Presupposition projection as a scope phenomenon*

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Abstract   The satisfaction theory of presupposition projection found in Heim 1983 has paved the way for a successful research tradition within dynamic semantics which has given rise to compositional analyses of a variety of projection behaviors. Since Geurts 1996, however, the promise of this research program has been called into question due to what Geurts dubs the "proviso problem": satisfaction theory generates incorrect predictions in cases in which a presupposition ends up filtered which should not have been. I show that the satisfaction account of presupposition projection is nevertheless in good shape by revealing that the observations of Geurts are valid only under certain basic assumptions about how semantic composition works. To illustrate this, I present a satisfaction account of presupposition projection that incorporates a notion of scope-taking based on *monads*. The resulting composition scheme provides a setting in which the proviso problem does not arise, thus lending support to the scope theory of presupposition projection.

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1 Introduction

Ever since Geurts 1996, the influential "satisfaction theory" of presupposition projection introduced by Heim (1983) has been known to suffer from certain descriptive inadequacies: specifically, it attributes weak, conditional presuppositions to many sentences which are felt by native speakers to have stronger, unconditional presuppositions.

(1) If Theo has a brother, he'll bring his wetsuit.
⤳ Theo has a wetsuit.

While (1) is usually judged to have the presupposition that Theo has a wetsuit, the account of Heim predicts it to have the conditional presupposition that Theo has a wetsuit *if he has a brother*, due to the semantics she attributes to conditional sentences. Geurts dubs this general difficulty for Heim, which pertains to a variety of operators, the "proviso problem".

The central aim of this paper is to explore the proviso problem, and, more generally, what I will call the problem of "trapped presupposition triggers", within a particular variant of the satisfaction account of Heim, but couched within a semantics that reifies undefinedness as a kind of semantic value, much like in trivalent logic accounts. My assumptions are largely faithful to the formulation of dynamic semantics of Heim and, more generally, the program of Stalnaker (1978) that regards utterance contexts as sets of worlds to be pruned by new assertions. Like Heim, I will regard a sentence's presuppositions to be given by its definedness conditions: in turn, a sentence S_1 presupposes a sentence S_2 iff the set of worlds at which S_1 is defined is a subset of the set at which S_2 is true.[1] My basic claim is that contemporary satisfaction accounts within the tradition of Heim suffer from the proviso problem because they do not fully exploit scope-taking strategies that the ambient compositional system has to offer. Once this point is recognized, it is fairly straightforward to upgrade the composition scheme so that it gives rise to the powerful notion of a *monad*. When semantic composition is viewed in terms of a monad, the proviso problem, as it is typically stated, does not arise.

More concretely, on the monadic view, presupposition projection arises from two aspects of interpretation acting in confluence. The first is that pre-

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[1] I will sidestep an analysis of anaphoric presuppositions in this paper. However, the proposed system is easily extended to countenance them; one may, for example, follow the approach of Charlow (2014) by using a State monad transformer.

supposition triggers may *take scope* above potential presupposition filters. This feature is obtained here by relying on an exceptional scope mechanism in the vein of Charlow 2020a,b; such a mechanism is defined in terms of monadic operators and can be used in tandem with roll-up pied-piping to achieve exceptional scope. The second feature is that the target of scope-taking may be identified with the environment in which a presupposition trigger's definedness conditions are *evaluated*. Thus if a presupposition trigger is contained within the scope of a presupposition filter, then it can escape the influence of the filter by taking scope above it.

One result of this new picture of presupposition is that sentences like (1) will be regarded as semantically ambiguous. (1), for example, will be taken *either* to presuppose that Theo has a wetsuit if he has a brother, *or* to presuppose that he has a wetsuit, depending on how the ambiguity is resolved. Indeed, this ambiguity will be regarded as a kind of scope ambiguity. From this point of view, the challenge of relating semantic presuppositions to the inferences which are actually observed is a problem of describing how pragmatic context drives the resolution of scope ambiguities.[2] To get a gist of the basic result, take (2), which can be understood with a conditional presupposition.

(2) If Theo is a scuba diver, he'll bring his wetsuit.
⤳ If Theo is a scuba diver, he has a wetsuit.

On the relevant reading, the presupposition of (2) has been weakened in comparison to (1) (as argued by Geurts). Thus on the scope theory, both (1) and (2) are semantically ambiguous. But while (1) is most easily understood with the noun phrase *his wetsuit* taking scope outside of the conditional, (2) can be understood so that it takes scope within the conditional's consequent clause.

The rest of this paper further motivates and develops this basic idea. Section 2 provides background on the satisfaction account and the common objections to it that go under the heading of the proviso problem. In Section 3, I present a version of the satisfaction account which involves a notion of scope-taking. While this version of the account shares its basic tenets with those falling within the framework initiated by Heim (1983), my proposal will ultimately incorporate some new (but minor) extensions, which are then recast in terms of a monadic semantics reminiscent of the one introduced in Charlow 2020b. This section also illustrates the extensions by analyzing presupposition projection out of conditionals, while Section 4 enriches the sys-

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2 Alongside the other factors (perhaps, relating to performance) which contribute to this task.

tem somewhat to analyze projection out of the clauses embedded by propositional attitude verbs. As will be seen throughout, the proviso problem and, more generally, the problem of trapped presupposition triggers does not arise in the new setting.

2 Satisfaction theory

Contemporary versions of the satisfaction theory are rooted in Karttunen 1974 and are broadly based on views about the pragmatics of discourse update found in Stalnaker 1973, 1974, and Stalnaker 1978. Karttunen identifies a variety of construction types that differ according to the conditions under which their presuppositions are satisfied. He gives the following characterization of the conditions under which a conditional sentence's presuppositions are satisfied, for example (here, 'X ∪ A' should be read as X ∪ {A}) (p. 185).

(3) Context X satisfies-the-presuppositions of "If A then B" just in case (i) X satisfies-the-presuppositions-of A, and (ii) X ∪ A satisfies-the-presuppositions-of B.

Karttunen defines a context as satisfying a sentence's presuppositions if it entails them. If, in (2), the pronoun *he* is taken to refer to Theo,

(2) If Theo is a scuba diver, he'll bring his wetsuit.

then the consequent of the conditional has the presupposition that Theo has a wetsuit. The principle stated in (3) determines that a context satisfies the presupposition of (2) if augmenting it with the sentence *Theo is a scuba diver* results in a new context that entails *Theo has a wetsuit*. As a result, any context satisfies the presuppositions of (2) if it entails *if Theo is a scuba diver, he has a wetsuit*. Such a context may, in particular, entail the stronger sentence *Theo has a wetsuit*.

   A notable feature of Karttunen's presentation is that it is agnostic about the presuppositions incurred by any given sentence. Karttunen merely describes the constraints that a linguistic context should satisfy in order to satisfy the presuppositions of an expression of a particular type. In general, the relevant constraints are meant not to determine what an expression's presuppositions *are*, but, rather, whether or not its presuppositions are *satisfied*, given some prior discourse. Such constraints therefore yield a notion of possible prior discourse: given the presuppositions of the atomic sentences

an expression contains, an utterance of the expression is predicted to be felicitous in some discourse contexts and not others. For instance, Karttunen's theory predicts that an utterance of (2) is felicitous in a context entailing that Theo has a wetsuit, as well as contexts entailing that he has a purple wetsuit, that he has a wetsuit and an airplane, or (weakly) that he has a wetsuit if he is a scuba diver. But Karttunen does not supplement his proposal by the principle that such constraints are themselves constitutive of an expression's presuppositions, that is, what is accommodated upon an utterance of the expression.

If one wishes to characterize accommodation, however, an account (such as Karttunen's) that remains agnostic about what is accommodated upon the utterance of a sentence with presuppositions incurs a loss. Indeed, a successful characterization of accommodation is needed to account for the empirical data for which semanticists generally hold their theories responsible. Such data are typically provided by linguistic diagnostic tests, including family-of-sentence tests (Chierchia & McConnell-Ginet 1990: Ch. 1, Section 3.2 and Ch. 6, Section 3.1).[3]

Thus the satisfaction account is supplemented in Heim 1983 in order to provide not only an account of the conditions which presuppositions impose on prior discourse, but also a hypothesis about the identity of the presuppositions themselves: Heim identifies an expression's presuppositions with the constraints it imposes. Her statement (p. 399, ex. 12) of what I will refer to as the 'Accommodation Principle' is as follows:[4]

(4)   Heim 1983: S presupposes p iff all contexts that admit S entail p.

For example, what it means for (2) to presuppose that if Theo is a scuba diver, he has a wetsuit is for it to be admissible in only the contexts which entail that if Theo is a scuba diver, he has a wetsuit. By identifying a sentence's presuppositions with the contexts in which the sentence is admissible, Heim is able to deliver an account of the presuppositions of complex sentences by stating how they give rise to local contexts for the sentences they contain. The Heimian rule for interpreting a sentence coordinated by *and*, for example, is the following one, where *c* is the context the coordinated sentence updates.

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3 And others; for example, the "Hey, wait a minute!" test of von Fintel (2004).

4 See also Geurts 1996: p. 264, ex. 4, where the principle is restated and given as a defining property of the satisfaction theory.

(5) $$c + [\![\varphi_r \; and \; \psi_s]\!] = c + [\![\varphi_r]\!] + [\![\psi_s]\!]$$

Idealizing a bit, let us take φ and ψ to be the at-issue contributions of φ_r and ψ_s, respectively, and r and s to be the sentences they presuppose. Given some context c, which is to be updated by the sentence $\varphi_r \; and \; \psi_s$, c must satisfy the presupposition r of φ_r, while $c + [\![\varphi_r]\!]$ must satisfy the presupposition s of ψ_s, in order for the update to be successful. Since Heim identifies the sentence's presupposition with thes constraints, we may determine the presupposition of φ_r and ψ_s to be $r \wedge (\varphi \to s)$. In this way, the Accommodation Principle in (4) serves to link an account of semantic definedness (delivered compositionally) to an account of accommodation.

2.1 The proviso problem

Let us consider the sentence in (1) again, in light of the satisfaction account of Heim.

(1) If Theo has a brother, he'll bring his wetsuit.

We would like to be able to analyze this sentence as presupposing that Theo has a wetsuit; indeed, that is what one tends to accommodate when this sentence is uttered by one's interlocutor. On a satisfaction account supplemented by the Accommodation Principle, we therefore require that (1) be defined if and only if Theo has a wetsuit. The proviso problem, as argued by Geurts (1996), is that on Heim's satisfaction account, the sentence is analyzed as having the weaker presupposition that Theo has a wetsuit if he has a brother. A context can be updated with (1) if and only if, if Theo has a brother in that context, then he has a wetsuit in that context. And so by the accommodation principle, Heim's account predicts that (1) presupposes that Theo has a wetsuit if he has a brother, in apparent conflict with the observation that an utterance of (1) produces the inference that Theo, in fact, has a wetsuit.

As Francez (2019) points out, the claim that satisfaction accounts make wrong predictions about the presuppositions of conditional sentences (among other kinds of sentence) only pertains to accounts incorporating something like the Accommodation Principle in (4). Lacking such a principle, a satisfaction account would only classify the contexts that a sentence can successfully update; that is, without further characterizing the inferences one tends to make when such a sentence is uttered. Indeed, Karttunen would predict

that (1) is acceptable in only the contexts entailing that if Theo has a brother, he has a wetsuit. This prediction seems plausible: if it is taken for granted in the context of discourse merely that Theo has a wetsuit if he has a brother, (1) should be acceptable;[5] conversely, it seems that at least this much need be taken for granted. Thus Karttunen's account remains sound.

