Semantics & Pragmatics Volume 15, Article 12: 1–20, 2022
https://doi.org/10.3765/sp.15.12

Scorekeeping in a chess game*

Bryan Pickel
University of Glasgow

Brian Rabern
University of Edinburgh

Submitted 2022-02-24 / First decision 2022-05-25 / Revision received 2022-06-06 / Accepted 2022-07-27 / Final version received 2022-08-02 / Published 2022-09-08 / Final typesetting 2023-12-13

Abstract There is an important analogy between languages and games. Just as a scoresheet records features of the evolution of a game to determine the effect of a move in that game, a conversational score records features of the evolution of a conversation to determine the effect of the linguistic moves that speakers make. Chess is particularly interesting for the study of conversational dynamics because it has language-like notations, and so serves as a simplified study in how the effect of an assertion depends on, as well as evolves, the scoreboard. In this paper, we offer a compositional semantics for chess notation and a simple formal picture for determining the full information conveyed by an entry. We will also discuss an alternative model resembling accounts of centered assertion.

Keywords: compositionality, common ground, semantic value, context, chess

1 Introduction

There is an important analogy, emphasized by Lewis (1979b), between languages and games. The appropriateness of and effect of a move in a particular game depends on the *score*, i.e. the various features of prior states of the game. For example, in chess, the possibility of moving a knight to a given location depends on the prior position of the chessboard, and the possibility of castling or capturing *en passant* depends on whether and how certain pieces have been moved. Likewise for moves in a language game: the appropriateness of and information conveyed by the utterance of a sentence in a

* Thanks to Wolfgang Schwarz, Mahrad Almotahari, Mike Ridge, Indrek Reiland, Beri Marušić, Elin McCready, and the anonymous referees. Dedicated to the memory of Landon Rabern and his chess engine Betsy.

©2022 Bryan Pickel and Brian Rabern
This is an open-access article distributed under the terms of a Creative Commons Attribution License (https://creativecommons.org/licenses/by/3.0/).

conversation depends on the “conversational score”, i.e. the various features of the context. So just as a scoresheet records features of the evolution of a game to determine the effect of a move in that game, a conversational score records features of the evolution of a conversation to determine the effect of the linguistic moves that speakers make.[1]

Comparisons between features of chess and features of language are ubiquitous in the history of philosophy and linguistics (e.g. Wittgenstein 1953: Section 31; Dummett 1959: p. 142; or de Saussure 1916: p. 88). However, chess is particularly interesting for the study of conversational dynamics because it has language-like notations. For example, the first move in a game might be recorded on the scoresheet as follows: 1.Nf3 Nf6. On this line the first entry (i.e. ‘Nf3’) describes White’s play (knight moves to the third row of column f) and the second (‘Nf6’) describes Black’s corresponding play. A sequence of entries preceding a given move can be used as a scoreboard to determine the legitimacy and effect of the move, and a complete scoresheet describes an entire game.

At first glance, chess notation seems to share a feature commonly ascribed to natural language. Both would appear to be *compositional*. In natural language, the meaning of a complex expression or sentence is held to be derivable from the meanings of its components and their arrangement.[2] This is meant to explain the fact that speakers can understand many novel sentences and the fact that their understanding is systematic. Thus, someone who understands the natural language sentence ‘the knight that took a bishop moved’, can also understand sentences that recombine its expressions such as ‘the bishop that took a knight moved’. Similarly with chess notation, someone who understands ‘Nf3’ and ‘Bb2’ can also understand ‘Bf3’ and ‘Nb2’.

As with sentences of natural language, the total assertoric effect of entries in chess notation depend on the *conversational* scoreboard. For example, an utterance of ‘a white car was moved’ does not specify the owner of the moved car. But the two sentence sequence ‘Every white car belongs to Mary. A white car was moved’, updates the conversational scoreboard with the information that a car belonging to Mary was moved. In the case of chess notation, some entries do not completely specify the move they record without background information regarding the prior position of the board. This background information is provided by the previous entries on the score-

1 This idea that utterances don’t just depend on the context but also alter the context is emphasized by Isard (1975) and Stalnaker (1970, 1973, 1978).

2 See Pagin & Westerståhl 2010 for standard formal definitions of compositionality.

card. Thus, chess notation provides a simplified study in how the effect of an assertion depends on, as well as evolves, the conversational scoreboard.

Szabó (2000) has argued that this dependence of the effect of an entry on the external factors is evidence that chess notation — specifically algebraic chess notation — is non-compositional, since the move recorded by an entry is not determinable as a function of the meanings of the syntactic components of that entry and their arrangement (cf. Szabó 2020: Section 4.1). It thereby provides a model for ways in which *compositionality* might fail. Szabó (2000: pp. 73–80) argues specifically that chess notation shows that standard arguments for the compositionality of natural language fall short. These standard arguments insist that compositionality explains the fact that humans can understand a large number of novel sentences and the fact that this understanding is systematic. Algebraic chess notation, says Szabó, is systematic and we can understand novel entries without having previously seen them. Yet, the notation, he alleges, is not compositional.

Szabó argues that the move recorded by an entry is not recoverable from the lexical meanings of the components alone but also depends on the state of the board when the move is taken. He suggests that in order to deliver a compositional semantics, one must embrace one of two implausible strategies. The first strategy is that the meaning of the constituent expressions ('N', 'f', '3') of an entry ('Nf3') must themselves depend on the board state.[3] The second strategy is to allow that the meaning of an entry fails to fully determine the move. Szabó rejects the first approach on the ground that it overly complicates the lexical semantics and he rejects the second approach on the ground that it "widens the gap between meanings of expressions and the information conveyed by their utterance" (Section 4.1).

