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Alternatives and attention in language and reasoning: A reply to Mascarenhas & Picat (2019)*

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Abstract In this paper, we employ an experimental paradigm using insights from the psychology of reasoning to investigate the question whether certain modals generate and draw attention to alternatives. The article extends and builds on the methodology and findings of Mascarenhas & Picat (2019). Based on experimental results, they argue that the English epistemic modal *might* raises alternatives. We apply the same methodology to the English modal *allowed to* to test different hypotheses regarding the involvement of alternatives in deontic modality. We find commonalities and differences between the two modals we tested. We discuss theoretical consequences for existing semantic analyses of these modals, and argue that reasoning tasks can serve as a diagnostic tool to discover which natural language expressions involve alternatives.

Keywords: reasoning, modals, alternatives

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1 Introduction

Recent efforts to seek convergence between natural language semantics and the psychology of reasoning have led to articulated theories of interpretive processes and general-purpose reasoning. In particular, the erotetic theory of reasoning of Koralus & Mascarenhas (2013, 2018) incorporates linguistic insights into a variant of the mental models theory of reasoning (Johnson-Laird 1983) to account for a wide range of failures of deductive reasoning. At the core of both the mental models and erotetic theories of reasoning is the idea that attentional mechanisms structure our mental representations of states of affairs, and that a semantics of alternatives is required to model the effects of attention in reasoning.

Disjunction and indefinites are generators of alternatives *par excellence*. They induce a particular kind of illusory inference whose extant accounts all agree must be due to the presence of alternatives, driving attention in ways that render tractable the space of possibilities to be considered, but that introduce opportunities for fallacious reasoning under certain well-understood conditions.

In this paper, we advance our understanding of the connection between the semantics of alternatives in language, attention, and failures of reasoning by looking at reasoning with two different modal constructions in English. Mascarenhas & Picat (2019) have shown experimentally that epistemic *might* gives rise to illusory inferences much like those associated with indefinites. The explanation they discuss has two key components. First, following Ciardelli, Groenendijk & Roelofsen (2009), *might* introduces alternatives as in inquisitive semantics or Hamblin semantics. Second, following the erotetic theory, reasoning with these alternatives is based on question-answer dynamics. Together, these two components predict the inferences they observe with *might*. They conjecture that all and only linguistic elements that generate alternatives as in inquisitive or Hamblin semantics will give rise to the particular class of fallacies that have been observed so far with disjunctions, indefinites, and *might*. The present paper examines this conjecture by extending their experimental paradigm to the English deontic possibility modal *allowed to*, which no extant theory argues is a generator of alternatives of the relevant kind. Our experiment includes both *might* and *allowed to*, providing a direct comparison with previous results and with each other.

We largely replicate the findings of Mascarenhas & Picat (2019) for *might*, but we observe a more complex picture for *allowed to*. On the one hand, it

shows fallacious conclusions structurally parallel to those found with *might*. On the other hand, *allowed to* fails to pattern with *might*, disjunctions, and indefinites in other well-established experimental results on the fallacious structures of central interest.

We explore the theoretical implications of these findings. Given the stability of the effect for *might*, we argue against the idea that the new data present evidence against the erotetic view. Rather, we outline how different extant theories of modality can account for the contrast we observe. Specifically, we discuss how certain possibility modals are conventionally associated with the generation of alternatives and that theories which incorporate this property in their analysis fare better when accounting for our data. As a result, we provide arguments from the psychology of reasoning that help winnow the conceptual space of theories of possibility modals in novel ways.[1]

1.1 Illusory inferences from disjunction

The erotetic theory of reasoning of Koralus & Mascarenhas (2013) incorporates linguistic insights into a variant of the mental-models theory of reasoning (Johnson-Laird 1983) to account for a class of attractive fallacies known as illusory inferences from disjunction. Consider the example in (1), greatly simplified from the original examples discovered by Walsh & Johnson-Laird (2004).

(1) **Premise 1**: John speaks English and Mary speaks French, or else Bill speaks German.
Premise 2: John speaks English.
Fallacious conclusion: Mary speaks French.

The reasoning problem in (1), along with a number of structurally identical problems, have acceptance rates between 80% and 85% (Walsh & Johnson-Laird 2004, Mascarenhas & Koralus 2017, Koralus & Mascarenhas 2018). Yet, the deductive inference in (1) is a fallacy. Suppose Bill speaks German (modeling premise 1) and John speaks English (premise 2), but Mary does not speak French. This is a model of the premises but not the conclusion, and the inference in (1) is thus invalid.