It is tempting to adopt an analogous stance toward the satisfaction account of Heim (1983) (and others which follow it). One might argue in the following way: since the account of Heim merely characterizes the admittance conditions on sentences with presuppositions — that is, in terms of the contexts whose update by such sentences is defined — it should be off the hook for the same reason as is Karttunen's account. (1) is expected to be admissible in a context in which Theo has a wetsuit if he has a brother. (Perhaps, in such a context, wetsuits are in high demand, due to the fact that the only way to successfully procure one is through one's brother.) The account of Heim predicts that, given such a context, nothing is accommodated upon an utterance of (1). Such a prediction seems intuitively correct. Meanwhile, nothing in Heim's account causes it to predict that utterances of (1) should be less felicitous in stronger contexts entailing that Theo has a wetsuit. So, no false prediction is made.

On further scrutiny, such an argument appears less secure. In particular, one of its premises is false: the account of Heim does *not* merely characterize the admittance conditions imposed on sentences with presuppositions. It also characterizes the presuppositions which are *inferred* upon utterances of such sentences. It does this by the accommodation principle in (4). In other words, besides describing constraints on discourse update in terms of admittance conditions (following Karttunen), Heim characterizes, in (4), a relation on *sentences*: the presupposition relation.

Meanwhile, despite the observation that utterances of (1) may be acceptable in contexts entailing the weak conditional presupposition, one tends to accommodate the stronger unconditional inference when such a rich context is unavailable. Why would that be? Since Karttunen's satisfaction account does not treat accommodation, it is in the clear. The innovation provided by the Accommodation Principle is that it characterizes presupposition as a type of inference, on a par with entailment and implicature: one sentence may presuppose another. The principle thus provides a hypothesis about the link between the relational characterization of presupposition and the defined-

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5 Thanks to Simon Charlow (p.c.) for pointing this out.

ness conditions of sentences. It is in this setting that the proviso problem crops up.

The proviso problem is well worth overcoming, as the Accommodation Principle allows semantic accounts of presupposition to ground themselves in characterizations of inference. In principle, an account without the principle may provide a useful framework within which to analyze the relation between sentences and the contexts they are able to update; these contexts, in general, amount to the entire histories of discourses, along with whatever relevant extra-linguistic events may occur surrounding them. But presuppositional inference (as revealed by the common linguistic diagnostics) is itself a rich and patterned phenomenon which a theory of meaning should be compelled to account for. My goal is to demonstrate that an account of presupposition projection in the vein of Heim 1983, but which is not susceptible to the proviso problem, is available with some minor extensions to the assumptions that semanticists normally make.

Finally, it is worth noting that problematic predictions would likely subsist in certain cases, even if contemporary satisfaction accounts abandoned the Accommodation Principle. In the analysis of propositional attitude verbs of Heim (1992), a verb of belief is predicted to trigger the presupposition that the individual denoted by its subject believes the presuppositions of its complement. Consider Heim's own example (p. 209, ex. 71) as an illustration.

(6)   John: I am already in bed.
      Mary: My parents think I am also in bed.

Mary's utterance is predicted, on Heim's account, to presuppose that her parents think that John is in bed, due to the presence of the presupposition trigger *also* in the scope of the propositional attitude verb *think*. But, in fact, her utterance appears only to presuppose that John is in bed: note that (6) itself has no lingering presuppositions and is, in particular, consistent with Mary's parents' ignorance of the truth of John's claim. Heim discusses (6) as a potential counterexample to the predictions of the satisfaction account. Thus a sentence whose main verb is *think* ought only be able to update contexts entailing a proposition about someone's beliefs, but such a proposition, as (6) shows, need not be entailed. Importantly, the problem arises independently of one's views about the Accommodation Principle; even lacking such a principle, Heim's account makes incorrect predictions about the kinds of contexts an utterance like Mary's can update. Since there is no entailment from the constraint on contexts that Mary's utterance appears to impose in

(6) (i.e., that John is in bed) to the constraint on contexts that it is hypothesized to impose by the satisfaction account (i.e., that Mary's parents believe John is in bed), Mary's utterance is not easily written off as meeting the required conditions of update. In contrast, a context in which an unconditional inference is true is also a context in which a corresponding conditional inference is true. The utterance of a sentence hypothesized to have conditional definedness conditions in a context that entails an unconditional proposition is thus accurately described by an account such as that of, say, Karttunen.

Examples such as (6) suggest that the original observations of Geurts characterize instances of a more general pattern. The pattern is that, in many cases, the definedness conditions imposed by presupposition triggers in embedded contexts project past presupposition filters (e.g., conditionals and propositional attitude verbs) which scope over these contexts. The inability of traditional accounts within the framework of satisfaction theory to account for this general pattern is the problem of trapped presupposition triggers: the presuppositions of these triggers are evaluated *locally* within the traditional satisfaction theory. Moreover, as (6) shows, the general problem persists regardless of the Accommodation Principle. Against this background, syntactic alternatives to the satisfaction account, for example, couched within discourse representation theory (Kamp 1981), may appear attractive. Indeed, the account of van der Sandt (1992) is designed to liberate presupposition triggers from their local contexts. More recently, Maier (2015) has shown how DRT may be used to characterize complex patterns of presupposition projection out of clauses embedded by propositional attitude verbs. My aim is to show that a similar kind of flexibility may be achieved within the satisfaction theory.

2.2 Ambiguity and the selection of presuppositions

Since Geurts 1996, satisfaction theoretic accounts of presupposition projection have often attempted to supplement the semantic analysis of projection with extra mechanisms, in order to account for the inferences which are actually observed, given some utterance. According to such accounts, the accommodated presupposition is determined by a property of the context in which the utterance is made, together with properties of the utterance itself. Singh (2007) thus characterizes accounts of the process of accommodation as, first, determining a hypothesis space of possible accommodations for a given utterance, and, second, determining the role that a given con-

text of usage plays in selecting from among the possible accommodations the presupposition which is, in fact, observed. Schlenker (2011) (summarizing Singh 2007[6]) calls the problem of determining such a hypothesis space the "strengthening problem" and that of selecting an accommodated presupposition from this space the "selection problem". In other terms, one might regard the first problem as that of determining an appropriate semantics for utterances — one which associates them, potentially, with more than one interpretation which, together, constitute a hypothesis space. One may then regard the second problem as that of resolving ambiguity, that is, by selecting an interpretation from this space. In the present case, ambiguous utterances are associated with meanings which differ only in their presuppositions, and the task of accommodation is that of choosing the correct meaning on some occasion of utterance.

Singh (2007) stipulates a representational component of the grammar that syntactically reifies the notation of Heim (1983): given an utterance context, represented as c, a sentence S is translated into some update to c, $c[\![S]\!]$ ($[\![S]\!]$ being the translation of S into some chosen logical language). This translation of English sentences is defined so that, for example, the conjunction S_1 *and* S_2 is rendered $c[\![S_1]\!][\![S_2]\!]$, and a hypothesis space of possible accommodations is determined from this translation in a manner which Singh elaborates. If r is the presupposition of S_1, for example, and s is the presupposition of S_2, the hypotheses space of possible accommodations is the set $\{r \wedge [\![S_1]\!] \rightarrow s, r \wedge s\}$; that is, either the presupposition usually delivered by the satisfaction theory, or the stronger presupposition $r \wedge s$. Thus Singh provides an account of the strengthening problem. It is up to extra-linguistic factors to determine how any one of the delivered alternatives is chosen; pragmatic mechanisms (along the lines explored by Beaver (1999), for example) may play a role.

Other accounts that focus on the strengthening problem, including, for example, Singh 2009, as well as Schlenker 2011, appear to recognize the complexity of the factors affecting accommodation in any given linguistic context, given the possibilities hypothesized to be available. Many such accounts aim to remain flexible in the face of this complexity by casting a wide net, over-generating. It is hoped that, by doing so, one may later discard the unwanted surplus by offering an answer to the selection problem. Any answer,

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6 And Singh 2008, 2009.

however, is likely to be highly sensitive to the speaker's communicative goals and, thus, pragmatically complicated.[7]

This paper likewise takes an approach oriented around flexibility: its strategy is to characterize the variation in accommodations observed across contexts by over-generating readings, which are then to be reined in in the context of a fuller account of the pragmatics of accommodation. My approach is relatively conservative, however, in that its flexibility arises purely from the compositional semantics of presupposition triggers and their containing expressions. It will become apparent that only a few minor changes to the compositional repertoire are necessary, in order to account for a range of projection behaviors. From this vantage point, fully characterizing accommodation amounts to accounting for the resolution of semantic ambiguity, which, more generally, may be influenced by any number of factors of a non-semantic origin.

That said, it may nonetheless be controversial whether or not an ambiguity-based account could provide a satisfactory solution to the problems observed by Geurts. Mandelkern (2016b) argues that pragmatic solutions according to which a sentence's semantic interpretation associates it with a conditional presupposition which is, in turn, strengthened into an unconditional presupposition make wrong predictions. He illustrates this with the following example (p. 396, ex. 14), giving the indicated judgment:[8]

(7)   ??John was limping earlier; I don't know why. Maybe he has a stress fracture. I don't know if he plays any sports, but if he has a stress fracture, then he'll stop running cross-country now.

According to Mandelkern, (7) is odd because it both asserts that the speaker has no knowledge of what sports John plays and presupposes that John runs cross-country (due to the trigger *stop*). Moreover, he argues, if the weaker conditional inference (expected by the satisfaction account) that John runs cross-country *if he has a stress fracture* were available, it should be preferred, in order to avoid this conflict of inferences. Apparently, (7) can only be understood with the stronger presupposition that John runs cross-country, causing it to sound strange.

Crucially, the type of account to which Mandelkern is responding takes there to be a conditional presupposition which arises from the semantics of

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7 Mayr & Romoli (2016) offer an account which aims to provide answers to both problems, as they pertain to conditional sentences.

8 See also Mandelkern 2016a for more similar challenges to such pragmatic accounts.

conditional sentences, together with a pragmatic mechanism which strengthens this conditional presupposition into an unconditional one. I propose not to adopt an account of this type, but rather one according to which a presupposition trigger in the consequent of a conditional sentence gives rise to a genuine semantic ambiguity. As for (7), I am committed to the view that there is a semantic interpretation of the sentence *if he has a stress fracture, then he'll stop running cross country now* on which it presupposes *if he has a stress fracture, he runs cross-country*, as well as an interpretation on which it presupposes *he runs cross-country*. No pragmatic inference to obtain the latter from the former is required.

Thus it should be demonstrated that a conditional presupposition is available for this sentence, in principle. Imagine that someone says (8).

(8) I saw John limping earlier. If he has a stress fracture, then I assume that he runs cross-country. Indeed, if he has a stress fracture, then he'll stop running cross-country now.

(8), it seems to me, is felicitous and, crucially, does *not* imply that John runs cross-country (note that it *can* be followed up with *but I don't know if he actually plays any sports*). Echoing an earlier observation, it is also possible to construct contexts in which (1) is understood with a conditional inference:

(9) We're going scuba diving later, and I don't know if Theo owns a wetsuit. But it seems that everyone who has a brother got a wetsuit for Christmas. So, if Theo has a brother, he'll bring his wetsuit.

The contexts provided in both (8) and (9) appear to increase the likelihood that a speaker presupposes the relevant conditional inference, compared to when the sentence with presuppositions is uttered in isolation; the effect is apparently sufficient to allow for a merely conditional presupposition to be understood. Still, it is natural to wonder why readings involving conditional presuppositions are difficult to obtain for these examples, in comparison to (2), where the conditional presupposition is relatively clear.

(2) If Theo is a scuba diver, he'll bring his wetsuit.