We insist that there is a gap between the meaning of an entry and the information it conveys in a context. Nonetheless, there are tight constraints between the two because a sequence of entries must fully determine the sequence of moves in the chess game it records. Moreover, this result can easily be delivered by assuming basic and common pragmatic principles that take us from the meaning of an entry to its assertoric effect. In this paper, we offer a compositional semantics for chess notation and a simple formal picture — derived from Stalnaker's pragmatics of assertion — for determining

3 For example, a piece term like 'N' could be construed as an indexical expression, which in a context (a game and move depth) picks out the set of knights of the contextually relevant color in the game of the context. In this way the meaning of an entry in context could be compositionally derived from the meaning of its constituents in context. This would be straightforward to implement in a standard Kaplan-style framework (see Kaplan 1989), but we agree with Szabó that this seems somewhat unnatural for this case.

the full information conveyed by an entry. We will also discuss an alternative model resembling accounts of centered assertion, and briefly comment on the not-at-issue and expressive content of chess notation.

2 Algebraic chess notation

A chessboard is an 8×8 matrix. The rows (*ranks*) are labeled with numerals $1-8$. And the columns (*files*) are labeled with lower-case letters $a-h$. So a rank and a file together specify one of the sixty-four squares in the matrix, for example the White queen standardly starts at $d1$—the 1^{st} rank on the d file. The pieces are represented by upper case letters: 'K' for king, 'Q' for queen, 'R' for rook, 'B' for bishop, and 'N' for knight. Pawns are sometimes represented by 'P' but more standardly the lack of a piece symbol indicates that a pawn is involved.[4]

The chess notation is then used to represent updates to the board. In most cases this is done by specifying a piece and where it moved to.[5] For example, an opening move might consist of White moving a knight from $g1$ to $f3$ followed by Black moving a knight from $g8$ to $f6$. This would result in the board state as in Figure 1.

There are also two atomic symbols for the special move of castling, namely 'o-o' for kingside (with rook $h1$ or rook $h8$) and 'o-o-o' queenside (with rook $a1$ or rook $a8$). There is also notation for indicating when a capture took place ('x') and whether it was *en passant*, when a pawn was promoted, or when check ('+') or checkmate ('#') results. For now we will just focus on the core part of the notation which represents where a piece moves to (and perhaps where it came from). While accommodating the further complications wouldn't be too difficult, they would also distract from our main points.[6]

4 There are variations on the notation that are also allowed. For example, in some countries alternative letters are sometimes used, for example in Germany one might use 'S' for a knight ('Springer'), or in France one might use 'F' for a bishop ('fou'). Figurines are also common such as '♘', so that the notation for White's move would be '♘f3'. We will follow the standard notation as prescribed by the International Chess Federation (FIDE 2018).

5 In chess terminology, one full *move* consists of both White's turn and Black's turn. Each turn is called a "half-move" (or a "ply" especially in connection with the depth of a computer's analysis in a game tree). But "move" is often used to describe half-moves as well as full moves depending on the context. We won't fuss too much over the strict terminology, but will be careful when its important and context doesn't disambiguate.

6 In fact, FIDE 2018 indicates that the notation for capture, check/checkmate, and *en passant* are all optional and needn't be included on the official scoresheet.

1.Nf3 Nf6

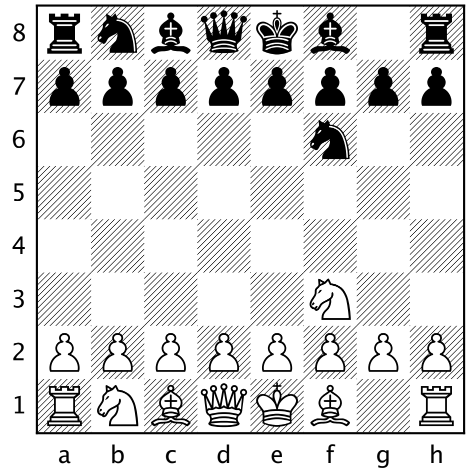

Figure 1 A standard opening

3 The case against compositionality

Szabó (2000, 2020) argues that algebraic notation is not compositional. The consideration motivating him is that the annotator communicates the chess game to the reader. That is, from a correct annotation of a game, the reader can reconstruct the game itself move by move.

> Someone who understands the Algebraic notation must be able to follow descriptions of particular chess games in it and someone who can do that must be able to tell which move is represented by particular lines within such a description.
>
> (Szabó 2020: Section 4.1)

Thus, Szabó assumes that the information communicated by an entry completely specifies the corresponding move. He then argues that the move represented by certain entries is not compositionally determined by the meanings of their syntactic constituents. In particular, Szabó points to entries that omit the square of origin, such as 'Nd7'. According to Szabó, knowing the meaning of 'N', 'd', and '7' is insufficient to know the move recorded. In particular, the entry compositionally determines that a knight moved to the square at the 7^{th} rank of the d file. But this leaves open important information such as what color the knight was, which particular knight it was, and where it departed from. Since the entry 'Nd7' represents the move, Szabó insists that the representational content of 'Nd7' is not compositionally determined.

As we have mentioned, Szabó (2000, 2020) draws a fairly radical conclusion from the case of chess notation. He takes it to show that the standard arguments for compositionality — namely the argument from novelty and the argument from systematicity — fail. In particular, Szabó thinks he has exhibited a case where the meaning of a composite expression is not determined by the meanings of its parts and their arrangement, but where speakers can readily understand novel expressions.