1 We use the term *possibility modal* purely descriptively to refer to modals like the English *might* and *be allowed to* as opposed to *must*.

1.2 Alternatives, mental models, and illusory inferences

Building on the mental-models explanation of these fallacies by Walsh & Johnson-Laird (2004), the erotetic theory of reasoning offers an account of this and related illusory inferences proposing that a question-answer dynamic is at the core of these fallacies. For ease of exposition, let us consider the logical structure behind the example in (1).

(2) **Premise 1**: $(a \wedge b) \vee c$
Premise 2: a
Fallacious conclusion: b

Following Hamblin semantics (Kratzer & Shimoyama 2002, Alonso-Ovalle 2006) and inquisitive semantics (Groenendijk 2008, Mascarenhas 2009), the erotetic theory of reasoning takes the disjunction in the first premise of (2) to *raise an issue*, putting forth two alternatives: $a \wedge b$ and c. The reasoner is now effectively entertaining a question, and will seek to find the most expedient way of answering or dispelling it. As it turns out, the second premise offers hints at an answer: the second premise a is *related* to the first alternative $a \wedge b$ rather than the second c, and the reasoner rushes to pick an answer: The right alternative from the first premise is $a \wedge b$, whence b follows by a mental-models analog of conjunction elimination.[2]

1.3 Alternatives in language beyond disjunction

If the attractiveness of fallacious schemata as in (2) is to be explained in terms of the presence of alternatives in the interpretation of the first (disjunctive) premise, then we expect other linguistic items that have been argued to raise alternatives of a similar kind to produce similar illusory inferences.

In the traditions of both Hamblin semantics (Kratzer & Shimoyama 2002) and inquisitive semantics (Ciardelli 2009), indefinites are akin to *wh*-questions.[3] An expression such as *some pilot* is assigned roughly the same mean-

2 This informal exposition of the erotetic theory of reasoning suffices for the purposes of this article, but it is crucial to note that the theory is given in a fully explicit form by Koralus & Mascarenhas (2013). They provide a full regimentation of the mental model theory in terms of truth-maker semantics (van Fraassen 1969, Fine 2012) with an inquisitive / Hamblin semantics for disjunction, and they define a small set of dynamic operations on mental models.

3 Alternatives of more than one kind play a role in semantics and pragmatics. This article concerns only those linguistic operators that encode alternatives in the sense of inquisi-

ing as *which pilot*. Accordingly, a sentence like *Some pilot writes poems* is analyzed as the set of all propositions of the form *x writes poems*, for *x* a pilot. In other words, the meaning of the question *Which pilot writes poems?* In this analysis, the erotetic theory predicts that the example below should give rise to an illusory inference.

(3) **Premise 1**: Some pilot writes poems.
Premise 2: John is a pilot.
Fallacious conclusion: John writes poems.

Mascarenhas & Koralus (2017) found that inferences like (3) were in fact attractive, with acceptance rates around 35%, significantly above the baseline for mistakes established by invalid controls without alternative-generating elements. This demonstrates the existence of the predicted illusory inferences with alternative-generating linguistic operators besides disjunction.

1.4 Reasoning vs. interpretation

Illusory inferences from *disjunction* as in (1) have acceptance rates around 85%, while illusory inferences with *indefinites* as in (3) are accepted around 35% of the time. Why the different acceptance rates, if both disjunctions and indefinites produce alternatives according to our best semantic theories?

Mascarenhas (2014) showed that the disjunction inferences are amenable to a pragmatic account. As an alternative to mental-models accounts we can derive the observed conclusion assuming an entirely classical reasoning module, acting on pragmatically strengthened meanings of the premises. Under most modern theories of scalar implicature (e.g. Sauerland 2004, Spector 2007) the first premise of the illusory inference from disjunction in (4a) is predicted to be interpreted as in (4b), a fact Spector (2007) already observed outside the context of reasoning problems.