On the scope theory, the conditional reading arises from a narrow-scope construal of the presupposition trigger, while the unconditional reading arises from a wide-scope construal. Which reading is brought out may depend heavily on contextual factors relating to background knowledge, which may influ-
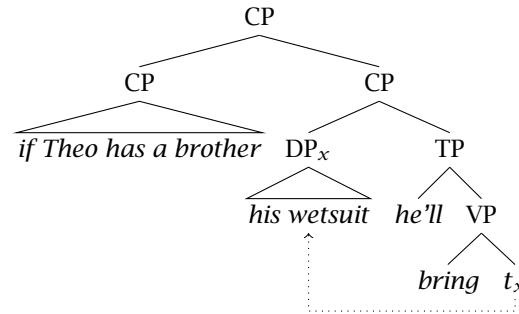
Figure 1 The covert syntax of (1): the local reading

ence which speaker presuppositions one considers to be plausible.[9] Or, there may also be a general bias toward wide-scope construals of presupposition triggers; that is, global accommodation (Heim 1983). All that said, it is not my purpose to determine which construals are available on which occasions; it should, however, be demonstrated that a reading predicted by the scope theory is available in *some* context.

3 The scope theory of presupposition projection

The main thrust of the scope theory of projection is that the presupposition of a given trigger projects to the constituent over which the trigger takes scope. This basic idea can be illustrated as follows. The presupposition of the trigger *his wetsuit* in (1)

(1) If Theo has a brother, he'll bring his wetsuit.

that is, *that Theo has a wetsuit*, can project either to the embedded clause in which the trigger originates, or to the conditional sentence itself. In case the trigger takes scope in the embedded clause, the presupposition projects locally, so that it is filtered by the conditional as a whole. The reading obtained from local projection of the presupposition can therefore be represented by the phrase marker in Figure 1. Accordingly, the scope theory associates this phrase marker with the reading of (1) whereon it presupposes that Theo has a wetsuit *if he has a brother*; that is, the unwanted weak reading. Of course, what we usually want in the case of (1) is an analysis according to which it presupposes simply that Theo has a wetsuit; that is, where the triggered pre-

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9 This point, as it relates to which presuppositions are accommodated, given an utterance, is expounded in Beaver 1999 and Beaver 2001: Section 9.3 (pp. 236–249).
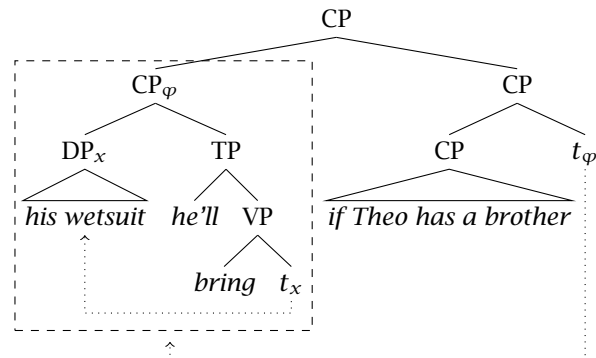
Figure 2 The covert syntax of (1): the global reading

supposition projects globally, past the conditional. To a first approximation, it may appear that the scope theory should obtain this reading by having the presupposition trigger *his brother* scope out of the consequent clause, up above the entire conditional sentence. Covert movement of quantifiers, however, is widely understood to generally be bounded by finite clauses; so, it would be preferable to have an alternative. To that end, we will follow Charlow (2020b), who invokes the strategy of *roll-up pied piping* (or "cyclic scope") in an analysis of the exceptional scope properties of indefinites. Rather than scope the exceptionally scoping element directly out of the clause in which it originates, we scope it to the edge of its containing clause, and then scope the clause *itself* to *its* containing clause's edge. Thus on the roll-up strategy, the presupposition trigger *his wetsuit* moves to the edge of the consequent clause, following which the consequent itself moves to the edge of the entire conditional. Figure 2 gives a schematic illustration. At no point, following this strategy, does anything escape the smallest finite clause in which it originates.

In the following pages, we will consider different interpretation schemes, all of which use the LFs illustrated in Figure 1 and Figure 2. All that will differ are the exact interpretation rules and lexical denotations involved. To start out, let us observe the semantic effect of presuppositional scope-taking, according to a simple interpretation scheme; namely, one using the rules of Functional Application and Predicate Abstraction from Heim & Kratzer 1998, coupled with a rule for interpreting traces.

(10) *Functional Application*   ([Heim & Kratzer 1998](): p. 105, ex. 13′)
      If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then for
      any assignment a, α is in the domain of $[\![\cdot]\!]^a$ if both β and γ are, and
      $[\![\beta]\!]^a$ is a function whose domain contains $[\![\gamma]\!]^a$. In that case, $[\![\alpha]\!]^a =$
      $[\![\beta]\!]^a([\![\gamma]\!]^a)$.

(11) *Predicate Abstraction*   ([Heim & Kratzer 1998](): p. 186, ex. 4)
      Let α be a branching node with daughters β and γ, where β dominates
      only a numerical index i. Then, for any variable assignment a, $[\![\alpha]\!]^a =$
      $\lambda x.[\![\gamma]\!]^{a[x/i]}$.

(12) *Traces and Pronouns Rule*   ([Heim & Kratzer 1998](): p. 111, ex. 9)
      If α is a pronoun or trace, a is a variable assignment, and $i \in dom(a)$
      then $[\![\alpha_i]\!]^a = a(i)$.

Note the caveat contained in the definition of Functional Application: that
the interpretation of a branching node is defined *if* the interpretations of
its daughters are. The import of this condition is that an expression inherits
the presuppositions of the expressions it contains, assuming we equate an
expression's presuppositions with its definedness conditions; if a contained
expression can't be interpreted (thus resulting in a presupposition failure),
then neither can the containing expression. Presupposition projection can
therefore be seen as an automatic side effect of applying Functional Appli-
cation.

Crucially, we wish to avoid presupposition projection in case we encount-
er a presupposition filter. To sidestep automatic projection past filters, let us,
for the moment, assume a kind of syncategorematic intepretation scheme.
Thus to complement the rules above, we may adopt the following rule in
order to interpret a material conditional.

(13) *Material Conditional Rule*
      Given a material conditional, $[[\textit{if } \varphi]\ \psi]$, and an assignment, a, if $[\![\varphi]\!]^a =$
      $\bot$, then $[\![[[\textit{if } \varphi]\ \psi]]\!]^a = \top$. If $[\![\varphi]\!]^a = \top$ and $[\![\psi]\!]^a$ is defined, then
      $[\![[[\textit{if } \varphi]\ \psi]]\!]^a = [\![\psi]\!]^a$. $[\![[[\textit{if } \varphi]\ \psi]]\!]^a$ is undefined either if $[\![\varphi]\!]^a$ is un-
      defined, or if $[\![\varphi]\!]^a = \top$ and $[\![\psi]\!]^a$ is undefined.

This rule makes a material conditional automatically true so long as its an-
tecedent is false. Thus presuppositions triggered in the consequent of the
conditional may only have an interpretive effect if the antecedent clause is

$$\begin{cases} \top & \mathsf{bro}(\mathsf{t}) \to \exists! x : \mathsf{suit}(x) \land \mathsf{have}(x)(\mathsf{t}); \mathsf{bring}(x)(\mathsf{t}) \\ \bot & \mathsf{bro}(\mathsf{t}) \land \exists! x : \mathsf{suit}(x) \land \mathsf{have}(x)(\mathsf{t}); \neg \mathsf{bring}(x)(\mathsf{t}) \\ \# & \mathsf{bro}(\mathsf{t}) \land \neg \exists! x : \mathsf{suit}(x) \land \mathsf{have}(x)(\mathsf{t}) \end{cases}$$

$\mathsf{bring}(\iota x : \mathsf{suit}(x) \land \mathsf{have}(x)(\mathsf{t}))(\mathsf{t})$

if   Theo has a brother
$\mathsf{bro}(\mathsf{t})$

his wetsuit
$\iota x : \mathsf{suit}(x) \land \mathsf{have}(x)(\mathsf{t})$

$\lambda x . \mathsf{bring}(x)(\mathsf{t})$

x *he'll bring t_x*
$\mathsf{bring}(g(t_x))(\mathsf{t})$
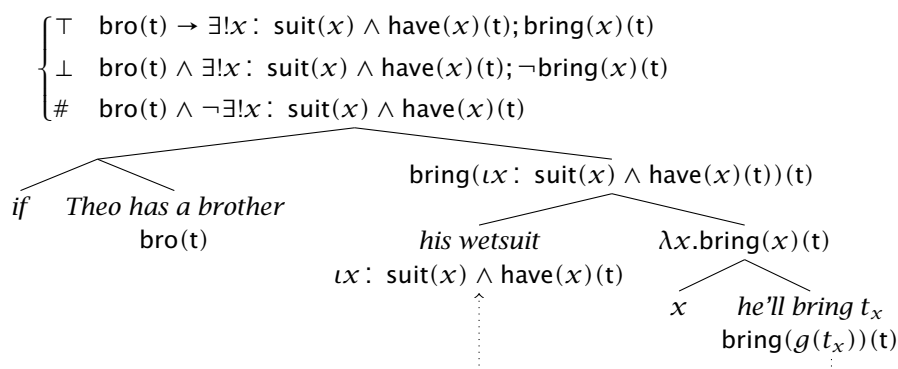
Figure 3   Interpreting (1) with Heim & Kratzer 1998: the local reading

true, in which case the interpretation of the consequent is calculated in the first place. The resulting projection properties for conditionals dictate that the interpretation of a conditional is defined just in case (a) the interpretation of its antecedent is defined, and (b) either its antecedent is false or the interpretation of its consequent is defined. The rule above thus gives rise to a pattern reminiscent of trivalent logics with "middle Kleene" semantics (as investigated in Peters 1979).[10] Meanwhile, it is a kind of extensional, static analog of the intensional, dynamic semantics for conditionals of Heim (1983), whose update rule has it that the semantic value of the consequent clause of a conditional is only considered at indices at which the antecedent clause is true.

The interpretation associated by $[\![\cdot]\!]^g$ with the LF of Figure 1 is derived in Figure 3 (where the interpretation of each node is provided relative to an arbitrary assignment g^{11}). For simplicity, anaphora are ignored, and the embedded subject pronoun of the consequent clause is interpreted as coreferential with *Theo*. Each step of the derivation invokes one of the rules provided above. Note that the presupposition trigger *his wetsuit* has raised out of its base-generated position, leaving behind the sentence with a free variable *he'll bring t_x*, which is interpreted by Predicate Abstraction. While this step ends up being semantically vacuous, it is included here, anyway, to foreshadow later examples. Most important to note here is the effect of the Material Conditional Rule, which derives the interpretation at the root of the derivation tree from its left granddaughters and its right daughter. The interpretation

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10 The "middle Kleene" terminology is adopted by Beaver & Krahmer (2001), where it is attributed to Krahmer (1994).

11 I adopt this convention throughout.

obtained is presented at the top of the tree in terms of the conditions under which it evaluates to $\top$, $\bot$, or # (i.e., undefined). In order to reduce clutter, I have adopted the following notation to represent uniqueness entailments. Given predicates P and Q, I will write

$$\exists x! : \; P(x); Q(x)$$

in place of the longer formula

$$\exists x! : \; P(x) \wedge \exists x : \; P(x) \wedge Q(x)$$

For instance, the truth condition represented at the top of Figure 3 is that if Theo has a brother, then (a) Theo has a unique wetsuit, and (b) Theo has a wetsuit that he'll bring. Thus according to Figure 3, (1) denotes $\top$ if, if Theo has a brother, then he has a unique wetsuit that he'll bring; it denotes $\bot$ if Theo has a brother, but he has a unique wetsuit that he won't bring; and it is undefined otherwise, that is, if Theo has a brother but lacks a unique wetsuit. These truth and definedness conditions are just as prescribed by the Material Conditional Rule, given the starting conditions associated with the consequent clause: its definedness condition (that Theo has a unique wetsuit) is checked *only* if the antecedent clause is true (i.e., if Theo has a brother).