We take a different lesson to follow from the case. What it shows is only that the meaning of an entry falls short of fully specifying the relevant move. So the situation is no immediate threat to compositionality or the arguments for it. Instead the compositional semantics needs to be supplemented by a pragmatic story. Szabó insists that this response comes at the price of an unfortunate gap between the meaning of an entry and the information conveyed by it. But once we see how easily the gap is bridged, we think one should readily pay this price. In fact, we think the case of chess notation serves as a good model for the standard interactions between compositional semantics and pragmatics. We will develop a semantics that derives the meaning of an entry on a scorecard from the meanings of its components and an off-the-shelf picture of the dynamics of assertion that combines with the semantic value of an entry to determine the total move.

In developing our picture, we take it that an account of chess notation should satisfy the following *desiderata*, which seem to motivate Szabó.

> Desideratum 1: A correctly annotated game record (a "scorecard") uniquely characterizes a chess game.
>
> Desideratum 2: A correct entry in a game record combined with the previous correctly annotated game record uniquely characterizes which piece moved where.

To fully satisfy these desiderata we appeal to two pragmatic components that go beyond the compositional semantics. First, the reader of the scorecard has more information available than merely the entry. They also have the information provided by the prior sequence of entries and common knowledge about the initial position and rules of a chess game. This information is essentially the *common ground* of the conversation. The common ground is what the writer and reader of a scorecard mutually take for granted for the purposes of the conversation at any given stage. For example, assume that the prior sequence of moves leading up to the board state in Figure 2 — and the fact that it is Black's move — are common ground.

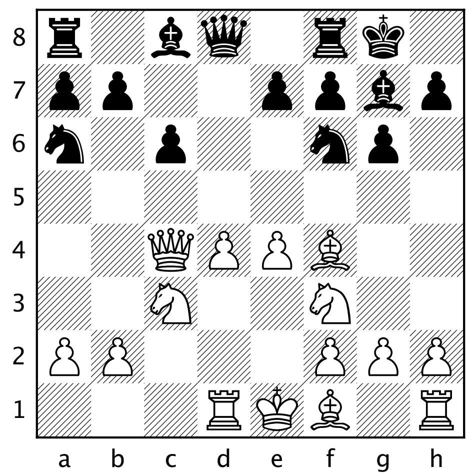

Figure 2 A game stage

If the next entry is 'Nd7' then the meaning of that entry plus the information in the common ground will determine that the Black knight at $f6$ moves to $d7$. We will offer a compositional semantics for algebraic chess notation according to which the entry 'Nd7' alone does not determine which knight moved to $d7$. However, this semantics will be paired with a pragmatic story that keeps track of how each successive entry on a scorecard updates the common ground. In a correctly annotated scorecard for a game leading up to Figure 2, the entry 'Nd7' together with this common ground will completely determine the relevant move because there is a single chess piece capable of making this move. This semantic and pragmatic story will completely parallel standard accounts of the effect of assertions of natural language sentences. It is therefore sufficient to rebut Szabó's charge that there is a cost to bridging the semantic value of an entry and the move recorded. Therefore, there is a plausible compositional semantics for chess notation.

Although the semantic story will rebut Szabó's specific worry that the reader of a scorecard must know the game history in addition to the entry in order to determine a move, there is a lingering complication. Specifically, a game history and an entry together do not always fully determine a move. For example, sometimes there is more than one piece — that share a color and a type — that can move to a given square. Consider the board state in Figure 3. In this case two Black knights are able to reach $d7$, so an entry such as 'Nd7', even supplemented with the game history, doesn't settle where the knight comes from, and so doesn't settle what transition takes place.

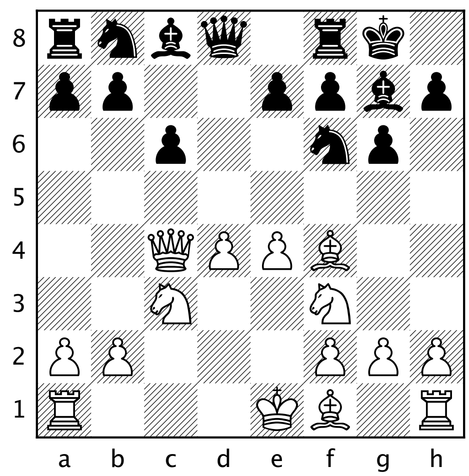

Figure 3 Two black knights can access $d7$

The entry would be a true description of either transition but it doesn't uniquely determine a transition. This is a problem because a reader should be able to reconstruct a game from its correctly annotated scorecard. If an entry leaves open which knight moves where, then this seems impossible.

The second pragmatic component of our picture concerns *correct* annotation. Correctly annotating a scorecard requires more than truth. It also requires the requisite quantity of information — essentially Grice's first maxim of quantity (Grice 1975). Thus, an entry is a correct annotation *only if* the entry together with the common ground determine a unique move. This additional requirement — that the entry and common ground together determine the move — is a standard rule for chess annotation.[7]

7 According to FIDE 2018 in a case where multiple pieces of the same color can move to the same square (e.g. such as in Figure 3) the start position of the moving piece must also be indicated, for example, by citing file ('Nbd7'), rank ('N6d7'), or full file+rank ('Nf6d7'). We will ignore the partial cases (i.e. just file or just rank) for ease of exposition.