(4) a. $(a \wedge b) \vee c$
b. $(a \wedge b \wedge \neg c) \vee (c \wedge \neg a \wedge \neg b)$

Assuming (4a) is interpreted as in (4b), and incorporating the second premise *a*, the conclusion *b* is no fallacy at all. It follows classically from the conjunction of the two premises. Crucially, no absolving implicature is predicted

tive semantics and Hamblin semantics. For example, we make no predictions for inference patterns with premises involving focus alternatives (Rooth 1996).

by any existing theory of scalar implicature for illusory inferences with *indefinites*. Recruiting experimental paradigms from psychosemantics into the domain of reasoning problems, Picat (2019) argues that, rather than being competing accounts, the erotetic theory and the scalar-implicature account are two possible routes leading to the same inference-making behavior in the case of the illusory inference from *disjunction*. For the indefinites case, only the erotetic theory predicts a fallacy, the scalar-implicature route is blocked, explaining its lower endorsement rate.

1.5 Deduction, norms of rationality, and probabilistic reasoning

The inference in (1) has the form of a deductive problem, and the experimental methodologies used to investigate it and related data points ostensibly rely on deduction and logical validity. Yet, a whole group of approaches to human reasoning exists that rejects the idea that logical validity plays any role in and of itself in human reasoning. Work in the "New Paradigm" tradition (see in particular Oaksford & Chater 2007) holds that the functional aim of the human reasoning faculty is not to track logical validity, but instead to deliver rational answers to decision problems solved under uncertainty. In this view, human reasoning relies on probabilistic inference, and the rational norm is the classical probabilistic calculus, instead of classical validity. Deductive validity emerges as a special case of the probabilistic calculus, but it is by no means the goal of the system. One particularly fruitful model, for example, proposes that naive reasoners consider the probability of the putative conclusion conditional on the conjunction of the premises, and check whether that *posterior probability* is higher than a contextually defined standard. If it is, they respond that the conclusion is indeed valid. Another view uses the notion of p-validity, where a conclusion will be deemed valid if its probability is greater than that of its premises, across probability distributions.

Interestingly, accounts in this vein of inferences as in (1) above are impossible, unless they make the same non-classical commitments as the erotetic theory regarding the interpretation of disjunctions and the dynamics of questions and answers. Consider the logical form of the inference as schematized in (2). The conjunction of the premises is as in (5a) below, which is equivalent in classical logic to (5b).

(5) a. $((a \wedge b) \vee c) \wedge a$
b. $(a \wedge b) \vee (a \wedge c)$

Since the classical probabilistic calculus is built atop classical logic, the equivalence above holds in the relevant probabilistic approaches. Consequently, we will get the following equality for the joint probability of the premises, where the comma ',' represents premise conjunction.

$$P((a \wedge b) \vee c, a) = P((a \wedge b) \vee (a \wedge c))$$

However, this means that this approach cannot distinguish between the observed conclusion b and an unobserved and altogether unintuitive conclusion of c, unless their prior probabilities were significantly different, which while possible is entirely unwarranted, certainly for the myriad examples of these inferences available in the literature. Consequently, a view of human reasoning that aims to interpret apparent mistakes as rational probabilistic inferences purely in terms of prior or posterior probabilities takes no account of these illusory inferences from disjunction.[4]

A final word on terminology is in order. As we have explained, the idea that there is such a thing as properly deductive reasoning among naive humans is a matter of debate. Our goal here is to discuss the crucial importance of *alternatives* and the semantics that generate them in reasoning and in language, and accordingly we are altogether agnostic as to whether the human faculty for reasoning targets probabilistic reasoning, deduction, or both. We will continue to use terms like "deduction" and "validity" in a descriptive sense, however, referring to the at least superficial nature of the tasks at hand, which are ostensibly about deduction and validity, and not about probabilistic inference. Additionally, we continue to refer to deviations from the classical norm of deduction as "fallacies," a term we also mean purely descriptively.

4 There are, however, good reasons to think that a probabilistic component is part of the phenomenon. Sablé-Meyer & Mascarenhas (2021) have shown that illusory inferences are possible with second premises that do not entail an element of the first premise, but are instead merely probabilistically connected to it. They show that the complete story involves combining the erotetic theory with a probabilistic strategy for selecting an alternative from the first premise based on the information in the second premise. Crucially, the role of alternatives as studied in linguistic semantics is no less central in their theory, and the probabilistic component is one of Bayesian confirmation theory, rather than the standard of rationality in "New Paradigm" probabilistic approaches. This more complex version of the erotetic theory is not needed for the cases of interest in this article, so we refrain from presenting it.