Importantly, the obtained meaning corresponds to the local reading, on which the existence and uniqueness presuppositions of *his wetsuit* have been weakened by the conditional. We therefore arrive at a conditional presupposition for the conditional sentence as a whole, in line with the predictions Geurts observes that satisfaction accounts usually make. The presupposition we have attributed to (1), in particular, is $\mathsf{bro}(\mathsf{t}) \rightarrow \exists! x : \; \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t})$. We have therefore achieved a kind of static reconstruction of the problem that has been taken to hinder satisfaction accounts within the dynamic tradition.

On the scope theory, we may avoid this reading in order to obtain the global one by invoking cyclic scope. To illustrate this, Figure 4 presents the interpretation associated by $[\![\cdot]\!]^g$ with the LF of Figure 2. Note that, because the consequent clause of the conditional has itself moved to take scope at the conditional's edge, the final interpretation is delivered by Functional Application between the clause and its scope. As a result, the consequent clause's definedness conditions are inherited, and its presuppositions project unfiltered; the presupposition associated with (1) by the interpretation derived in Figure 4 is therefore the unconditional presupposition $\exists! x : \; \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t})$.

$$\begin{cases} \top & \exists! x \colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t}); \mathsf{bro}(\mathsf{t}) \to \mathsf{bring}(x)(\mathsf{t}) \\ \bot & \exists! x \colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t}); \mathsf{bro}(\mathsf{t}) \wedge \neg\mathsf{bring}(x)(\mathsf{t}) \\ \# & \neg\exists! x \colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t}) \end{cases}$$

bring($\iota x \colon$ suit(x) $\wedge$ have(x)(t))(t) $\lambda\varphi.\mathsf{bro}(\mathsf{t}) \to \varphi$

his wetsuit $\lambda x.\mathsf{bring}(x)(\mathsf{t})$ φ bro(t) $\to g(t_\varphi)$

$\iota x \colon$ suit(x) $\wedge$ have(x)(t)

x *he'll bring* t_x *if*   *Theo has a brother* t_φ

bring($g(t_x)$)(t)   bro(t) $g(t_\varphi)$
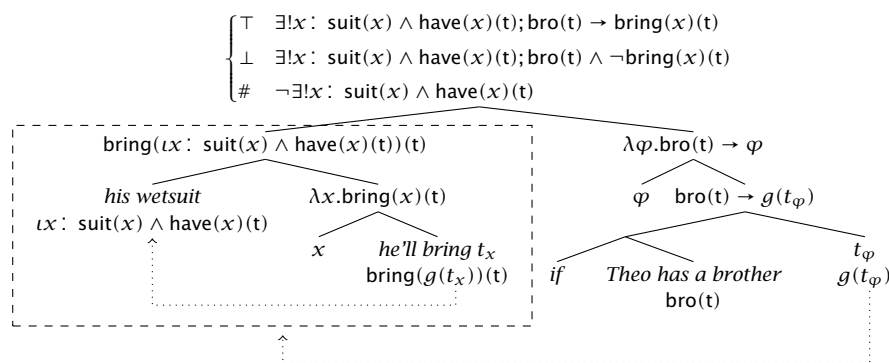
Figure 4   Interpreting (1) with Heim & Kratzer 1998: the global reading

Remarkably, we have obtained this result within a satisfaction-theoretic setting, with only the compositional principles of Heim & Kratzer (1998) as our bedrock. Thus the true innovation of the foregoing account is the invocation of covert movement of clause-level categories, which has in turn afforded us roll-up pied piping as a scope-taking strategy. The take-away intended by this demonstration is that the scope theory of presupposition projection is largely already present in semantic grammars developed within the tradition of Heim & Kratzer. At the same time, Charlow (2020a,b) motivates cyclic scope on independent grounds in an account of indefinites' ability to take scope past island boundaries (see also Elliott 2023 for an account of intensionality that relies on cyclic scope). Hence, the features of the scope theory of presupposition projection that are properly *new* will consist mainly in certain technical details of the account, which will be elucidated in the rest of this paper.

The remainder of this section aims to address an uncomfortable feature of the scope theory, as presented: that it is not compositional. Note, in particular, the Material Conditional Rule, which gave a syncategorematic statement of the conditions under which conditional sentences are true. A fuller account of the constructions that act as presupposition filters in English would require, within our static system, that we state one such rule for each construction. This is due to the characteristic feature of filters (as well as plugs) that their meanings may be defined if the meaning of one of their sub-constituents is not. A straightforward way of making the system compositional is to introduce a means of representing "failure to denote" explicitly, as a semantic value, #. Thus in addition to our usual types (e, t, $e \to t$, etc.),

we may introduce for each type α its possibly-undefined variant $\alpha_\#$, which we can call 'maybe α'. Given this addition, the full system of types is

(14) $$T ::= e \mid t \mid T \to T \mid T_\#$$

thus furnishing our type system with $e_\#$, $t_\#$, $e_\# \to t$, $(e \to t)_\#$, $e_{\#\#}$, etc., in addition to the types we already had. In general, for any type α, $\alpha_\#$ has all the inhabitants of α, but with the addition of the undefined value #.[12]

As a further illustration, take the individual-denoting metalanguage expression $\iota x : \mathsf{suit}(x) \land \mathsf{have}(x)(\mathsf{t})$ which translates *Theo's wetsuit*. We might consider this expression to be of type e in a typical setting, but in a setting with maybe types, it is of type $e_\#$: if Theo has a unique wetsuit w, then the expression denotes w. Otherwise, it denotes #; that is, it is "undefined".

Importantly, adding maybe types follows an already-common strategy for representing undefined values in semantic theories. Perhaps, the most well known of these are trivalent logic accounts, as in, for example, Peters 1979, Blamey 1986, Beaver & Krahmer 2001, George 2008 (among many others), which are inspired by a tradition going back to Łukasiewicz 1920, Bochvar 1938, and Kleene 1938, 1952. While trivalent logics effectively posit the values of the type $t_\#$, other attempts to integrate logic with a representation of undefinedness have generalized it to entities (Coppock & Beaver 2015), as well as function types (Lepage 1992, Lapierre 1992, Haug 2014). What these accounts have in common is that, by representing undefinedness, they provide a source of control over its behavior in complex expressions. Maybe types provide such control, as well, while being very permissive about the types of values whose undefinedness they can describe. Thus they allow an account of semantic definedness to be provided systematically, in terms of very general principles of semantic composition.

With the cyclic scope-taking strategy in the background, let us introduce two type-shifting operators that will allow us to compose English expressions whose semantic type might be $\alpha_\#$ (for some type α) in a way that is scopally mindful. These operators, $\eta_\#$ (pronounced 'unit') and $\star_\#$ (pronounced 'bind'),

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12 Technically, $\alpha_\#$ may be regarded as the disjoint union of α with some other type # having a single inhabitant, # (i.e., if we take # to be the unit type): $\alpha_\# = \alpha + \#$. When representing an inhabitant of a disjoint union, one typically marks whether it derives from an inhabitant of the type on the left or of the type on the right. For example, both t (for *Theo*) and # can be seen to derive inhabitants of $e_\#$ as $\mathtt{InL}(\mathsf{t})$ and $\mathtt{InR}(\#)$, respectively. It reduces notational clutter, however, to leave the injection constructors $\mathtt{InL}$ and $\mathtt{InR}$ implicit. In general, it will be clear whether some value of type $\alpha_\#$ is a left injection (a value of type α) or a right injection (#), so the interpretation of *Theo* of type $e_\#$ can be written simply t.

$$\begin{cases} \top & \mathsf{bro(t)} \rightarrow \exists! x\colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t}); \mathsf{bring}(x)(\mathsf{t}) \\ \bot & \mathsf{bro(t)} \wedge \exists! x\colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t}); \neg\mathsf{bring}(x)(\mathsf{t}) \quad : t_\# \\ \# & \mathsf{bro(t)} \wedge \neg\exists! x\colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t}) \end{cases}$$

$\lambda\psi.\mathsf{bro(t)} \Rightarrow \psi : t_\# \rightarrow t_\#$

if
$(\Rightarrow) : t_\# \rightarrow t_\# \rightarrow t_\#$

$\mathsf{bro(t)} : t_\#$
$\Big| \eta_\#$

Theo has a brother
$\mathsf{bro(t)} : t$

$$\begin{cases} \top & \exists! x\colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t}); \mathsf{bring}(x)(\mathsf{t}) \\ \bot & \exists! x\colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t}); \neg\mathsf{bring}(x)(\mathsf{t}) \quad : t_\# \\ \# & \neg\exists! x\colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t}) \end{cases}$$

$\lambda k.((\iota x\colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t})) \star_\# k)$
$: (e \rightarrow t_\#) \rightarrow t_\#$
$\Big| \star_\#$

his wetsuit
$\iota x\colon \mathsf{suit}(x) \wedge \mathsf{have}(x)(\mathsf{t}) : e_\#$

$\lambda x.\mathsf{bring}(x)(\mathsf{t}) : e \rightarrow t_\#$

$x \quad \mathsf{bring}(g(t_x))(\mathsf{t}) : t_\#$
$\Big| \eta_\#$

he'll bring t_x
$\mathsf{bring}(g(t_x))(\mathsf{t}) : t$
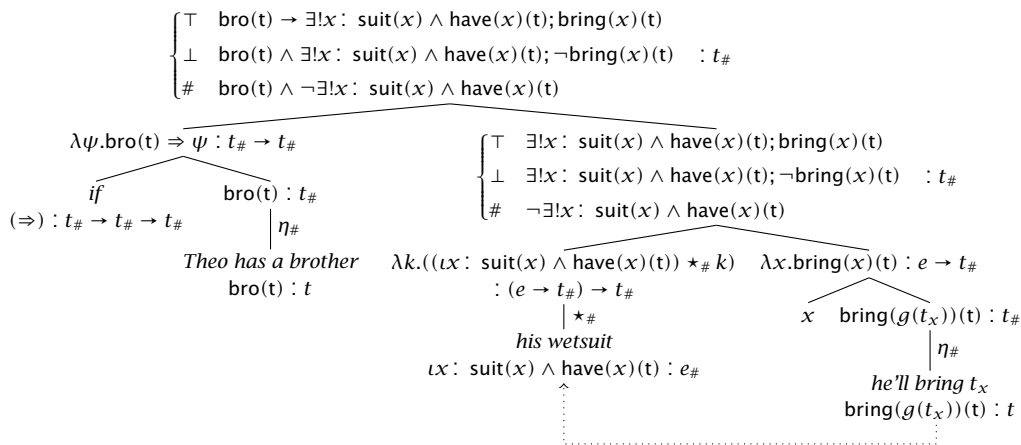
Figure 5 Interpreting (1) with Heim & Kratzer 1998 and #: the local reading

have the following type signatures and definitions (where α and β are arbitrary types).

(15)
$$\eta_\# : \alpha \rightarrow \alpha_\#$$
$$\eta_\#(v) = v$$

$$(\star_\#) : \alpha_\# \rightarrow (\alpha \rightarrow \beta_\#) \rightarrow \beta_\#$$
$$v \star_\# k = k(v) \qquad\qquad (v \neq \#)$$
$$\# \star_\# k = \#$$

Thus $\eta_\#$ changes the type of an expression without otherwise affecting its meaning: it takes an ordinary value of type α and lifts its type to $\alpha_\#$, in order to suggest that it *could have been* be undefined, even though it isn't. $\star_\#$ lifts an expression of type $\alpha_\#$ (which, therefore, may be undefined) into one which takes scope over contexts of type $\alpha \rightarrow \beta_\#$, in order to return a possibly-undefined value of type $\beta_\#$. We will use $\star_\#$, therefore, in order to lift a presupposition trigger (something with definedness conditions) into a scope-taker.

