivorous/some tigers are herbivorous) at the assertive/at-issue level. The contrast in the only examples (48b) and (49b) is thus explained on the basis of the fact that while (48b) merely has an assertive content which conflicts with the common ground, (49b) has a presupposition which conflicts with the common ground. In (49a) (and (49c)) the common ground-conflicting information is, again, conveyed fully with at-issue material, which is why they are not odd. Following this generalization, our core hypothesis that SIs are presuppositions explains the oddness of (48a): the exhaustive inference is conveyed at a presuppositional level.
Presuppositional exhaustification

To sum up, the shortcoming of Magri’s system in light of (48)-(49) is that it is not sensitive to the distinction between presuppositional content and at-issue content for the purpose of explaining oddness. Fortunately, this problem can be easily fixed by reformulating Magri’s theory in terms of pex.

5.3 pex and oddness

According to our account of exhaustification, pex triggers an asserted vs. presupposed entailment structure that is the mirror image of that triggered by only. The definition of pex is repeated from (2) in a simplified form:

(51) \[ \text{pex}(\phi) = \left\{ \begin{array}{ll} \text{presupposition:} & \bigwedge \neg \psi : \psi \in \text{Excl}(\phi) \cap R \\ \text{assertion:} & \phi \end{array} \right. \]

Using pex we can formulate a refined version of Magri’s theory that doesn’t overgenerate oddness. Specifically, we can maintain his (I) and (II) from Section 5.1 — the core elements of Magri’s theory — yet drop the overly strong oddness filter in (III). Instead, we introduce the weaker oddness filter in (52) which is sensitive to the interaction between the common ground and different ingredients of meaning:

(52) **Revised oddness filter.** Let \( \phi \) be a sentence with a presupposition \( p \) and assertion \( p' \) (notation: \( \phi = \overline{pp'} \)). Then asserting \( \phi \) is odd in \( c \) if

(i) \( p \) is inconsistent with \( c \), or

(ii) \( p \) and \( p' \) are each individually consistent with \( c \), but \( p \land p' \) is inconsistent with \( c \).

According to (52), an assertion of \( \phi \) is odd if it has a presupposition that is inconsistent with the common ground. It is also odd if the update of the common ground with the presupposed and assertive content of \( \phi \) is inconsistent with the common ground, while the presupposition and assertive content each on its own is consistent with the common ground. (52) is an extension of a reasonable and independently justified constraint on accommodation. As various theorists have pointed out, presupposed information
that is inconsistent with the common ground cannot be accommodated. If a speaker wants to convey information that challenges commonly-held beliefs, they must do so fully at the at-issue level of meaning. Unlike Magri’s original oddness filter, this filter straightforwardly allows interlocutors to assert weird beliefs even against hypothesized common grounds relative to which they are inconsistent — but such weird or conspiratorial contents should be fully fronted to the at-issue content of assertions (cf. Heim 1992).

Given `pex` and the revised oddness filter in (52), we can explain the target oddness patterns. Consider first examples (48a) and (49a), repeated in (53) and (54), given their LFs with `pex` in (53a) and (54a). In each case, the underlined part represents the part of the content that is presupposed. (53a) presupposes that it is not the case that all tigers are carnivorous. Since this presupposition is inconsistent with the common ground in \( C \), it is correctly predicted to be odd by the revised oddness filter. In contrast, (54a) presupposes that it is not the case that all tigers are herbivorous. This presupposition is consistent with (in fact entailed by) the common ground in \( C \). Since it is the assertive component alone in (54a) that mismatches the common ground, the revised oddness filter does not rule it as odd (even if it is judged to be obviously false by the interlocutors).

\[
\text{(53) } \#\text{Some tigers are carnivorous.}\n\]
\[
\text{(a) } \text{pex}[\text{some tigers are carnivorous}]
\]
\[
\begin{align*}
&= \neg \text{all tigers are carnivorous} \land \text{some tigers are carnivorous}
\end{align*}
\]

\[
\text{(54) } \text{Some tigers are herbivorous.}\n\]
\[
\text{(a) } \text{pex}[\text{some tigers are herbivorous}]
\]
\[
\begin{align*}
&= \neg \text{all tigers are herbivorous} \land \text{some tigers are herbivorous}
\end{align*}
\]