So, on the picture we develop, the compositionally derived semantic value of a correct entry does not determine the move being made. At least, it doesn't do so by itself. However, supplemented with the common ground, there will be only one move compatible with an entry in a correctly annotated game. Specifically, the rule for adding the content of an entry in a scorecard to the common ground established by the previous description and knowledge of the original state does uniquely characterize which piece moved where. We will provide a compositional semantics, specify bridge principles for content, and provide the formal update rules.

4 The syntax and compositional semantics of chess

In this section, we provide a syntax and semantics for the language of chess. The syntax is straightforward. The basic vocabulary contains five pronounced piece terms and an unpronounced piece term '$\varnothing$' for pawn.[8] The basic vocabulary also include rank and file terms.

- Piece terms: K, Q, B, N, R, $\varnothing$
- File terms: a, b, c ,d, e, f, g, h
- Rank terms: 1, 2, 3, 4, 5, 6, 7, 8

We offer three formation rules. One rule (p) derives an expression for a square from a term for a file and a rank. Another rule (r) derives an expression for a move from a term for a piece and a term for a square. A final rule (r') derives an expression for a move from a term for a piece and two terms for squares.

p: Take a file term α and a rank term n to a square term $p(\alpha, n)$

r: Take a piece term Γ and square term σ to an entry $r(\Gamma, \sigma)$

r': Take a piece term Γ and two square terms σ_1 and σ_2 to an entry $r'(\Gamma, \sigma_1, \sigma_2)$

For example, the entry 'Nd7' derives from r applied to a term for a piece 'N' and a term for a square 'd7'. The term for a square 'd7' derives from applying the rule p to the term for a file 'd' and rank '7'.

We now offer a semantics. An entry on a scorecard can be evaluated for truth relative to a game (or game history) and a particular stage of that game.

8 This piece term could be eliminated in favor of a non-branching phrase structure rule.

At each stage of a game the board will be in a particular state, i.e. a particular arrangement of pieces on the board. Each square in a board state can be occupied by one of the six types of White pieces, {♙, ♘, ♗, ♖, ♕, ♔}, one of the six types of Black pieces, {♟, ♞, ♝, ♜, ♛, ♚}, or it can be empty, $\otimes$.[9] So each square can be assigned one of these 13 different elements:

$$D = \{\text{♙}, \text{♘}, \text{♗}, \text{♖}, \text{♕}, \text{♔}, \otimes, \text{♟}, \text{♞}, \text{♝}, \text{♜}, \text{♛}, \text{♚}\}$$

A board state is an 8×8 matrix whose elements are drawn from D. Each element of a matrix will be notated using a number n, $1-8$, paired with letter α, $a-h$. In order to correspond to ranks and files on a chessboard we will assume that row 1 is at the bottom, and row 8 at the top, whereas column a is leftmost and h is rightmost. The occupant of row n and column α of matrix $\mathfrak{s}$ will be written $\mathfrak{s}[n, \alpha]$ (or $\mathfrak{s}[z]$, if z is an ordered pair of a row and a file). Compare the board state given by the matrix on the left with the starting position $\mathfrak{s}_0$ of a standard game (in Figure 4), and note that $\mathfrak{s}_0[1, a] =$ ♖.

$$\mathfrak{s}_0 = \begin{bmatrix} \text{♜} & \text{♞} & \text{♝} & \text{♛} & \text{♚} & \text{♝} & \text{♞} & \text{♜} \\ \text{♟} & \text{♟} & \text{♟} & \text{♟} & \text{♟} & \text{♟} & \text{♟} & \text{♟} \\ \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes \\ \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes \\ \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes \\ \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes & \otimes \\ \text{♙} & \text{♙} & \text{♙} & \text{♙} & \text{♙} & \text{♙} & \text{♙} & \text{♙} \\ \text{♖} & \text{♘} & \text{♗} & \text{♕} & \text{♔} & \text{♗} & \text{♘} & \text{♖} \end{bmatrix}$$

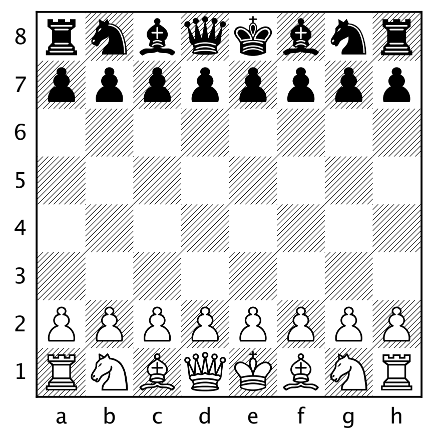

Figure 4 Modeling board states

9 This is essentially following Shannon's (1950) model of a board state (Section 5): "A square on a chessboard can be occupied in 13 different ways: either it is empty (0) or occupied by one of the six possible kinds of White pieces (P=1, N=2, B=3, R=4, Q=5, K=6) or one of the six possible Black pieces (P=-1, N=-2, ..., K=-6). Thus, the state of a square is specified by giving an integer from -6 to +6. The 64 squares can be numbered according to a co-ordinate system…The position of all pieces is then given by a sequence of 64 numbers each lying between -6 and +6."

A game history $\mathfrak{g}$ is a sequence of board states: $\mathfrak{g} = (\mathfrak{g}_0, \mathfrak{g}_1, \mathfrak{g}_2 \dots, \mathfrak{g}_n, \dots)$. A move-depth $m \in \mathbb{N}$ (or ply) is the number of board states after the initial position. So, $\mathfrak{g}_m$ is the board state in the m^{th} position of the game $\mathfrak{g}$.