2 Illusory inferences with modals

2.1 Illusory inferences with epistemics: Mascarenhas & Picat (2019)

Mascarenhas & Picat (2019) pursued the hypothesis that there is a strong connection between inquisitive-semantics/Hamblin-style alternatives and illusory inferences by testing the English epistemic possibility modal *might*. They based their conjecture on an analysis of *might* by Ciardelli, Groenendijk & Roelofsen (2009) in the framework of inquisitive semantics, involving *attentive content*, which would draw the listener's attention to a single proposition. Consider the sentence in (6).

(6) John might be in London.

Ciardelli, Groenendijk & Roelofsen (2009) argue for a version of inquisitive semantics where a sentence may give rise to alternatives displaying proper inclusion between them. They propose that the meaning of (6) should be seen as identical to (7a) in their version of inquisitive semantics, where $\top$ is the tautology. Accordingly, they argue that (6) gives rise to the alternative set in (7b).

(7) a. John is in London $\vee$ $\top$
b. {John is in London, $\top$}

Crucially, the sentence as a whole is not a tautology in inquisitive semantics. It is informationally idle in that an update with this sentence will not exclude possibilities from any common ground. Nevertheless, it contains two distinct alternatives, it is not identical to the interpretation of the tautology, and is therefore not equivalent to it in inquisitive terms. Ciardelli, Groenendijk & Roelofsen (2009) argue that the semantic contribution of a *might* sentence is not to provide information or raise a *bona fide* question, but simply to *draw attention* to a single possibility. Since one of the alternatives is the trivial alternative, to which it makes no sense to draw attention, a *might*-sentence in this semantics offers *one* alternative for consideration, namely: in the case of (6), the proposition that *John is in London*.[5]

5 Ciardelli (2009) and Ciardelli, Groenendijk & Roelofsen (2009) stress that their analysis is not meant to capture epistemic uses of *might*. They argue that this may be an advantage of their analysis, as *might* has been argued to differ from other epistemic operators. However, we see no principled reason not to assume that different epistemic operators differ in their sensitivity to alternatives.

Consider now a conjunction embedded under *might* as in (8a), which in the view just presented is to be analyzed as (8b).

(8) a. $might(a \wedge b)$
b. $(a \wedge b) \vee \top$

The logical structure in (8b) is a special case of the first premise of standard illusory inferences from disjunction reviewed above: $(a \wedge b) \vee c$. If this analysis is on the right track, then we expect the schema in (9) to give rise to illusory inferences.

(9) **Premise 1**: $might(a \wedge b)$
Premise 2: a
Fallacious conclusion: b

Mascarenhas & Picat (2019) used reasoning tasks to test the hypothesis that *might* introduces alternatives and thus invites illusory inferences. The target reasoning problems had the structure in (9), instantiated in (10).

(10) Miranda might play the piano and be afraid of spiders.
Miranda plays the piano.
Fallacious conclusion: Miranda is afraid of spiders.

If the fallacious conclusion in (10) is an illusory inference resulting from erotetic reasoning, a certain pattern is predicted, also based on what previous research on indefinites and disjunction has revealed. First, if the fallacy is indeed attractive, the rate of "yes"-responses to the conclusion of critical targets containing modals should be higher than the rate of "yes"-responses to invalid control problems not containing *might*.

The well-established cases of the fallacy of interest display order effects where the attractiveness of the fallacy is mitigated when the order of the premises is reversed, as would be expected if the fallacy relies on a question-answer dynamic (Mascarenhas & Koralus 2017, Koralus & Mascarenhas 2018). Thus, an effect of order is predicted for the cases at hand should erotetic reasoning be involved. Additionally, fallacious conclusions should disappear altogether if the two premises are combined into one sentence, "flattening" the question-answer structure.

To test these predictions, Mascarenhas & Picat (2019) included reasoning problems with the structure in (10) in their study, as well as items with the order of the two premises reversed, and a "flat" structure version that

combines the truth-conditional information in the two premises into a single sentence.

Additionally, they considered the possibility that the first premise alone might suffice to prompt the conclusion, a plausible hypothesis since the first premise certainly raises the probability of the embedded conjunction. If fallacious conclusions in the canonical case in (10) were entirely attributable to the first premise alone, then these fallacious conclusions would not constitute a case of the illusory inferences of interest.