Consider next the only examples in (48b) and (49b), repeated in (55) and (56). According to standard accounts, only \( \phi \) presupposes its prejacent, \( \phi \), and asserts the negation of each excludable focus alternative of \( \phi \). In the case of (55), we get the presupposition that some tigers are carnivorous, which is consistent with (in fact entailed by) the common ground in \( C \). In addition, the common ground-conflicting information that not all tigers are carnivorous is fully included in the assertive content. It is thus easy to see that (55) is

34 Recall Stalnaker 1978 assumed a stronger oddness condition than (52); namely that asserting \( \phi \) is odd if \( c \) doesn’t entail \( p \). For our present purposes we could adopt Stalnaker’s strong filter, but Lewis 1979 and others show that all cases but the core case in (52) involve further complications that we want to sidestep here.
Presuppositional exhaustification

not filtered out by the revised oddness filter. In the case of (56), we get the presupposition that some tigers are herbivorous, which is inconsistent with C and is thus marked as odd by the revised oddness filter. Both of these predictions are adequate.\(^{35}\)

(55) Only SOME tigers are carnivorous.
    a. only [SOME tigers are carnivorous]
       = some tigers are carnivorous \(\land\) \(\neg\) all tigers are carnivorous

(56) #Only SOME tigers are herbivorous.
    a. only [SOME tigers are herbivorous]
       = some tigers are herbivorous \(\land\) \(\neg\) all tigers are herbivorous

The cases in (48c) and (49c), repeated in (57)-(58), are also handled adequately. The key observation, in these cases, is that all the common ground-mismatching content is asserted, and so the revised oddness filter doesn't predict them to be odd:

(57) Some but not all tigers are carnivorous.
    a. = some tigers are carnivorous \(\land\) \(\neg\) all tigers are carnivorous

(58) Some but not all tigers are herbivorous.
    a. = some tigers are herbivorous \(\land\) \(\neg\) all tigers are herbivorous

Finally, let us reconsider Magri’s original examples, (46a) and (46b). We focus on (46a), repeated here in (59). Recall the basic set up. From the audience’s perspective, the common ground \(C\) is hypothesized to include the information that Italians come from the same country. From that it follows that the ‘some’ sentence is contextually equivalent to its ‘all’ alternative, so that if the former is relevant, so is the latter.

(59) #Some Italians come from a warm country.
    a. \textbf{pex}[some Italians come from a warm country]
       = \(\neg\) all Italians warm country \(\land\) some Italians warm country

Based on (59a), we get the presupposition that not all Italians come from a warm country. Although the common ground \(C\) entails that all Italians come

\(^{35}\) Strictly speaking (55a) and (56a) are also parsed with \textbf{pex} in the matrix. But we assume that in this case \textbf{pex} doesn’t negate any relevant alternative (since the alternative triggered by \textit{some} are already captured by \textit{only}), so we ignore it. The same applies to (57)-(58).
from the same country, it is compatible with the possibility that Italy is warm and also it is not warm. Accordingly, the presupposition in (59a), taken on its own, is strictly compatible with $C$. In addition, the at-issue content of (59a), taken on its own, is also compatible with $C$. Crucially, however, the conjunction of the presupposed and at-issue content of (59a) is incompatible with $C$. Accordingly, the revised oddness filter predicts correctly that it should be filtered out.

At this point, it is worth reemphasizing the basic intuition behind our revised oddness filter. Speakers may sometimes want to communicate information that is inconsistent with what they can reasonably predict interlocutors will take as part of the common ground. To do this effectively, and get the audience to revise the common ground, they should express that mismatching content as part of the assertive content alone. As we saw, this is what is achieved in all the intuitively non-odd examples (54), (55), (57) and (58). Accordingly, we predict that a counterpart of a Magri-style sentence that has, relative to a specific common ground $C$, the same entailments as (59), except that it makes the controversial information more at-issue, is expected to be intuitively less odd. This prediction is attested by the contrast between examples like (60a) and (60b). Compared to (60a), asserting (60b) results in a more direct and successful way to call for revision of $C$, specifically, of the belief that Italians come from the same country:

$$C: \text{stable background knowledge}$$

\begin{align*}
\text{a. } & \#\text{Some Italians come from a warm country.} \\
\text{b. } & \text{Some but not all Italians come from a warm country.} \\
\text{c. } & \#\text{Some Italians come from a warm country. ??For a few actually come from northern Europe, which is cold.} \\
\text{d. } & \text{Some but not all Italians come from a warm country. For a few actually come from northern Europe, which is cold.}
\end{align*}

The target contrast between (60a) and (60b) is brought out by the corresponding contrast in the continuations that each supports, as illustrated in (60c) and (60d). Although relative to $C$, (60a) and (60b) have the same truth-conditions, only the explicit 'some but not all' variant makes the 'not all' entailment at-issue, which explains why a continuation which targets that entailment as a topic is licensed in (60d) but not in (60c).