With these tools in hand, we may provide the meaning of conditional sentences categorematically. Let us introduce a connective, $\Rightarrow$, of type $t_\# \rightarrow t_\# \rightarrow t_\#$, with the truth conditions of material implication, to serve as the meaning of *if*.

(16)
$$\top \Rightarrow \psi = \psi$$
$$\bot \Rightarrow \psi = \top$$
$$\# \Rightarrow \psi = \#$$

Such a meaning for the conditional connective allows it to combine with its antecedent and consequent using Functional Application, thus allowing us to dispense with the Material Conditional Rule. Meanwhile, it achieves an equivalent interpretive effect: if the antecedent denotes #, then so does the conditional as a whole, while if it denotes $\bot$, the conditional is true regardless of the truth of the consequent (or whether or not it is defined). Only if the conditional's antecedent denotes $\top$ do we need to check the meaning of the consequent in the first place. The local reading of (1) is derived again in Figure 5. Each node of the derivation tree is annotated with its semantic type. Because of the addition of maybe types to our type system, movement of the presupposition trigger is no longer semantically vacuous, but type-driven. The type $e_\#$ with which it starts out precludes it from composing with the verb through Functional Application, so its type must instead be lifted to $(e \rightarrow t_\#) \rightarrow t_\#$ via $\star_\#$ so that it may take scope over and compose with the remaining sentential context of type $e \rightarrow t_\#$.

In a similar manner, we may derive a new interpretation for the LF of Figure 2, in which the presupposition trigger takes cyclic scope (see Figure 6). Here, too, movement is type-driven: the presupposition trigger *his wetsuit* first moves to the edge of the clause in which it originates, once its type is lifted from $e_\#$ to $(e \rightarrow t_{\#\#}) \rightarrow t_{\#\#}$; likewise, the clause itself moves to the edge of the entire conditional, once its type is lifted from $t_{\#\#}$ to $(t_\# \rightarrow t_\#) \rightarrow t_\#$. Note that the high type of the consequent clause — $t_{\#\#}$ — is what drives it to take scope at the edge of the conditional; this type is the result of the two applications of $\eta_\#$ that take place within the moved presupposition trigger's scope. By ensuring that the clause itself has semantic type $t_{\#\#}$, these two type shifts provide the impetus driving the clause from its initial type of t to its final type of $(t_\# \rightarrow t_\#) \rightarrow t_\#$, which, in turn, drives it to take scope over the conditional, percolating up the presupposition trigger's definedness conditions.

Note, crucially, that both of the $\eta_\#$ type shifts in the consequent clause of Figure 6 occur *within* the scope of the presupposition trigger, inside the clause to whose edge it has taken scope. Thus the meaning of the consequent clause according to the wide-scope derivation is *different* from the meaning this clause is assigned by the narrow-scope derivation in Figure 5: in the

Figure 6 Interpreting (1) with Heim & Kratzer 1998 and #: the global reading

narrow-scope derivation, there is only one application of $\eta_{\#}$ in the scope of the presupposition trigger. By comparison, the semantic type of the clause in Figure 6 is $t_{\#\#}$: it returns a *possibly undefined* possibly undefined truth value — defined (as a possibly undefined truth value) if Theo has a unique wetsuit and undefined otherwise.

Indeed, an alternative "wide-scope" derivation could be given which similarly applies $\eta_{\#}$ only once inside the scope of the presupposition trigger, and then once again outside its scope to obtain the same type $t_{\#\#}$ compatible with wide scope. Doing so would allow the derivation of the consequent clause in Figure 5 to be reused in a derivation which gives it wide scope, that is, after applying an $\eta_{\#}$. It can be seen from the definitions of $\eta_{\#}$ and $\star_{\#}$, however, that such a derivation would cause the meaning of consequent clause to reconstruct, producing an equivalent result to the narrow-scope derivation.[13]

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13 This fact exemplifies the law relating $\eta_{\#}$ and $\star_{\#}$ of Left Identity, which will be introduced in Section 4.

In sum, the account under consideration relates the projection behavior of the presuppositions triggered by a given expression to the syntactic context over which that expression takes scope. Scope-taking, moreover, may be achieved either directly or cyclically. In the latter case, we observe scope-taking past scope islands, which, under the present account, generates readings on which presuppositions project past these islands. As above, when the scope island is constituted by a presupposition filter, we obtain a global reading of the moved presupposition trigger.

Before moving on, let us briefly note that the proposals we have outlined are fairly minor, in the sense that the flexible projection behavior just witnessed is chiefly the result of the composition rule of Functional Application, together with type-driven covert movement—two principles of semantic analysis that many, following Heim & Kratzer (1998), take for granted. Three aspects of the account are relatively novel, however. The first is the addition of undefined values to the type system, in terms of "maybe" types; these values give us a means of assigning interpretations to presupposition filters that allow them to manage presupposition failure among their constituents on their own terms, by adopting such types for their arguments.

The second is pied-piping as a scope-taking strategy, and the third is the small collection of type shifts $\eta_\#$ and $\star_\#$. These latter additions mirror proposals recently made in the literature on phenomena seeming to involve exceptional scope (Charlow 2020a,b, Elliott 2023). Charlow, in particular, proposes that indefinite noun phrases take exceptional scope out of finite clauses by first taking scope within their smallest containing finite clause and then pied-piping the clause for further scope-taking—an aspect of his proposal which dovetails with his assumption that indefinites give rise to *alternatives*; for example, the noun phrase *some man* denotes the set of men— its type is thus $\mathsf{s}e$.[14] Alternatives, moreover, are percolated up the semantic derivation to where the indefinite takes its scope by movement in combination with the application of the type shifts η_s, which turns a value into its corresponding singleton set, and $\star_\mathsf{s}$, which turns a set into a scope-taker over set-valued functions.

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14 In fact, Charlow studies indefinites in terms of a grammar in which they denote functions from assignments onto such sets, a detail which I ignore here.

(17)
$$\eta_{\mathsf{S}} : \alpha \to \mathsf{S}\alpha$$
$$\eta_{\mathsf{S}}(v) = \{v\}$$

$$(\star_{\mathsf{S}}) : \mathsf{S}\alpha \to (\alpha \to \mathsf{S}\beta) \to \mathsf{S}\beta$$
$$m \star_{\mathsf{S}} k = \bigcup\nolimits_{v \in m} k(v)$$

The covert scopal treatment of indefinites pioneered by Charlow is the inspiration for the scope-taking strategy used here in the analysis of presupposition triggers; indeed, the scopal properties of presupposition triggers and indefinites appear fairly well aligned.[15]

4 Presuppositional scope-taking: The general case

The last section gave a first approximation to a general account of presupposition projection by treating it as a scope phenomenon. We'll now enrich the account somewhat to cover a broader array of phenomena; in particular, presupposition projection out of intensional contexts, for example, those created by modals and propositional attitude verbs. Thus we will investigate a grammatical framework here very close to the satisfaction theoretic account of presupposition projection proposed in Heim 1983.[16] The salient difference between the current account and that of Heim will be the reliance on scope-taking in terms of covert roll-up pied-piping.

What the compositional strategies investigated in this paper have in common is that they give rise to a structure known in category theory and functional programming as a *monad*. Monads were popularized for the design of programs in functional languages by Wadler (1992), and were eventually introduced to semanticists by Shan (2002). In addition to the work of Charlow (2014, 2020a,b), there have been a number of applications of monads in linguistic semantics, including to anaphora (Giorgolo & Unger 2009, Unger 2012), conventional implicature (Giorgolo & Asudeh 2012), and intensionality (Elliott 2023). As Charlow highlights, monads are particularly useful for semantic analyses that treat scope. A monad is a *functor* on types, that is, a

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15 See also Elliott 2023, in which cyclic scope is adopted as a strategy to account for the readings available for noun phrases within intensional contexts, for example, the scope of a propositional attitude verb. Covert scope-taking drives the choice of intensional context in which a given predicate is evaluated, in this case.

16 The current account effectively combines intensionality with a trivalent semantics; that is, by replacing t with $t_{\#}$. See George 2014 for a reconstruction of the essential components of Heim's theory within a trivalent setting.

(Left Identity)	$\eta(v) \star k \;=\; k(v)$
(Right Identity)	$m \star \lambda x.\eta(x) \;=\; m$
(Associativity)	$(m \star n) \star o \;=\; m \star \lambda x.(n(x) \star o)$

Figure 7   The Monad Laws

map M from types to types, associated with two operators, η and $\star$ having the following type signatures (for any α and β):[17]

(18)
$$\eta : \alpha \to M\alpha$$
$$(\star) : M\alpha \to (\alpha \to M\beta) \to M\beta$$

In addition, the operators should satisfy the monad laws, which are presented in Figure 7. It is because of these laws that monads provide a convenient semantics for scope-taking. Left Identity says that lifting a value into the monad (via η) and then turning it into a scope-taker (via $\star$) results in semantic reconstruction (it is the same as not having taken scope at all). Right Identity guarantees that invoking η in the scope of m does nothing meaningful (besides ensure type-correctness). As Charlow (2020b) illustrates, Associativity is important to the derivation of *exceptional* scope properties; for example, an indefinite m which has taken scope at the edge of a clause n, which itself takes scope over another clause o, can, in fact, act as though n and o have first been composed, in order for m to take scope over both.

4.1   Adding intensionality

We can take the next step by adding a form of intensionality to the grammar. First, let us add a type of indices i to our atomic types; i may be, for example, the type of possible worlds, world-assignment pairs, situations, etc. Now, instead of the monad $(\cdot)_\#$, which provided a semantics for presuppositional scope-taking, we'll use a new monad (which we can call '$I_\#$') allowing for presupposition and intensionality to be treated at once. $I_\#\alpha$ is defined as

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17 In category theory, a functor is a map from a category to a category satisfying certain laws (which happen to be implied by the monad laws). M should be an *endofunctor* on the category of types, that is, a functor from this category to itself.

$$\lambda i. \begin{cases} \lambda j.\mathsf{bring}(u)(\mathsf{t})(j) & \mathsf{suit}(u)(i) \wedge \mathsf{have}(u)(\mathsf{t})(i) \wedge \exists! x\colon \mathsf{suit}(x)(i) \wedge \mathsf{have}(x)(\mathsf{t})(i) \\ \# & \neg\exists! x\colon \mathsf{suit}(x)(i) \wedge \mathsf{have}(x)(\mathsf{t})(i) \end{cases} \colon \mathtt{I}_\#(i \to t)$$

$$\lambda k.((\lambda i.\iota x\colon \mathsf{suit}(x)(i) \wedge \mathsf{have}(x)(\mathsf{t})(i)) \star_{\mathtt{I}_\#} k) \qquad \lambda x, j, i.\mathsf{bring}(x)(\mathsf{t})(i) \colon e \to \mathtt{I}_\#(i \to t)$$
$$\colon (e \to \mathtt{I}_\#(i \to t)) \to \mathtt{I}_\#(i \to t)$$

$$\Big|\, \star_{\mathtt{I}_\#} \qquad\qquad\qquad\qquad x \quad \lambda j, i.\mathsf{bring}(g(t_x))(\mathsf{t})(i) \colon \mathtt{I}_\#(i \to t)$$
his wetsuit

$$\lambda i.\iota x\colon \mathsf{suit}(x)(i) \wedge \mathsf{have}(x)(\mathsf{t})(i) \colon \mathtt{I}_\# e \qquad\qquad \Big|\, \eta_{\mathtt{I}_\#}$$
Theo will bring t_x
$$\lambda i.\mathsf{bring}(g(t_x))(\mathsf{t})(i) \colon i \to t$$

Figure 8   A simple sentence with intensions

$i \to \alpha_\#$: the type of functions from indices into values which may possibly be undefined.