The semantics will be relativized to a game $\mathfrak{g}$ and move m. That is, an entry such as 'Nd7' or 'Nf6d7' may be true in one game at one stage but false in a different game or at a different stage. Whether the piece moved is white or black will depend on the active color, which is determined by whether the move is odd or even. Thus, the semantics for piece expressions must be as follows.

$$[\![\mathrm{K}]\!]^{\mathfrak{g},m} = \begin{cases} ♔, \text{if } m \text{ is odd,} \\ ♚, \text{if } m \text{ is even.} \end{cases} \qquad [\![\mathrm{N}]\!]^{\mathfrak{g},m} = \begin{cases} ♘, \text{if } m \text{ is odd,} \\ ♞, \text{if } m \text{ is even.} \end{cases}$$

$$[\![\mathrm{Q}]\!]^{\mathfrak{g},m} = \begin{cases} ♕, \text{if } m \text{ is odd,} \\ ♛, \text{if } m \text{ is even.} \end{cases} \qquad [\![\mathrm{R}]\!]^{\mathfrak{g},m} = \begin{cases} ♖, \text{if } m \text{ is odd,} \\ ♜, \text{if } m \text{ is even.} \end{cases}$$

$$[\![\mathrm{B}]\!]^{\mathfrak{g},m} = \begin{cases} ♗, \text{if } m \text{ is odd,} \\ ♝, \text{if } m \text{ is even.} \end{cases} \qquad [\![\varnothing]\!]^{\mathfrak{g},m} = \begin{cases} ♙, \text{if } m \text{ is odd,} \\ ♟, \text{if } m \text{ is even.} \end{cases}$$

The semantics for square terms derive straightforwardly from the semantics for rank and file terms. Specifically, if α is a file term, then $[\![\alpha]\!]^{\mathfrak{g},m}$ = the α column of $\mathfrak{g}_m$. If n is a rank term, then $[\![n]\!]^{\mathfrak{g},m}$ = the n row of $\mathfrak{g}_m$. And, if $\sigma = p(\alpha, n)$ is a square term, then $[\![\sigma]\!]^{\mathfrak{g},m} = \langle [\![n]\!]^{\mathfrak{g},m}, [\![\alpha]\!]^{\mathfrak{g},m} \rangle$.

The semantics for entries is also straightforward. An entry such as 'Nf6d7' should indicate that there was a knight on $f6$ at the prior board state, that there is no longer a knight at $f6$, and that there is a knight on $d7$ at the present board state. Similarly, 'Nd7' indicates that at the previous board state there was a knight on some position, that there is no longer a knight in that position, and that a knight is at $d7$ of the present board state. This will be derived using the following rules.

If $\phi = r(\Gamma, \sigma)$ is an entry, then $[\![\phi]\!]^{\mathfrak{g},m} = 1$ iff

(i) $[\![\Gamma]\!]^{\mathfrak{g},m} = \mathfrak{g}_m[[\![\sigma]\!]^{\mathfrak{g},m}]$, and

(ii) $[\![\Gamma]\!]^{\mathfrak{g},m} = \mathfrak{g}_{m-1}[z]$, and

(iii) $\mathfrak{g}_m[z] = \otimes$, for some square z.

If $\phi = r'(\Gamma, \sigma_1, \sigma_2)$ is an entry, then $[\![\phi]\!]^{\mathfrak{g},m} = 1$ iff

(i) $[\![\Gamma]\!]^{\mathfrak{g},m} = \mathfrak{g}_m[[\![\sigma_2]\!]^{\mathfrak{g},m}]$, and

(ii) $[\![\Gamma]\!]^{\mathfrak{g},m} = \mathfrak{g}_{m-1}[[\![\sigma_1]\!]^{\mathfrak{g},m}]$, and

(iii) $\mathfrak{g}_m[[\![\sigma_1]\!]^{\mathfrak{g},m}] = \otimes$.

An entry will be written in a context which determines a game and a move. An entry will be true in a context c if and only if it is true at the game and the move of the context (i.e. $\mathfrak{g}_c$ and m_c).

Truth: An entry ϕ is true in context c if and only if $[\![\phi]\!]^{\mathfrak{g}_c,m_c} = 1$

It will be observed that — even fixing m — this semantics does not fully determine the move described by an entry such as 'Nd7'. This entry tells us only that some (let us say Black) knight moves to $d7$ from a position that it no longer occupies. The prior position of the knight is left open.

5 The pragmatic effect of an assertion

Recall Desideratum 1 and Desideratum 2, a correctly annotated scorecard should uniquely determine a game and the information contained on a scorecard up to a given entry plus the information given by the entry should completely determine which piece was moved where. However, we've just seen that an entry on a scorecard on its own does not tell us which piece was moved where, even given knowledge of the active color. The question is, can we explain how the information expressed by one entry combined with the information expressed by the preceding entries determine which piece moved where? Ideally, this information should follow from absolutely minimal principles of pragmatics because the knowledge of which piece moved where is fairly automatic and straightforwardly deduced.

Stalnaker's pragmatic picture of the effect of an assertion on what is mutually presupposed in a conversation is ideally situated to explain how this additional information is conveyed (see, e.g., Stalnaker 1970, 1973, 1978). For Stalnaker the *common ground* of a conversation is the set of propositions mutually taken for granted as background information. Propositions for Stalnaker are sets of possible worlds, and the intersection of the common ground is the *context set* — the set of worlds that are open possibilities in the conversation. An assertion adds a proposition, i.e. the assertoric content of the sentence uttered, to the common ground, and thereby removes worlds from the context set (assuming the assertion was informative).