---

36 A reviewer asks whether our \textit{pex}-based refinement of Magri’s theory of oddness needs an independent account of why—given again normal adult information—variants of Magri-sentences like (i) seem felicitous (or at least improve in acceptability):
Russell (2012) discussed a similar oddness pattern, illustrated in (61) below, which we argue provides additional evidence for \textit{pex}. Consider the pattern in (61). Given the polarity of the interjection (a negative one, as in ‘oh no!’) and the goal in $\mathcal{C}$, to make sense of (61d) interpreters should compute its ‘not all’ implicature. Interestingly, this doesn’t rescue the plain ‘some’ as-

(i) Some Italians come from a warm country, but not all of them do.

Our theory commits us to the presence of an embedded \textit{pex}, as in (ia). Still, since the assertive content of (ia) as a whole directly conflicts with the common ground, (ia) is not filtered out by our revised oddness filter. As a result (and as in other cases of conflicts between an assertive content and the hypothesized common ground), its assertion may successfully signal to interlocutors that they should revise their initial hypothesis about the content of the common ground. Evaluated against a conservatively revised common ground (which doesn’t entail that all Italians come from the same country), the ‘not all’ alternative and the ‘some’ prejacent of the embedded \textit{pex} are no longer contextually equivalent. As a result, the embedded ‘not all’ implicature is strictly optional. If that implicature is not computed, (ia) is expected to be felicitous. If it is computed, however, shouldn’t we expect the second clause to feel oddly redundant, contrary to fact? No, because even in this case explicitly asserting the ‘not all’ content has various non-trivial functions: (i) it makes salient and at-issue content that was not at-issue by the end of processing the first clause, and (ii) that shift in prominence increases the likelihood that the speaker may succeed in signaling to other interlocutors that they should revise even stable elements of the common ground. Now, one may worry that our response actually reveals an even deeper problem. Consider a simple ‘some...not all’ sequence, such as (ii), in any normal context in which the ‘all’ alternative is likely to be recognized as relevant:

(ii) Some of the students passed. But not all of them did.

Why doesn’t the ‘not all’ continuation feel redundant? The answer can’t be exactly as before, since the point here is that the speaker is not trying to induce any deep revisions to the common ground. However, from the perspective of our \textit{pex}-based theory, there are still various non-trivial reasons for explicitly asserting the ‘not all’ content. In practice, interlocutors can often be misaligned with respect to key elements of the common ground, the questions under discussion, and so on: by explicitly asserting the ‘not all’ content, the speaker can guarantee that it will be computed by others, which in some cases may be important even if the risk of miscommunication is small. In addition, the speaker may wish to make the ‘not all’ content at issue due to broader discourse factors and strategies (e.g., to make it especially salient, ready for subsequent anaphoric dependencies, etc). From this perspective, parallel sequences with standard presupposition triggers—e.g. \textit{John knows it is raining, and it is}—will feel more degraded to the degree that the computation of the presupposition in the first part is deterministic (e.g., it is quite unlikely that a typical interlocutor may miss that ‘$A$ knows $p$’ entails $p$, though sometimes even these sequences might improve in acceptability if, say, they are used to establishing appropriate anaphoric dependencies).
assertion in (61d) from oddness. This is not what we would expect, in this kind of context, if it had the same truth-conditions and presupposed vs. assertive structure as its ‘some but not all’ counterpart in (61a). The acceptability of (61b) and (61c), which make the ‘not all’ entailment at issue, suggests that the negative interjection, in this context, targets the ‘not all’ entailment.

(61) Goal in C: we want our students to pass
   a. Crap! Some but not all of our students passed!
   b. Crap! Only some of our students passed!
   c. Crap! Not all of our students passed!
   d. ◼Crap! Some of our students passed!