(19)
$$\eta_{\mathtt{I}_\#} \colon \alpha \to \mathtt{I}_\#\alpha$$
$$\eta_{\mathtt{I}_\#}(v) = \lambda i.v$$

$$(\star_{\mathtt{I}_\#}) \colon \mathtt{I}_\#\alpha \to (\alpha \to \mathtt{I}_\#\beta) \to \mathtt{I}_\#\beta$$
$$m \star_{\mathtt{I}_\#} k = \lambda i. \begin{cases} k(m(i))(i) & m(i) \neq \# \\ \# & m(i) = \# \end{cases}$$

In this case, the unit $\eta_{\mathtt{I}_\#}$ makes a value trivially index-sensitive by allowing it to read in an index, only to ignore it. The function $\star_{\mathtt{I}_\#}$, lifts an intensional meaning into a scope-taker by allowing it to read in an index, evaluate itself at that index, and (so long as it is defined there) feed the resulting value to its scope, which is then evaluated at the same index. Note that $\mathtt{I}_\#$ is therefore an "intensionalized" version of $(\cdot)_\#$: $\mathtt{I}_\#\alpha = i \to \alpha_\#$, while $\eta_{\mathtt{I}_\#}(v) = \lambda i.\eta_\#(v)$ and $m \star_{\mathtt{I}_\#} k = \lambda i.m(i) \star_\# \lambda x.k(x)(i)$. The only difference, now, is that we have an index to pass along while meanings are composed via the monadic operators.[18]

The derivations given above in an extensional setting are easily recast in the intensional one: any basic expression whose semantic type ended in t before now has a semantic type ending in $i \to t$ (e.g., the semantic type of *bring* is now $e \to e \to i \to t$); in addition, any basic expression previously of type $\alpha_\#$ (for some type α) is now of type $\mathtt{I}_\#\alpha$. To illustrate, consider the derivation of *Theo will bring his wetsuit* in Figure 8. What we end up with is a meaning of type $\mathtt{I}_\#(i \to t) = i \to (i \to t)_\#$; in order to get something

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18 Techinically, $\mathtt{I}_\#$ is obtained by applying a Reader monad transformer, where the environment is provided by an index, to the maybe monad $(\cdot)_\#$.

more propositional, we can evaluate this result into something of type $i \rightarrow t_{\#}$, using the following evaluation function $(\cdot)^{\not z}$:[19]

(20)
$$(\cdot)^{\not z} : \mathtt{I}_{\#}(i \rightarrow t) \rightarrow \mathtt{I}_{\#}t$$
$$\varphi^{\not z} = \lambda i. \begin{cases} \varphi(i)(i) & \varphi(i) \neq \# \\ \# & \varphi(i) = \# \end{cases}$$

Given some proposition φ which reads in an index, possibly in order to return an intension (of type $i \rightarrow t$), $\varphi^{\not z}$ is an intension, which may be undefined at some indices, gotten by identifying the index read first with the index read second. If we evaluate the meaning derived at the root of the derivation in Figure 8, we obtain the following result:

(21)
$$\lambda i. \begin{cases} \top & \exists! x : \mathsf{suit}(x)(i) \wedge \mathsf{have}(x)(\mathsf{t})(i); \mathsf{bring}(x)(\mathsf{t})(i) \\ \bot & \exists! x : \mathsf{suit}(x)(i) \wedge \mathsf{have}(x)(\mathsf{t})(i); \neg\mathsf{bring}(x)(\mathsf{t})(i) \\ \# & \neg\exists! x : \mathsf{suit}(x)(i) \wedge \mathsf{have}(x)(\mathsf{t})(i) \end{cases}$$

This is the proposition that is true at an index i if Theo brings his unique wetsuit at i, false at i if he does not, and undefined at i if he has no unique wetsuit at i.

4.2 Scoping above propositional attitude verbs

In the framework of Heim (1992), the semantics of propositional attitude verbs is presented in terms of conditions on the doxastic information state associated with the propositional attitude holder. Verbs of belief are analyzed as constraining the doxastic information state to satisfy some prejacent, and verbs of desire are analyzed as giving rise to a preference-based semantics, according to which doxastic information states that satisfy the prejacent are preferred to those that do not. The meaning of the verb *believe*, for instance, is taken to be a context-change potential: given some proposition φ as its prejacent (which is also a context-change potential) and an agent a, it prunes the input context to include only indices i at which the agent's doxastic information state entails the proposition; that is, such that $\mathsf{dox}_{a,i} + \varphi = \mathsf{dox}_{a,i}$. For example, in order for the sentence in (22) to be defined on an input context, the update $\mathsf{dox}_{\mathsf{t},i} + \varphi$ must be defined at every

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19 The "lightning" symbol is used in order to be consistent with the notation from the appendix of Charlow 2020b, which provides a function with a similar purpose, though tailored to alternatives.

index i in the context, where φ is the context-change potential denoted by *Theo lost his wetsuit*.

(22) Theo believes he lost his wetsuit.

Since the prejacent itself is defined only at contexts in which Theo has a wetsuit, the complete update is defined at i only if $\mathsf{dox}_{t,i}$ entails that Theo has a wetsuit; that is, if Theo believes at i that he has a wetsuit. The latter is, therefore, the presupposition that Heim's account predicts for (22). It does seem that (22) can be understood to presuppose *Theo believes he has a wetsuit*; note, for example, that (23) is most easily understood as being presupposition-free.

(23) Theo believes he has a wetsuit, and he believes he lost his wetsuit.

Due to the preference-based semantics for verbs of desire, Heim is also able to account for the fact that such verbs may give rise to presuppositions of *belief*. For example, the presupposition of the second sentence of (24) is satisfied in the context created by the first sentence.

(24) Theo believes he has a wetsuit. He hopes his wetsuit is dry.

The meaning of verb *hope* is like that of *believe*, except that the context-change potential it provides prunes input contexts to include only indices i at which updating the agent's doxastic information state with the prejacent is "preferred" to updating it with the prejacent's negation. The second sentence of (24), for example, will be true at an index i if $(\mathsf{dox}_{t,i} + \varphi) >_{t,i} (\mathsf{dox}_{t,i} + \neg\varphi)$ (where $>_{t,i}$ is the relevant preference relation on information states, and φ is the context-change potential denoted by *Theo's wetsuit is dry*). In order for such an update to be defined at i, $\mathsf{dox}_{t,i} + \varphi$ must be defined, just as for the case of the verb *believe*. The second sentence of (24) is therefore predicted (apparently, correctly) to presuppose that Theo believes he has a wetsuit.[20]

Importantly, there is a reading of (22) (as well as of the second sentence of (24)) according to which the presupposition triggered by *his wetsuit* appears to be unaffected by the surrounding context. Thus the presupposition of (22) is satisfied if it is placed into the context in (25).

(25) Theo has a wetsuit, and he believes he lost his wetsuit.

Likewise, the presupposition of the second sentence of (24) appears to be satisfied if the sentence is placed into the context in (26).

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20 Though, see Maier 2015 for discussion.

(26)   Theo has a wetsuit. He hopes his wetsuit is dry.

Such examples present difficulties for Heim's account, which, on its own, lacks a mechanism to give rise to such readings. Moreover, the problem appears to generalize to other presupposition triggers: recall (6), which featured the presupposition trigger *also*.

(6)   John: I am already in bed.
       Mary: My parents think I am also in bed.

In this case, the presupposition triggered by *also* — that John is in bed — appears to be unaffected by the surrounding context created by *think*.

As a reviewer points out, Heim suggests that *de re* readings of the presupposition triggers in these examples may play a role in their presuppositions being understood globally. Indeed, if the relevant presupposition trigger denotes an intension (i.e., a function from indices to extensions), and if the trigger's index is co-valued with that of the propositional attitude verb itself (rather than the index controlled by the verb), the definedness conditions of the trigger will percolate: consider the fact that $\mathsf{dox}_{a,i} + \varphi(i)$ will be undefined at the index i whenever the update $\varphi(i)$ is undefined. As a result, one may wonder if supplementing Heim's account with technology to obtain *de re* readings would be sufficient.

Importantly, the present account *provides* such technology. Moreover, by treating presupposition and intensionality in terms of the same monad $\mathsf{I}_{\#}$, it predicts that a *de re* reading of the nominal restriction of a presuppositional determiner and a global presupposition for the noun phrase it restricts will, in fact, always coincide, as will a *de dicto* reading and a local presupposition. This prediction seems to be correct, at least for the examples we have been considering; note that (22) can be understood either to presuppose *Theo has a wetsuit* or *Theo believes he has a wetsuit*. In the former case, the presupposition projects globally, and *his wetsuit* must be understood *de re*: an actual wetsuit belonging to Theo is presupposed to exist. In the latter case, the presupposition projects locally, and *his wetsuit* is understood *de dicto*: the speaker need not be committed to the existence of a wetsuit — only Theo's belief in one. Other theories of intensionality, when combined with Heim's account, ought to make similar predictions for the cases at hand, for the reasons discussed in the previous paragraph. Thus the case that a theory of intensionality should make it coincide with presupposition projection in principle (as does the one argued for here) should be made on independent
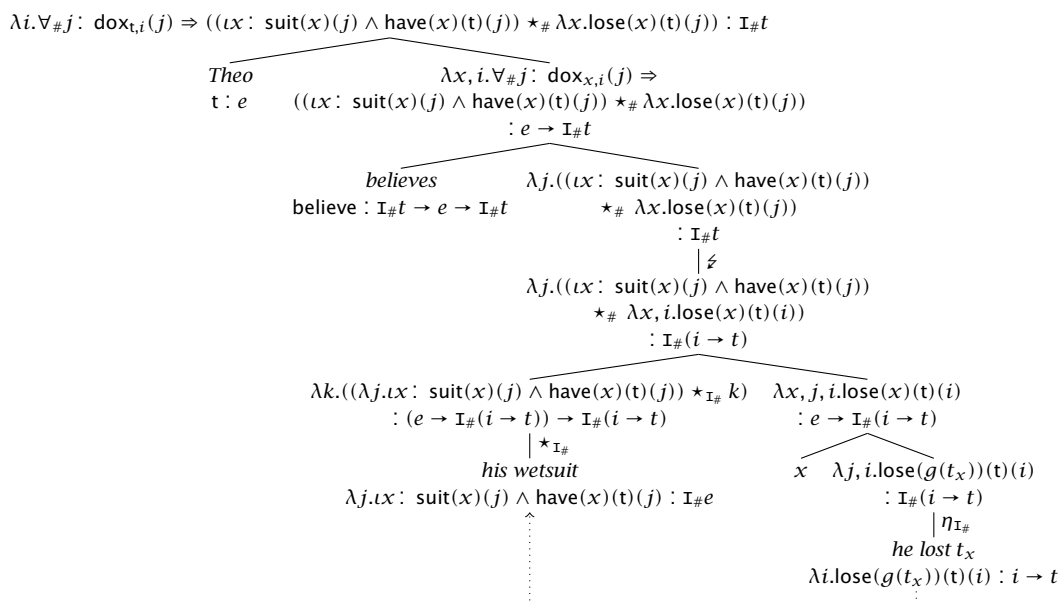
$\lambda i.\forall_{\#}j \colon \mathsf{dox}_{\mathsf{t},i}(j) \Rightarrow ((\iota x \colon \mathsf{suit}(x)(j) \land \mathsf{have}(x)(\mathsf{t})(j)) \star_{\#} \lambda x.\mathsf{lose}(x)(\mathsf{t})(j)) \colon \mathbf{I}_{\#}t$

$\begin{array}{cc} \textit{Theo} & \lambda x,i.\forall_{\#}j \colon \mathsf{dox}_{x,i}(j) \Rightarrow \\ \mathsf{t} \colon e & ((\iota x \colon \mathsf{suit}(x)(j) \land \mathsf{have}(x)(\mathsf{t})(j)) \star_{\#} \lambda x.\mathsf{lose}(x)(\mathsf{t})(j)) \\ & \colon e \to \mathbf{I}_{\#}t \end{array}$