In scoring a chess game, a player is trying to specify a unique chess game. That is, they are trying to specify the unique sequence of board positions that constituted that game. Therefore, the most natural correlate of a possible world in this semantics is a game history. An agent filling out a scorecard seeks to characterize a game up to uniqueness by the sequence of entries constituting the scorecard. To properly read the scorecard, the audience must know some background facts. They must know the initial position of the chessboard. We labelled the standard starting position as $\mathfrak{s}_0$. Thus, for standard chess the context set must be a subset of the set of games whose initial position is $\mathfrak{s}_0$, or $\{\mathfrak{g} : \mathfrak{g}_0 = \mathfrak{s}_0\}$. To understand the entries, the audience must also know the rules for movement of the pieces.

In effect, the common ground determines a *game tree* for chess – a standard device in game theory, initially formalized and put to use by Zermelo (1913).[10] In a game tree the nodes are board states and the directed edges are moves. A maximal chain in a tree, a branch, is a complete game.[11] An update will exclude branches from this tree. A complete scorecard should narrow the tree to a single branch.

We have characterized entries in a scorecard as true or false relative to a game and a move-depth. Given that a context set is a set of games, each entry in the register must update the set by excluding those games incompatible with its *game content*. The game content of an entry in a scorecard at a move-depth is the set of games compatible with the truth of the entry at the move-depth of the context (m_c). That is:

The *game content* of ϕ in $c = \{\mathfrak{g} : [\![\phi]\!]^{\mathfrak{g},m_c} = 1\}$.

The assertion of an entry restricts the context set to those games compatible with the entry's game content.

Game update rule:

Context set S updated with ϕ in $c = S \bigcap \{\mathfrak{g} : [\![\phi]\!]^{\mathfrak{g},m_c} = 1\}$.

10 See Osborne & Rubinstein 1994 for standard definitions.

11 The longest possible chess game is finite assuming standard terminating rules and has a move-depth over 10,000, so the number of possible chess games, i.e. number of terminal nodes in the standard chess game tree, is very large. Shannon (1950) famously estimated that the number is at least 10^{120} — that was only for a depth of 80, so it is actually much larger than that.

Given that the scorecard is properly filled out, information contained in the entry together with the context set should completely determine which move is made.

For example, consider a scorecard whose first entry is 'Na3'. This entry restricts the set of games $\mathfrak{g}$ such that there is a White knight on square $a3$ at board state $\mathfrak{g}_1$ and for some square z, there was a White knight on z at state $\mathfrak{g}_0$ but not at state $\mathfrak{g}_1$. The context set includes the fact that $\mathfrak{g}_0 = \mathfrak{s}_0$ and the fact that only the knight on $b1$ is capable of moving to $a3$. Thus, the player filling out the scorecard can exploit the context set to shorten the entries. An update by 'Na3' in this context restricts the context set exactly as would an update by the entry 'Nb1a3'.

Thus, Szabó is correct, that "staring at" an entry such as 'Na3' will not determine which piece moved where. But given a simple picture of conversational dynamics, the entry itself plus the context set will determine which piece moved where. So there is no reason to regard chess notation as non-compositional. The semantic value of an entry is determined by the semantic values of its constituents, and then the information conveyed by an entry in a scorecard is completely determined by its semantic value, the prior entries in the scorecard, and the rules of chess.[12]

6 Scorekeeping a chess puzzle

Chess is a game of perfect information. But a scorekeeper may not have that perfect information. A scorekeeper may chance upon a game without knowing how many moves in it is, but observe the board state and the active color. They may then fill out a scorecard from that point on. Similarly, chess puzzles (i.e. both so-called "studies" and "problems") often provide merely a board state and an active color.[13] But players may use algebraic notation to record the solution.

12 Notice that one can use chess notation to play chess purely via language. This is what happens with chess *sans voir*, "blindfold chess", or chess on horseback. Each utterance updates the common ground in the way we have specified.

13 Standardly, "problems" are distinguished from "studies". See for example, Nunn (2002: p. xi): "Basically, a chess problem is a composed position together with a target which must be achieved in a specified number of moves (e.g. mate in two, selfmate in three, etc.). There should be a unique solution achieving the target and it is the solver's task to uncover this solution, which is usually well hidden. A study is again a composed position, but in this case the objective is either to win or to draw, without limit on the number of moves." Our points here apply to both sorts of compositions.

For example, consider Paul Morphy's famous mate in two problem (see Figure 5).

White to move & Mate in 2

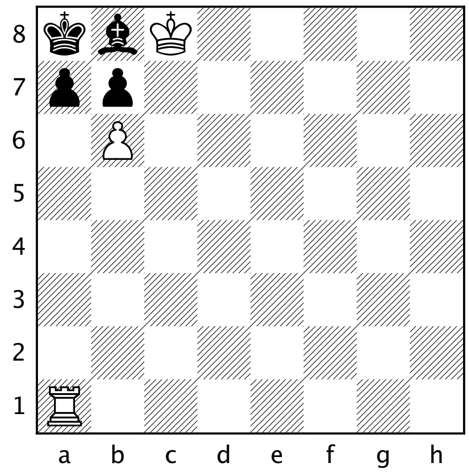

Figure 5 A chess puzzle

A solution to this problem would be recorded as '1.Ra6…'.[14] In these situations, the problem solver or scorekeeper may not know the history, and in particular they may not know the move-depth. Therefore, they may not be in a position to determine the *game content* of an entry, which is the set of games that are compatible with the truth of that entry at that move-depth. For instance, the game content of 'Ra6' in the tenth position of the scorecard in a normal game is the set of games where a rook occupies $a6$ at state $\mathfrak{g}_{10}$. If 'Ra6' occurs in a scorecard but we do not know the move-depth, all that can be determined by the entry alone is that *at some point* a rook (of the active color) moved to $a6$.