An analogous puzzle arises if we consider instead an interjection with a different polarity (a positive one, as in ‘thank goodness!’). To make sense of (62c), given the positive polarity of the interjection and the goal in C, the ‘some’ implicature should be computed. Interestingly, this somehow doesn’t rescue the ‘not all’ assertion in (62c) from oddness. This is not what we would expect, in this context, if that assertion had the same truth-conditions and presupposed vs. assertive structure as the assertion of its ‘not all but some’ counterpart in (62b). The oddness of (62d), which we have independent reason to think makes the ‘some’ entailment non-at-issue, suggests that the positive interjection, in this context, tries but fails to target the ‘some’ entailment.

(62) Goal in C: we don’t want all our students to fail
   a. Whew! Some of our students passed!
   b. Whew! Not all but some of our students passed!
   c. ◼Whew! Not all of our students passed!
   d. ◼Whew! Only some of our students passed!

Russell (2012) makes two important observations. First, for these mini-discourses to be felicitous, the assertive content of the continuations must be aligned, given the relevant goals or desires, with the polarity of the interjection. Secondly, ‘implicated’ content somehow doesn’t seem to have assertive status. Given pex, we have a natural explanation for Russell’s second observation: implicated content is presupposed, hence not at-issue. In (61d), the ‘not all’ inference, even if computed, is not at-issue, thus can’t be the target for the negative interjection. Similarly, in (62c) the ‘some’ inference,
Presuppositional exhaustification

even if computed, is also not at-issue, thus can’t be the target for the positive interjection.³⁷

Summing up, in this section we presented an additional argument for the hypothesis that SIs are incorporated at the presuppositional level, based on one well-known feature of presuppositions: namely, that when presuppositions are inconsistent with the common-ground, they are in general hard to accommodate and the corresponding assertions tend to result in oddness judgments. This conclusion complements the result of our previous puzzles, which took advantage of a different property of presuppositions: namely, the unique ways in which presupposed material projects from certain embedded environments and the specific conditions required for such material to be locally accommodated under negation.

6 pex and the status of its presupposition content

We have argued that by treating SIs as a species of non-at-issue and specifically presuppositional content, our pex-based theory resolves various extant puzzles about the behavior of SIs. Yet even if those admittedly somewhat complex applications seem promising, one might still worry that pex negatively affects the treatment of ordinary SIs. Consider again a standard example of a scalar enrichment:

(63) \( C: \) compatible with none, some but not all, or all students having passed.
   a. A: Did any students pass?
   b. B: Some students did.
   c. pex[Some students passed] = \( \neg \)all students passed \( \land \) some students passed

³⁷ Following Russell (2012), one might try to capture patterns like (61)-(62) by revising the notion of relevance, e.g., by requiring that the prejacent of exh should maximize relevance (relative to prior expectations) and thus must not be less relevant to the conversation than any SIs it gives rise to. To serve the domain of alternatives restricting function that relevance serves under a Magri-style theory with obligatory exh, this would have to be implemented as saying that alternatives that are at least as relevant as the prejacent are not exhaustified. In the cases considered in this section, obligatory exh with such a revised notion of relevance might make the same predictions as pex. However, by capturing an analogous distinction in terms of presupposed/non-at-issue vs. assertive content, we in addition predict specific projection patterns for SIs that help solve the puzzles concerning negation and ‘some under some’ sentences. In contrast, this approach based on relevance doesn’t directly make any predictions about the projection behaviour of SIs, and thus doesn’t provide a uniform solution to our three puzzles.
In simple dialogues like (63), A can understand B’s assertion as implicating that ‘not all students passed’—without thereby holding that the common ground itself entails that. Yet many argue that, in typical cases, the presuppositions of utterances must be entailed by the common ground. Indeed, standard tests for presupposition-hood such as the ‘Hey Wait A Minute’ test (HWAM!; Shanon 1976, von Fintel 2004) fail for such ordinary SIs, as suggested by the oddness of A’s reply in (64c):

(64)  
   a. A: Did any students pass?  
   b. B: Some students did.  
   c. A: #Hey wait a minute! I didn’t know that not all of them did!

However, there are many cases in which presuppositions don’t have to be strictly entailed by the common ground and can be easily (globally) accommodated. One notable case, due to Shanon (1976), is the prejacent of only, which is widely taken to be presupposed rather than asserted (as this helps explain, among other things, its projection in cases like not only John was at the party). Consider the dialogue (65): B’s response in (65b) is perfectly felicitous even if it is not part of the common ground that John was at the party.38

(65)  
   C: A doesn’t know whether John or anyone other than B was at the party  
   a. A: Who went to the party?  
   b. B: Only John was there!