$\begin{array}{cc} \textit{believes} & \lambda j.((\iota x \colon \mathsf{suit}(x)(j) \land \mathsf{have}(x)(\mathsf{t})(j)) \\ \mathsf{believe} \colon \mathbf{I}_{\#}t \to e \to \mathbf{I}_{\#}t & \star_{\#} \lambda x.\mathsf{lose}(x)(\mathsf{t})(j)) \\ & \colon \mathbf{I}_{\#}t \end{array}$

$\Big| \not\leftleftarrows$

$\lambda j.((\iota x \colon \mathsf{suit}(x)(j) \land \mathsf{have}(x)(\mathsf{t})(j))$
$\star_{\#} \lambda x,i.\mathsf{lose}(x)(\mathsf{t})(i))$
$\colon \mathbf{I}_{\#}(i \to t)$

$\begin{array}{cc} \lambda k.((\lambda j.\iota x \colon \mathsf{suit}(x)(j) \land \mathsf{have}(x)(\mathsf{t})(j)) \star_{\mathbf{I}_{\#}} k) & \lambda x,j,i.\mathsf{lose}(x)(\mathsf{t})(i) \\ \colon (e \to \mathbf{I}_{\#}(i \to t)) \to \mathbf{I}_{\#}(i \to t) & \colon e \to \mathbf{I}_{\#}(i \to t) \end{array}$

$\Big| \star_{\mathbf{I}_{\#}}$

$\textit{his wetsuit}$

$\lambda j.\iota x \colon \mathsf{suit}(x)(j) \land \mathsf{have}(x)(\mathsf{t})(j) \colon \mathbf{I}_{\#}e$

$\begin{array}{cc} x & \lambda j,i.\mathsf{lose}(g(t_x))(\mathsf{t})(i) \\ & \colon \mathbf{I}_{\#}(i \to t) \end{array}$

$\Big| \eta_{\mathbf{I}_{\#}}$

$\textit{he lost } t_x$

$\lambda i.\mathsf{lose}(g(t_x))(\mathsf{t})(i) \colon i \to t$

Figure 9   Interpreting (22): the local reading

grounds. I will not make a case for a uniform treatment here, but will rather illustrate that the very same mechanisms the current theory provides for scoping presupposition triggers out of conditionals are easily adapted to the propositional attitude setting.[21]

Within the current framework, we can, in fact, give a standard semantics to propositional attitude verbs in the style of Hintikka 1969. To do so, it is useful to introduce some notation. In addition to the connective ⇒ from above, we need a quantifier, $\forall_{\#}$, of type $(\alpha \to t_{\#}) \to t_{\#}$; that is, whose scope may possibly be undefined at some values. We will generally write $\forall_{\#}x \colon \varphi(x)$ in place of $\forall_{\#}(\lambda x.\varphi(x))$. $\forall_{\#}$ acts as follows:

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21 See Romoli & Sudo 2009 (also Geurts 1998 and Maier 2015) for discussions of this very issue and arguments that intensionality and presupposition should be treated uniformly. It is also pertinent that Elliott (2023) argues for a monadic treatment of intensionality from observations that do not appear to involve presupposition triggers in an essential way.
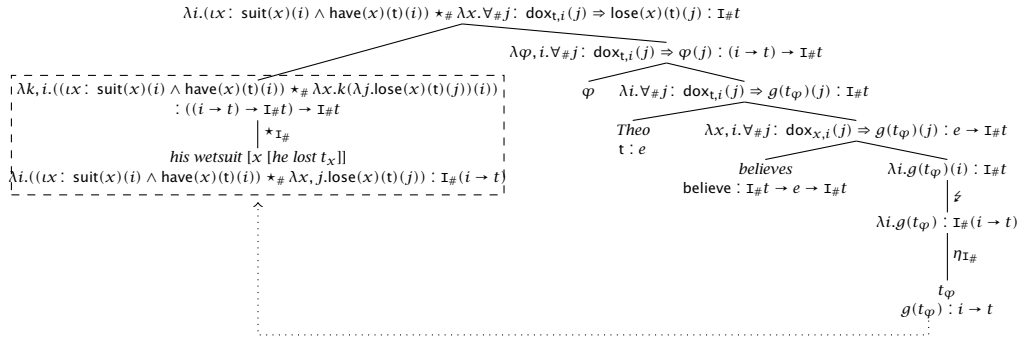
Figure 10 Interpreting (22): the global reading

$$
(27) \qquad \forall_{\#}x\colon \varphi(x) = \begin{cases} \top & \forall x\colon (\varphi(x) = \top) \\ \bot & \exists x\colon (\varphi(x) = \bot) \wedge \neg\exists x\colon (\varphi(x) = \#) \\ \# & \exists x\colon (\varphi(x) = \#) \end{cases}
$$

That is, a formula quantified by $\forall_{\#}$ is true if the scope of $\forall_{\#}$ is true at every possible value of the bound variable, and it is undefined if the scope is undefined at some possible value; otherwise, it is false. Finally, we introduce a constant $\mathsf{dox}\colon e \to i \to i \to t$, which, given an individual, encodes the doxastic accessibility relation associated with it. We can now state the semantics of the verb *believe*.

$$
(28) \qquad \begin{aligned} &\mathsf{believe}\colon \mathrm{I}_{\#}t \to e \to \mathrm{I}_{\#}t \\ &\mathsf{believe} = \lambda\varphi, x, i.\forall_{\#}j\colon \mathsf{dox}_{x,i}(j) \Rightarrow \varphi(j) \end{aligned}
$$

Corresponding to the ambiguity of presupposition projection in (22), two derivations of its meaning can be given. On the one hand, there is a derivation in which the embedded clause does not scope out, given in Figure 9. It is straightforward to check that its resulting definedness conditions are that Theo must have a unique wetsuit at all doxastically accessible worlds (and that he need not have one at any others). This is because the connective '⇒' is defined to ignore undefinedness in its consequent when its antecedent is

false. Indeed, these definedness conditions ought to correspond to the truth conditions of *Theo believes he has a wetsuit.*[22]

On the other hand, there is a derivation of (22) in which the embedded clause, though it is derived in a similar manner, scopes out; this is given in Figure 10 (compare with Figure 9). In this case, the definedness condition of the meaning which results is that Theo must have a unique wetsuit at the world of evaluation. Assuming there is such a wetsuit, the sentence is taken to be true at the evaluation world if Theo lost the wetsuit at all of his doxastically accessible worlds; that is, if he believes he lost it. In other words, the at-issue content of the embedded clause has simply reconstructed, even though its presuppositions have been evaluated globally.[23] The scope account, therefore, predicts (22) to have a reading whereon it presupposes nothing about Theo's beliefs.[24]

Finally, without giving a full-fledged account of the semantics of *also*, it is possible to see how a derivation of (6) could roughly go. For illustrative purposes, let us assume that *also* has the following meaning, setting aside analyses of anaphora and association with focus.

(29)
$$\text{also} : (e \to i \to t) \to e \to \mathtt{I}_{\#}(i \to t)$$
$$\text{also} = \lambda P, x, i. \begin{cases} P(x) & P(\mathtt{j})(i) \\ \# & \neg P(\mathtt{j})(i) \end{cases}$$

Here, I have simply written the anaphoric component of the meaning of *also* into its presupposition: given the property denoted by the verb phrase it modifies, it presupposes that this property holds of John. The sentence *I am also in bed* should then receive the interpretation in (30), given appropriate meanings for the other words (e.g., assuming that *I* refers to Mary).

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22 That is, while an analysis of indefinites is not given here, a suitable one will allow this sentence to be interpreted as

$$\lambda i. \forall_{\#} j : \text{dox}_{\mathtt{t},i}(j) \Rightarrow \exists x.\text{suit}(x)(j) \wedge \text{have}(x)(j) : \mathtt{I}_{\#} t$$

23 This is due to the law Left Identity.

24 As mentioned, similar contrasts can be observed among verbs of desire: the sentence *Theo hopes his wetsuit is dry* can be understood to presuppose either that Theo has a wetsuit or that he believes he has one. It is possible to see that similar derivations would be responsible for describing the ambiguities observed in such cases.

(30)
$$\lambda i. \begin{cases} \text{inBed(m)} & \text{inBed(j)}(i) \\ \# & \neg\text{inBed(j)}(i) \end{cases} : \text{I}_\#(i \to t)$$

also thus turns the entire complement of *think* into a presupposition trigger. Two options now become available: the complement may remain in-situ and undergo evaluation via $(\cdot)^{\natural}$ (analogous to the derivation in Figure 9), or it may take scope above the matrix verb (analogous to the derivation in Figure 10). In the former case, *think* combines with the complement and the subject, in order to produce the presupposition that Mary's parents think John is in bed. In the latter case, the at-issue meaning of the complement, inBed(m), semantically reconstructs, while the presupposition is evaluated globally; thus the entire sentence is analyzed as entailing that Mary's parents think Mary is in bed and presupposing merely that John is in bed.

5 Conclusion

One of the great innovations of the satisfaction account of presupposition projection is that it fits the description of presupposition projection behavior squarely within the program of compositional semantics; projection is simply a side effect of semantic composition. What Geurts showed is that this program may not have been as sure-footed as originally hoped: satisfaction-account analyses of the presuppositions of complex constructions are too deterministic, leading to filtration when it is unwanted. I hope to have shown that this worry is ultimately unfounded. The *traditional* satisfaction account is essentially based on simple functional application, strapping presupposition triggers down into their local contexts and forcing them to undergo evaluation. The upgraded satisfaction account I have presented is monadic, allowing presupposition triggers to scope freely.

Given the progress made in understanding presupposition within the satisfaction account, it is important to point out that the monadic strategy is backwards compatible: analyses of individual expressions couched within a traditional satisfaction-theoretic setting based on functional application may still exist comfortably within a monadic setting, which additionally provides a flexible scope-taking mechanism. Indeed, although semanticists are not typically accustomed to type systems like the one presented in this paper, it should be fairly straightforward to port its main ideas into a more conventional setting. In the framework of Heim, discourse contexts are regarded as sets of indices (functions of type $i \to t$). Sentence denotations may

then be viewed as context-change potentials, which, following, for example, Rothschild (2011), can be encoded as functions from sets to sets (i.e., of type $(i \rightarrow t) \rightarrow i \rightarrow t$). Notably, η and $\star$ are defined for this kind of lifting, as well:[25]

(31)
$$\eta : \alpha \rightarrow (i \rightarrow t) \rightarrow \alpha$$
$$\eta(v) = \lambda c.v$$

$$(\star) : ((i \rightarrow t) \rightarrow \alpha) \rightarrow$$
$$(\alpha \rightarrow (i \rightarrow t) \rightarrow \beta) \rightarrow$$
$$(i \rightarrow t) \rightarrow \beta$$
$$m \star k = \lambda c.k(m(c))(c)$$

For example, *Theo lost his$_1$ wetsuit* can be taken to denote the following context change potential (i_1 here denotes the individual that the index i assigns to the discourse referent 1):

(32) $\lambda c : \forall i \in c : \mathsf{suit}(i_1)(i) \wedge \mathsf{have}(i_1)(\mathsf{t})(i).c \cap \lambda i.\mathsf{lost}(i_1)(\mathsf{t})(i)$

Meanwhile, updating a context with a sentence is just a matter of applying the context-change potential denoted by the sentence to the context. So an update with *Theo lost his wetsuit* will only be defined on contexts in which Theo has a unique wetsuit.

Complex sentences like the ones studied here can then be given derivations that manipulate more familiar-looking meanings, but which crucially involve scope-taking in terms of η and $\star$. Given the promise of scope-taking for accounts of presupposition projection, it will be interesting to see what analyses it may yield.