Of course, some information about the past history of a game can be figured out from the current board state. And some problems rely essentially on retrograde analysis (cf. Smullyan 1979). For example, it might be provable that the rook or king had to have previously moved, so that it can be determined that castling is thereby not a legal move. Or it might be provable that a given pawn must have arrived at its location in way allowing for *en passant*

14 Or more explicitly: 1.Ra6 bxa6 2.b7# or 1.Ra6 Bσ 2.Rxa7#.

capture, and so on. But, in general, a board state could have been accessed in many different ways, at many different move distances from the initial position. Therefore, it is desirable to generalize our account to allow for the possibility that one can score a chess game without knowing the history.[15]

To accommodate these contexts it is natural to generalize Stalnaker's account of assertion to allow cases where we are ignorant of both the game and the move-depth, just as we might generalize the notion of the context set from a set of worlds to a set of world-time pairs, or centered worlds. The context set will be a set of pairs of games and move numbers compatible with the background information. In other words, the context set can be construed as a set of nodes in a game tree. Rather than restricting the context set by the *game content*, an entry will instead restrict the context set by the *game-stage content* of an entry, which is the set of pairs of games and move numbers compatible with the truth of that entry.

$$\text{The } \textit{game-stage content} \text{ of entry } \phi \text{ in } c = \{(\mathfrak{g}, m) : [\![\phi]\!]^{\mathfrak{g},m} = 1\}$$

An entry ϕ in context set S does two things. First, it advances the move. Second, it carries information about the board state at that next move. We therefore suggest the following rule:

Game-stage update rule:

$$\text{Context set } S \text{ updated with } \phi = \{(\mathfrak{g}, m) : (\mathfrak{g}, m-1) \in S \wedge [\![\phi]\!]^{\mathfrak{g},m} = 1\}.$$

Including pairs of games and moves in the content of an entry obviously bears some analogy to accounts of centered content, according to which the content of a sentence is a pair of a possible world and some other parameter such as a time, place, or person (cf. Lewis 1979a, Ninan 2010). But because move position in the scorecard evolves according to regimented rules, it also bears analogy to dynamic views according to which

15 In the case of chess studies, the set-up will often convey more information than the current board state. For example, there are *ad hoc* conventions regarding the acceptability of *en passant* and castling. Importantly, for our purposes, these conventions do not determine the specific move depth or full game history. Moreover, in the case of retrograde puzzles, where algebraic chess notation is also used, these assumptions may be explicitly lifted or the conventions may even interact in interesting ways. For example, in some cases the exact history of a board position remains underdetermined, and different solutions are required for the alternative possible histories. Thanks to a referee from *Semantics and Pragmatics* for raising this point.

some discourse parameter evolves over the course of a conversation (see Groenendijk & Stokhof 1991). For example, Heim (1982) proposed that when an indefinite description such as ‘a knight’ is used, it adds an entity to a discourse parameter — these are available antecedents for subsequent anaphoric pronouns and definites. Analogously, each entry in a chess scorecard systematically advances the move-depth. In fact, one could fold up our static story into a dynamic package, if one preferred (see Rothschild & Yalcin 2017).

7 Conclusion

We take this paper to have shown that it is plausible to construct a compositional semantics for chess notation that explains the total information conveyed by an entry in a chess card in terms of standard conversational dynamics. An assertion ‘Nd7’ affirms that a knight is now on the square $d7$, but it does not uniquely determine where the knight previously was. Instead, this information will fall out of the content of the assertion together with the common ground, if the game has been correctly recorded. This information is conveyed just as the entire current board position follows from the previous entries on the scorecard. As a result, Szabó’s example of algebraic notation gives little evidence that human beings can systematically understand non-compositional notational systems. Szabó had provided another example of arithmetical decimal notation, which Dever (2003) has argued has a natural compositional interpretation. For this reason, we take it that the ability of speakers to understand many novel sentences and their ability to do so systematically still does provide some evidence for the thesis that natural languages are compositional.

The appearance of a failure of compositionality is due entirely to the fact that the move recorded by an entry in a scorecard depends on both the semantic meaning of an entry and on the conversational scoreboard. Contrary to Szabó, we take algebraic chess notation to be a good example of the standard division of labor between semantics and pragmatics. In particular, the total informational effect of an entry depends both on its semantic value and on the common ground established by mutual acceptance of the rules of the game, the initial position, and the previous entries on the scorecard. This is precisely analogous to the fact that the total information conveyed by an assertion depends both on the semantic value of the sentence asserted and on the common ground. Algebraic chess notation therefore provides a simplified model of the evolution of the information content of a discourse. It also provides a model of how the informational common ground can evolve even when relativized beyond a possible world (or game) parameter which might be extended to model *de se* communication.

The semantics we have offered covers the majority of the descriptive dimensions of chess notation. It can easily be extended to cover idioms such as the notation for castling. Interestingly, chess notation also arguably contains symbols for not-at-issue and expressive content. For example, in addition to notation for a move additional symbols can be added to convey that the move is a capture or that it results in check. Since these symbols are optional and the information they carry already follows from the facts about which pieces moved where, these seem more like asides or appositives (e.g., 'Nxg5' seems to be saying a knight moves to $g5$, *a square occupied by an opponent piece*). In addition to the descriptive language used to record chess games the notation also often includes symbols for evaluative comments such as "?" for a bad move and "!" for a good move. A full account of chess notation could therefore provide a model for the not-at-issue and expressive content of an assertion.