Another famous example, illustrated in (66), concerns the presuppositions of possessives:

(66)  
   C: A and B both believe that A doesn’t know whether B has a sister  
   a. A: Why are you leaving?  
   b. B: I have to pick up my sister at the airport.

As we said earlier, even in these cases there are substantive constraints on global accommodation. One is the inspiration for our revised oddness filter: accommodated content should not be too controversial, i.e., should not be inconsistent with the common ground (Karttunen 1974, Lewis 1979, von

---

38 We thank Kai von Fintel (p.c.) for the observation about only and for pointing out its relevance to this discussion.
Presuppositional exhaustification

Fintel 2008, Del Pinal 2021, a.o.). Another condition is that, given the target assertion and common ground, interlocutors should be able to figure out what specific proposition is the potential candidate for accommodation. Both conditions are satisfied by the replies in (65b), (66b), and also by ordinary cases of scalar enrichments such as B’s reply in (63b). Interestingly, whenever global accommodation can be naturally employed, HWAM! responses tend to sound a bit weird:

(67)  a. A: Who went to the party?  
       b. B: Only John was there.  
       c. A: ??Hey wait a minute! I didn’t know John was there!

(68)  a. A: Why are you leaving?  
       b. B: I have to pick up my sister at the airport.  
       c. A: ??Hey wait a minute! I didn’t know you had a sister!

We conclude, then, that there is a class of expressions which trigger presuppositions which tend to be automatically globally accommodated when their content is consistent but not entailed by the common ground. pex and various other overt exhaustifiers arguably belong to this class.

Skeptics may feel that this can’t be the whole story. Although the HWAM! responses in (67) and (68) are a bit odd, they are perhaps more acceptable than the HWAM! response in (64). This raises several questions. Why do some easy-to-accommodate presuppositions work better at HWAM!-style tests than typical SIs? What distinguishes, relative to their behavior in HWAM! tests, presupposed SIs from other presuppositions that tend to be systematically accommodated? To properly address this worry, we would need a more complete account of the class of non-at-issue content we are after, explaining why it projects as it does, and how it relates to global HWAM-style tests. We leave this as an open challenge, but sketch below a tentative line of response.

A key difference between pex and standard presupposition triggers is that the SIs triggered by pex are subject to Relevance. Compared to standard triggers (e.g. again, knows, only, possessives), pex is less deterministic concerning what information, if any, is presupposed since only relevant formal alternatives are subject to exhaustification (cf. (2a) and (1c)). Consider a case like (63). Suppose A is reluctant to accommodate the (potential) ¬∀ presupposition of B’s assertion. Instead of voicing a HWAM!-style response, A has another option, namely, assume that the ∀-alternative of B’s utterance is not relevant. Restricting the set of relevant alternatives so as to exclude a
problematic presupposition is, arguably, less disruptive to the flow of conversation than explicitly challenging it with a 'HWAM!' response. Insofar as interlocutors try to minimize the disruption of information exchange, the HWAM response will thus be dispreferred. Crucially, there is no parallel move available for sentences with triggers like \textit{again}, \textit{only}, and possessives. Their presuppositions are entrenched and cannot be dismissed as irrelevant: if a hearer refuses to accommodate them, they have little choice but to halt the conversation and protest.

This preliminary account of the difference between \texttt{pex} and other presupposition triggers in HWAM!-style environments makes some interesting predictions, which we think are borne out. We said that, in plain cases like (64), HWAM!-style responses are odd because, due to relevance, the targeted SI is optional. To test this, consider parallel dialogues, such as (69), in which the SI targeted by the HWAM-response is obligatory. The prediction is that in such cases, the HWAM! response should improve in acceptability:

(69) \hspace{1cm} C: Normal background information.
    a. A: How was your trip across Europe?
    b. B: (I learned that) #/?? Some Italians come from a beautiful country!
    c. A: Hey wait a minute! I didn't know that Italians don't all come from the same country.

To be clear, our prediction is not that each line in this dialogue is natural; it is only that the HWAM! response improves in acceptability compared to its counterpart in plain cases such as (64). We think this prediction is correct, but further empirical work is needed to corroborate this.