Summing up, they tested the four conditions in (11) in a between-subjects design, due to the high degree of similarity between the stimuli in the conditions.

(11)
a. CANONICAL $might(a \wedge b), a \vdash b$
b. P1 $might(a \wedge b) \vdash b$
c. FLAT $a \wedge might(b) \vdash b$
d. REVERSED $a, might(a \wedge b) \vdash b$

They found that canonical and reversed targets showed significantly more "yes"-responses than "no"-controls, displaying the fallacious conclusion they predicted. Canonical and reversed targets were also more accepted than flat targets, which in turn were no different than invalid controls, showing as expected that the mistake disappears if the question-answer dynamic is destroyed. They also found that canonical targets were more accepted than P1 targets, proving that the fallacy cannot simply be explained in that the first premise raises the probability of the embedded conjunction. They failed to find an effect of the order of premises 1 and 2 being reversed, however; canonical and reversed targets were not significantly different. Mascarenhas & Picat (2019) tentatively attribute this null result to their experiment's being underpowered to detect an effect of the size witnessed in earlier studies for canonical vs. reversed illusory inferences, a drop in endorsement of the fallacy in the order of ten percentage points (Mascarenhas & Koralus 2017). Lastly, they found that the rate of fallacies with *might* was comparable to that with indefinites, which suggests, they argue, that the fallacy is based on erotetic reasoning alone and not on scalar implicatures.

Mascarenhas & Picat (2019) conclude that the findings are in line with Ciardelli's (2009) proposal that *might* introduces inquisitive/Hamblin alternatives but could also be made sense of from of a Kratzerian view on possibility modals together with the assumption that existential quantifiers raise alternatives.

2.2 Illusory inferences with deontics

Mascarenhas & Picat (2019) argue convincingly that *might* produces fallacious inferences. With the exception of the null result for order effects between premises, these fallacious inferences have properties particular to illusory inferences with disjunction and indefinites of interest, lending weight to the idea that the same mechanism is at play.

However, their results are merely compatible with the strong hypothesis that all and only inquisitive-semantics/Hamblin-style alternatives give rise to illusory inferences of the relevant kind in reasoning. To test this more ambitious hypothesis, we need to extend the methodology of connecting alternative generation with inference-making behavior to more cases. In particular, it is important to find a case that is close enough to the existing cases to justify a suspicion that illusory inferences might be found, while being distinct enough analytically to support substantive theoretical considerations. We propose adding deontic modals to the mix.

Extending Mascarenhas & Picat's (2019) paradigm to deontic modality addresses several questions left open by their study, and can potentially inform our theories of modality and reasoning. To our knowledge, no theory of *deontic* possibility modals exists that argues that they are alternative generators in the sense introduced above. Yet, they are very close in form and meaning to epistemic modals (Kratzer 1977, 1991, 2012), for which such a theory might in principle be formulated, as outlined by Mascarenhas & Picat (2019). We return to these theoretical considerations at the end of this article.

2.3 Experiment — *be allowed to* versus *might*

2.3.1 Methods

Design and Materials Like Mascarenhas & Picat (2019), we used an inference-making task to test a series of predictions related to the alternative-generating power of epistemic and deontic possibility modals.

The target reasoning problems involved one of two modals, epistemic *might* or deontic *allowed to*. They instantiated one of the three structures in (12). We treated STRUCTURE as a between-subjects factor due to the sentences in the three conditions being very similar. We fully crossed this factor with the factor MODALITY, which we treated as a within-subjects factor. As

a result, a given participant saw only one of the structure conditions in (12), but saw both modals, in different items.[6]

(12) a. CANONICAL might have/was allowed to $(a \wedge b), a \vdash b$
b. FLAT $a \wedge$ might have/was allowed to $(a \wedge b) \vdash b$
c. REVERSED a, might have/was allowed to $(a \wedge b) \vdash b$

We give examples of each structure with epistemic and deontic modality in (13).

(13) a. John {might have stolen / was allowed to steal} from the rich and give(n) to the poor; John stole from the rich ⊢ John gave to the poor. CANONICAL
b. John stole from the rich, and he {might have stolen / was allowed to steal} from the rich and give(n) to the poor ⊢ John gave to the poor. FLAT
c. John stole from the rich; John {might have stolen / was allowed to steal} from the rich and give(n) to the poor ⊢ John gave to the poor. REVERSED

We also varied which of *a* vs. *b* from the first premise appeared in the second premise, controlling for any potential order effects from the embedded conjunction. This last factor ORDER was tested within subjects.