References

Dever, Josh. 2003. Book review: Problems of compositionality. *Philosophical Review* 112(2). 254–258. https://doi.org/10.1215/00318108-112-2-254.

Dummett, Michael. 1959. Truth. *Proceedings of the Aristotelian Society* 59(1). 141–62. https://doi.org/10.1093/aristotelian/59.1.141.

FIDE. 2018. Laws of chess. In *Fédération internationale des échecs (FIDE) handbook*. Chapter E.01. Accessed: 2021-09-May. https://handbook.fide.com/.

Grice, Paul. 1975. Logic and conversation. In Peter Cole & Jerry Morgan (eds.), *Speech acts* (Syntax and Semantics 3), 41–58. New York: Academic Press. https://doi.org//10.1163/9789004368811_003.

Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100. https://doi.org/10.1007/BF00628304.

Heim, Irene. 1982. *The semantics of definite and indefinite noun phrases*. Amherst, MA: University of Massachusetts dissertation. Republished as Heim 2011. https://scholarworks.umass.edu/dissertations/AAI8229562.

Heim, Irene. 2011. *The semantics of definite and indefinite noun phrases*. https://semanticsarchive.net/Archive/jA2YTJmN/Heim%20Dissertation%

20with%20Hyperlinks.pdf. Republished version of Heim 1982 typeset by Anders Shoubye and Ephraim Glick.

Isard, Stephen. 1975. Changing the context. In Edward L. Keenan (ed.), *Formal semantics of natural language*, 287–296. Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9780511897696.019.

Kaplan, David. 1989. Demonstratives. In Joseph Almog, John Perry & Howard Wettstein (eds.), *Themes from Kaplan*, 481–563. Oxford: Oxford University Press.

Lewis, David. 1979a. Attitudes *de dicto* and *de se*. *The Philosophical Review* 88(4). 513–543. https://doi.org/10.2307/2184843.

Lewis, David. 1979b. Scorekeeping in a language game. *Journal of Philosophical Logic* 8(1). 339–359. https://doi.org/10.1007/BF00258436.

Ninan, Dilip. 2010. *De se* attitudes: Ascription and communication. *Philosophy Compass* 5(7). 551–567. https://doi.org/10.1111/j.1747-9991.2010.00290.x.

Nunn, John. 2002. *Solving in style*. London: Gambit Publications.

Osborne, Martin J. & Ariel Rubinstein. 1994. *A course in game theory*. Cambridge, MA: MIT Press.

Pagin, Peter & Dag Westerståhl. 2010. Compositionality: Definitions and variants. *Philosophy Compass* (3). 250–264. https://doi.org/10.1111/j.1747-9991.2009.00228.x.

Rothschild, Daniel & Seth Yalcin. 2017. On the dynamics of conversation. *Noûs* 51(2). 24–48. https://doi.org/10.1111/nous.12121.

de Saussure, Ferdinand. 1916. *Cours de linguistique générale*. Lausanne: Payot. Published in English as *Course in general linguistics* (Translated by Wade Baskin) in 1959 by Philosophical Library.

Shannon, Claude E. 1950. Programming a computer for playing chess. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 41(314). 256–275. https://doi.org/10.1080/14786445008521796.

Smullyan, Raymond M. 1979. *The chess mysteries of Sherlock Holmes*. New York: Alfred A. Knopf.

Stalnaker, Robert. 1970. Pragmatics. *Synthese* 22(1-2). 272–289. https://doi.org/10.1007/BF00413603.

Stalnaker, Robert. 1973. Presuppositions. *Journal of Philosophical Logic* 2(4). 447–457. https://doi.org/10.1007/BF00262951.

Stalnaker, Robert. 1978. Assertion. In Peter Cole (ed.), *Pragmatics* (Syntax and Semantics 9), 315–332. New York: Academic Press. https://doi.org/10.1163/9789004368873_013.

Szabó, Zoltán Gendler. 2000. *Problems of compositionality*. New York: Routledge. https://doi.org/10.4324/9781315054438.

Szabó, Zoltán Gendler. 2020. Compositionality. In Edward Zalta & Uri Nodelman (eds.), *The Stanford encyclopedia of philosophy* (Fall 2020 Edition). https://plato.stanford.edu/archives/fall2020/entries/compositionality/.

Wittgenstein, Ludwig. 1953. *Philosophical investigations*. Oxford: Wiley-Blackwell.

Zermelo, Ernst. 1913. Über eine Anwendung der Mengenlehre auf die Theorie des Schachspiels. In Ernest William Hobson & Augustus Edward Hough Love (eds.), *Proceedings of the fifth international congress of mathematicians*, vol. 2, 501–504. Reprinted as Zermelo 2010.

Zermelo, Ernst. 2010. On an application of set theory to the theory of the game of chess. In Heinz-Dieter Ebbinghaus, Craig G. Fraser & Akihiro Kanamori (eds.). Trans. by Paul B. Larson, *Collected works/ Gesammelte Werke: Volume 1 – Set theory, miscellanea/ Mengenlehre, Varia* (Schriften der Mathematisch-naturwissenschaftlichen Klasse der Heidelberger Akademie der Wissenschaften 21). Berlin: Springer. https://doi.org/10.1007/978-3-540-79384-7_9.

Bryan Pickel
69 Oakfield Avenue, Glasgow
G12 8LP, United Kingdom
bryan.pickel@glasgow.ac.uk

Brian Rabern
40 George Square, Edinburgh
EH8 9JX, United Kingdom
brian.rabern@ed.ac.uk