To be sure, constructing examples to adequately test this prediction is not easy. For example, a reviewer pointed out to us that there is another kind of variant of (64), illustrated in (70), which also ensures that the $\forall$-alternative is relevant, and yet the parallel HWAM! response feels just as inadequate:

(70) \hspace{1cm} a. A: Did all students pass?
    b. B: Some students did.
    c. A: #Hey wait a minute! I didn't know that not all of them did!

In (70), A's question practically guarantees that the $\forall$-alternative is relevant and that A doesn't know if it holds. In addition, there is clearly a reading of (70) in which B's response in (70b) implicates that not all students passed,
Presuppositional exhaustification

which in our account is modelled as a reading in which the \( \neg \forall \) information is presupposed (and may thus be accommodated by A).\(^{39}\) It would then seem to follow that A’s HWAM! response should be felicitous, contrary to fact. However, there is an independent confound which explains why, in this kind of case, the HWAM! response is infelicitous. Namely, that A’s HWAM! response is pointlessly redundant: for the information that A doesn’t know that not all students passed was already conveyed by A’s previous question *Did all students pass?* Indeed, a similar effect can be observed in parallel cases with standard presupposition triggers:

(71)

\begin{enumerate}
  \item A: Did John pass?
  \item B: I just discovered/found out that he did!
  \item A: #Hey wait a minute! I didn’t know that John passed!
\end{enumerate}

In (71), B’s response triggers an informative, non-trivial presupposition. Yet just as in cases like (70), A can’t felicitously follow up with a HWAM! response due to its obvious and pointless redundancy relative to the information conveyed by A’s original question.

To conclude this section, we should clarify what precise commitments we are incurring in treating the SIs triggered by \( \text{pex} \) as ‘presuppositions’. Our suggestion can be summarized as the claim that there is a strong family resemblance between SIs and standard presuppositions, albeit an imperfect one:

(i) SIs project from certain embedded environments in the ways expected of standard presuppositions. In addition, they can be locally accom-

\(^{39}\) The assumption that A can accommodate the presupposition in B’s response in dialogues like (70) is not in tension with the observation that in certain sequences of assertions (esp., by a single speaker), ignorance seems to block the use of accommodation. Here’s a characteristic example:

(i) D: It’s possible that John passed. #The fact that he passed is amazing.

Why don’t we just accommodate the presupposition of D’s second assertion? Contextualized in the way that supports the oddness judgment, it is part of the common ground that D did not get any relevant new information in the time between D’s two assertions. As a result, the first assertion (usually exhaustified relative to ‘possible’) conveys that D doesn’t fully believe that John passed, and the second one that D believes, at the same time/relative to the same information state, that John passed. In contrast, the dialogue in (70) doesn’t convey any information that is in tension with the attribution that B believes the \( \neg \forall \) proposition, which is why we can accommodate that presupposition of B’s assertion, without thereby representing B as an incoherent agent.
modated under similar licensing conditions as standard presuppositions. This is the lesson of the *some-under-some* and the exhaustification under negation and other DE operators puzzles discussed in Sections 2–4.

(ii) When SIs are obligatorily triggered, they can’t be globally accommodated (and the corresponding utterances are judged infelicitous) if they are inconsistent with the common ground. This parallels the constraints on global accommodation exhibited by other kinds of presupposition triggers. This is the lesson of the Magri-style oddness cases discussed in Section 5.

(iii) Interpretations enriched with SIs are felicitous in contexts where the SIs are compatible with yet not entailed by the common ground. In this respect, SIs are closer to non-presupposed, non-at-issue content, such as parenthetics, than to standard cases of presupposed, non-at-issue content (yet as we saw above, some presupposition triggers such as overt exhaustifiers and some possessives generate presuppositions that also tend to be systematically accommodated).

Focusing just on (ii)–(iii), one may be tempted to adopt a version of the prejacent is the assertive content and the excluded alternatives are simply non-at-issue (as opposed to strictly presupposed). Yet (i)—i.e., the projection puzzles—suggest that the species of non-at-issue content we are dealing with is that of standard presuppositions. Clearly, (i) and (iii) pull in different directions within the taxonomy of non-at-issue content. Still, we modeled SIs as closer to presuppositional content than to non-presuppositional, non-at-issue content for two reasons: first, we do not yet know of any theoretical implementations of the latter that predicts the observed projection behavior, and secondly, we argued in this section that there is a subclass of fairly standard presupposition triggers that behave like the prejacent is the assertive content and the excluded alternatives are simply non-at-issue (as opposed to strictly presupposed). Having said that, given a formal model of non-at-issue, non-presupposed content that captures the target projection behavior of SIs, one could re-formulate the prejacent is the assertive content and the excluded alternatives are simply non-at-issue (as opposed to strictly presupposed). From our perspective, this would amount to a welcome version of the main hypothesis we are advancing in this paper.
Presuppositional exhaustification