References

Beaver, David. 1999. Presupposition accomodation: A plea for common sense. In Lawrence S. Moss, Jonathan Ginzburg & Rijke de Maarten (eds.), *Logic, language, and computation*, vol. 2, 21–44. Stanford: CSLI Publications. https://doi.org/d5345d21c4ddc20d61552928ae64f332b3ff2936.

Beaver, David. 2001. *Presupposition and assertion in dynamic semantics* (Studies in Logic, Language and Information). Stanford: CSLI Publications. https://semanticsarchive.net/Archive/jU1MDVmZ.

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25 This lifting, notably, is a instantiation of the Reader monad.

Beaver, David & Emiel Krahmer. 2001. A partial account of presupposition projection. *Journal of Logic, Language and Information* 10(2). 147–182. https://doi.org/10.1023/A:1008371413822.

Blamey, Stephen. 1986. Partial logic. In Dov Gabbay & Franz Guenthner (eds.), *Handbook of philosophical logic: Alternatives in classical logic*, vol. III (Synthese Library 166), 1–70. Dordrecht: Springer Netherlands. https://doi.org/10.1007/978-94-009-5203-4_1.

Bochvar, Dmitry A. 1938. On a three valued calculus and its application to the analysis of contradictories. *Matematicheskii Sbornik* 4(2). 287–308. Engish translation: Bochvar 1981. https://www.mathnet.ru/eng/sm5749.

Bochvar, Dmitry A. 1981. On a three-valued logical calculus and its application to the analysis of the paradoxes of the classical extended functional calculus. Trans. by Merrie Bergmann. *History and Philosophy of Logic* 2(1-2). 87–112. https://doi.org/10.1080/01445348108837023.

Charlow, Simon. 2014. *On the semantics of exceptional scope.* New York: New York University dissertation. https://semanticsarchive.net/Archive/2JmMWRjY.

Charlow, Simon. 2020a. Static and dynamic exceptional scope. Rutgers University, Manuscript. https://ling.auf.net/lingbuzz/004650.

Charlow, Simon. 2020b. The scope of alternatives: Indefiniteness and islands. *Linguistics and Philosophy* 43(4). 427–472. https://doi.org/10.1007/s10988-019-09278-3.

Chierchia, Gennaro & Sally McConnell-Ginet. 1990. *Meaning and grammar: An introduction to semantics.* Cambridge: MIT Press.

Coppock, Elizabeth & David Beaver. 2015. Definiteness and determinacy. *Linguistics and Philosophy* 38(5). 377–435. https://doi.org/10.1007/s10988-015-9178-8.

Elliott, Patrick D. 2023. A flexible scope theory of intensionality. *Linguistics and Philosophy* 46(2). 333–378. https://doi.org/10.1007/s10988-022-09367-w.

von Fintel, Kai. 2004. Would you believe it? The king of France is back! (Presuppositions and truth-value intuitions). In Marga Reimer & Anne Bezuidenhout (eds.), *Descriptions and beyond*, 315–341. New York: Oxford University Press. https://doi.org/10.1093/oso/9780199270514.003.009.

Francez, Itamar. 2019. How not to project the satisfaction theory of projection (on Karttunen) or: Who has a proviso problem? In Cleo Condoravdi & Tracy Holloway King (eds.), *Tokens of meaning: Papers in honor of Lauri*

Karttunen, 187–198. Stanford: CSLI Publications. https://ling.auf.net/lingbuzz/004219.

George, B. R. 2008. Predicting presupposition projection: Some alternatives in the strong Kleene tradition. Manuscript. https://semanticsarchive.net/Archive/DY0YTgxN/.

George, B. R. 2014. Some remarks on certain trivalent accounts of presupposition projection. *Journal of Applied Non-Classical Logics* 24(1-2). 86–117. https://doi.org/10.1080/11663081.2014.911521.

Geurts, Bart. 1996. Local satisfaction guaranteed: A presupposition theory and its problems. *Linguistics and Philosophy* 19(3). 259–294. https://doi.org/10.1007/BF00628201.

Geurts, Bart. 1998. Presuppositions and anaphors in attitude contexts. *Linguistics and Philosophy* 21(6). 545–601. https://doi.org/10.1023/A:1005481821597.

Giorgolo, Gianluca & Ash Asudeh. 2012. ⟨$M, \eta, \star$⟩ monads for conventional implicatures. *Sinn und Bedeutung* 16(1). 265–278. https://ojs.ub.uni-konstanz.de/sub/index.php/sub/article/view/425.

Giorgolo, Gianluca & Christina Unger. 2009. Coreference without discourse referents: A non-representational DRT-like discourse semantics. In Barbara Plank, Erik Tjong Kim Sang & Tim Van de Cruys (eds.), *The 19th meeting of Computational Linguistics in the Netherlands* (LOT Occasional series 14), 69–81. Utrecht: LOT, Netherlands Graduate School of Linguistics. https://dspace.library.uu.nl/handle/1874/297140.

Haug, Dag Trygve Truslew. 2014. Partial dynamic semantics for anaphora: Compositionality without syntactic coindexation. *Journal of Semantics* 31(4). 457–511. https://doi.org/10.1093/jos/fft008.

Heim, Irene. 1983. On the projection problem for presuppositions. *The West Coast Conference on Formal Linguistics (WCCFL)* 2. 114–125.

Heim, Irene. 1992. Presupposition projection and the semantics of attitude verbs. *Journal of Semantics* 9(3). 183–221. https://doi.org/10.1093/jos/9.3.183.

Heim, Irene & Angelika Kratzer. 1998. *Semantics in generative grammar*. Malden, MA: Blackwell.

Hintikka, Jaakko. 1969. Semantics for propositional attitudes. In John W. Davis, Donald J. Hockney & W. Kent Wilson (eds.), *Philosophical logic* (Synthese Library 20), 21–45. Dordrecht: Springer. https://doi.org/10.1007/978-94-010-9614-0_2.

Kamp, Hans. 1981. A theory of truth and semantic representation. In Jeroen A. Groenendijk, Theo M. V. Janssen & Martin B. J. Stokhof (eds.), *Formal methods in the study of language* (Mathematical Centre Tracts 135), 277–322. Amsterdam: Mathematisch Centrum.

Karttunen, Lauri. 1974. Presuppositions and linguistic context. *Theoretical Linguistics* 1(1–3). 181–194. https://doi.org/10.1515/thli.1974.1.1-3.181.

Kleene, Stephen Cole. 1938. On a notation for ordinal numbers. *Journal of Symbolic Logic* 3(4). 150–155. https://doi.org/10.2307/2267778.

Kleene, Stephen Cole. 1952. *Introduction to metamathematics.* Amsterdam: North Holland.

Krahmer, Emiel J. 1994. Partiality and dynamics: Theory and application. *Amsterdam Colloquium* 9. 391–410. Institute for Logic, Language and Computation (ILLC), Department of Philosophy, University of Amsterdam.

Lapierre, Serge. 1992. A functional partial semantics for intensional logic. *Notre Dame Journal of Formal Logic* 33(4). 517–541. Publisher: Duke University Press. https://doi.org/10.1305/ndjfl/1093634484.

Lepage, François. 1992. Partial functions in type theory. *Notre Dame Journal of Formal Logic* 33(4). 493–516. https://doi.org/10.1305/ndjfl/1093634483.

Łukasiewicz, Jan. 1920. O logice trójwartościowej. *Ruch Filozoficzny* (5). 170–171. English translation: Łukasiewicz 1970.

Łukasiewicz, Jan. 1970. On three-valued logic. In Ludwik Borkowski (ed.). Trans. by O. Wojtasiewicz, *Selected works by Jan Łukasiewicz.* Amsterdam: North–Holland.

Maier, Emar. 2015. Parasitic attitudes. *Linguistics and Philosophy* 38(3). 205–236. https://doi.org/10.1007/s10988-015-9174-z.

Mandelkern, Matthew. 2016a. A note on the architecture of presupposition. *Semantics and Pragmatics* 9(13). 1–24. https://doi.org/10.3765/sp.9.13.

Mandelkern, Matthew. 2016b. Dissatisfaction theory. *Semantics and Linguistic Theory (SALT)* 26. 391–416. https://doi.org/10.3765/salt.v26i0.3804.

Mayr, Clemens & Jacopo Romoli. 2016. Satisfied or exhaustified: An ambiguity account of the Proviso Problem. *Semantics and Linguistic Theory (SALT)* 26. 892–912. https://doi.org/10.3765/salt.v26i0.3961.

Peters, Stanley. 1979. A truth-conditional formulation of Karttunen's account of presupposition. *Synthese* 40(2). 301–316. https://doi.org/10.1007/BF00485682.

Romoli, Jacopo & Yasutada Sudo. 2009. *De re/de dicto* ambiguity and presupposition projection. *Sinn und Bedeutung* 13(2). 425–438. https://doi.org/10.18148/sub/2009.v13i2.563.

Rothschild, Daniel. 2011. Explaining presupposition projection with dynamic semantics. *Semantics and Pragmatics* 4(3). 1–43. https://doi.org/10.3765/sp.4.3.

van der Sandt, Rob A. 1992. Presupposition projection as anaphora resolution. *Journal of Semantics* 9(4). 333–377. https://doi.org/10.1093/jos/9.4.333.

Schlenker, Philippe. 2011. The Proviso Problem: A note. *Natural Language Semantics* 19(4). 395–422. https://doi.org/10.1007/s11050-011-9072-z.

Shan, Chung-chieh. 2002. Monads for natural language semantics. *European Summer School in Logic, Language and Information student session (ESSLLI) 2001.* 285–298. https://doi.org/10.48550/arXiv.cs/0205026.

Singh, Raj. 2007. Formal alternatives as a solution to the proviso problem. *Semantics and Linguistic Theory (SALT)* 17. 264–281. https://doi.org/10.3765/salt.v17i0.2960.

Singh, Raj. 2008. *Modularity and locality in interpretation.* Cambridge, MA: Massachusetts Institute of Technology dissertation. https://dspace.mit.edu/handle/1721.1/45895.

Singh, Raj. 2009. Symmetric and interacting alternatives for implicature and accommodation. Manuscript, MIT.

Stalnaker, Robert. 1973. Presuppositions. *Journal of Philosophical Logic* 2(4). 447–457. http://www.jstor.org/stable/30226075.

Stalnaker, Robert. 1974. Pragmatic presuppositions. In Milton K. Munitz & Peter K. Unger (eds.), *Semantics and philosophy*, 197–214. New York: New York University Press. Reprinted in Stalnaker 1999b.

Stalnaker, Robert. 1978. Assertion. In Peter Cole (ed.), *Pragmatics* (Syntax and Semantics 9), 315–332. New York: Academic Press. Reprinted in Stalnaker 1999a.

Stalnaker, Robert. 1999a. Assertion. In *Context and Content: Essays on Intentionality in Speech and Thought*, 78–95. Oxford: Oxford University Press. https://doi.org/10.1093/0198237073.003.0005.

Stalnaker, Robert. 1999b. Pragmatic Presuppositions. In *Context and Content: Essays on Intentionality in Speech and Thought*, 47–62. Oxford: Oxford University Press. https://doi.org/10.1093/0198237073.003.0003.

Unger, Christina. 2012. Dynamic semantics as monadic computation. In Manabu Okumura, Daisuke Bekki & Ken Satoh (eds.), *New frontiers in artificial*

Intelligence: JSAI-isAI 2011 workshops, LENLS, JURISIN, ALSIP, MiMI, Takamatsu, Japan, December 1-2, 2011. Revised selected papers (Lecture Notes in Computer Science 7258), 68–81. Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-642-32090-3_7.

Wadler, Philip. 1992. The essence of functional programming. *The 19th ACM SIGPLAN-SICACT symposium on Principles of programming languages.* 1–14. https://doi.org/10.1145/143165.143169.

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