7 Summary and outlook

To summarize, our proposal to treat the Scalar Implicatures (SIs) triggered by a grammatical exhaustivity operator as having non-at-issue — specifically presuppositional — status, helps explain two kinds of puzzling observations about the behavior of SIs. The first concerns the projection behavior of embedded SIs, including why SIs tend to be absent under negation (and in other downward entailing environments) yet arise under intonation patterns like those used to make standard presuppositions at-issue under negation, and why they project in the intricate ways they do in ‘some under some’ sentences. The second puzzle concerns why SIs are computed in cases in which the resulting enriched readings (but not the non-enriched literal readings) clash with the common ground and yet (unlike parallel sentences that make the SIs explicit) can’t in general be used by speakers to get listeners to radically revise their hypotheses about the common ground. By replacing the flat standard exhaustification operator with \( \text{pex} \) — a presupposition trigger with respect to any negated (relevant) alternatives of the prejacent — we have the key component of a uniform account of these properties of SIs. For each of our puzzles, there are piecemeal patches or stipulations that can refine the predictions of standard grammatical accounts of \( \text{exh} \), but none of them seems to work as a uniform solution to our puzzles. In contrast, once we adopt \( \text{pex} \), we need only appeal to general and independently motivated principles concerning presupposition projection and limits on the global accommodation of presupposition.

An important question is how our proposal relates to the triggering problem for presuppositions and in general with current accounts of the division between assertive and other kinds of content. The triggering problem is the question of how we can predict which aspects of the meaning of a lexical item are presupposed. As far as we know, this is still an open problem (Abrusán 2011, Tonhauser et al. 2013, Tieu, Schlenker & Chemla 2019). But note that our \( \text{pex} \) is less problematic for approaches to the triggering problem inspired by Schlenker’s (2007) work than standard \( \text{exh} \) is. If the presupposition-assertion division is ignored, our \( \text{pex} \), the standard \( \text{exh} \), and the standard lexical entry for \( \text{only} \) are all equivalent. Schlenker develops a pragmatic analysis that implies that a lexical item \( \alpha \) with a conjunctive meaning such as \( p \land q \) can only occur if either \( p \) or \( q \) is presupposed. Both \( \text{only} \) and \( \text{pex} \) are consistent with Schlenker’s proposal, but \( \text{exh} \) is not: \( \text{only} \) presupposes its prejacent and asserts exhaustivity; \( \text{pex} \) asserts the prejacent and presupposes exhaustivity;
and \texttt{exh} doesn't have any presuppositions and asserts both the prejacent and the negation of any excludable alternatives. Schlenker's (2007) intuition that a contribution must not lead to multiple novel inferences or answer different questions remains one of the leading intuitions guiding current approaches. From this theoretical perspective, the hypothesis that \texttt{pex} is a presupposition trigger is not surprising. Indeed, we speculate that the difference between \texttt{only} and \texttt{pex} might in fact be a useful test case for theorizing on the typology of inferences.

\section*{References}


Presuppositional exhaustification


Velleman, Dan, David Beaver, Emilie Destruel, Dylan Bumford, Edgar Onea & Liz Coppock. 2012. It-clefts are it (inquiry terminating) constructions. *Semantics and Linguistic Theory (SALT)* 22. 441–460. [https://doi.org/10.3765/salt.v22i0.2640](https://doi.org/10.3765/salt.v22i0.2640).

Winter, Yoad. 2019. On presupposition projection with trivalent connectives. *Semantics and Linguistic Theory (SALT)* 29. 582–608. [https://doi.org/10.3765/salt.v29i0.4644](https://doi.org/10.3765/salt.v29i0.4644).

---

Itai Bassi
ZAS
Schützenstraße 18
10117 Berlin, Germany
bassi@leibniz-zas.de

Guillermo Del Pinal
E305 South College,
150 Hicks Way
Amherst, MA 01003
gdelpinal@umass.edu

Uli Sauerland
ZAS
Schützenstraße 18
10117 Berlin, Germany
sauerland@leibniz-zas.de