In sum, we manipulated 3 factors: MODAL with two levels (deontic or epistemic), STRUCTURE of the reasoning problem with three levels (canonical, flat, reversed) and in which ORDER the two conjuncts in premise 1 appeared in premise 2/conclusion (order "left": left conjunct of premise 1 appeared in premise 2, right conjunction in conclusion; order "right": right conjunct of premise 1 appeared in premise 2, left conjunct appeared in conclusion). Thus, we had $2 \times 2 \times 3 = 12$ conditions. We created 6 items per within-subjects condition (4 in total, DEONTIC–LEFT, DEONTIC–RIGHT, EPISTEMIC–

6 There were two minimal changes in the material compared to that tested by Mascarenhas & Picat (2019). First, we minimally changed the *flat* structure condition. This was because the more extreme version of the flat condition *a and* MODAL*(b)* used previously prompts additional inferences in the deontic case that are absent from the epistemic case. Specifically, *a and allowed to b* may give rise to the implicature *not allowed to a* given that *a* is a salient excludable alternative. This reading is not available for *might*, as *not might a* contradicts *a*, and therefore cannot be excluded innocently (Fox 2007). We also changed the tense to the past form. We did this to avoid the ambiguity of the present tense in English, and a possible interference with type of modality. We did not expect this change to affect epistemic *might*.

LEFT, EPISTEMIC–RIGHT). These 24 critical items were distributed across 4 experimental lists for each group factor (STRUCTURE) separately, resulting in 12 sub-experiments that participants were randomly assigned to.

Besides target items, we used 12 control items and 12 baseline items. Control items served the purpose of eliminating unfocused participants. They used valid and invalid conclusions based on *modus ponens* and *disjunctive syllogism*. Baseline items used conjunction elimination and were used to establish a baseline for error rates.

Participants and Procedure We recruited 183 participants via Prolific; 56% were female and their mean age was 33 (ranging from 20 to 60, SD = 13.2). Participants were assigned to one of 12 experimental lists and each solved 48 reasoning problems. The procedure was the same as used in Mascarenhas & Picat 2019. Participants were presented with premises and a proposed conclusion. They had to evaluate whether the conclusion followed from the premises. Before the core of the experiment began, we explained and exemplified the concepts of valid and invalid conclusion.

Our exclusion criteria were the same as in Mascarenhas & Picat 2019. We excluded people who reported having often taken notes, having taken at least one graduate-level course in semantics/pragmatics, or who answered fewer than 50% of control questions correctly. With these exclusion criteria we removed 26 participants (14.2%) and analyzed data from 157 participants.

2.3.2 Results

We analyzed the results with generalized linear mixed-effects models, using the `lme4` package and `glmer` function in `R`. We report the most complex converging model. Fixed effect factors were MODAL with levels deontic/epistemic (reference level: deontic), ORDER OF CONJUNCTS with levels left/right (reference level:left), STRUCTURE with levels canonical, flat, reversed (reference level: canonical). All fixed effects were dummy-coded. We calculated contrasts with pairwise comparisons using the `emmeans` package in `R` and used Holm correction for multiple comparisons. We report both corrected and uncorrected p-values.

First, we looked at whether deontic modals give rise to fallacious inferences of the form *modal(a and b); a* $\vdash$ *b*. We compared the rate of falla-

Comparison	Est.	Std.Err	p-value	Corrected p-value
Deontic canon. vs. no-base.	6.9	1.5	$p < 0.0001$	$p < 0.001$
Epist. canon. vs. no-base.	6.2	1.5	$p < 0.0001$	$p < 0.01$

Table 1 Summary of output from the model testing the influence of inference type on responses.

cies with canonical structures to "no"-baselines for both modals separately.[7] Our analysis shows that the rate of "yes"-responses differs significantly from "no"-baselines for both modals; see Figure 1 and Table 1.

To test whether the two modals are affected differently by STRUCTURE, we looked at the interaction between STRUCTURE and MODAL in our analysis. We compared the maximally converging model with the interaction term (RESP ~ MODAL*STRUCTURE + (1 | SUBJECT) + (1 | ID)) to a model without the interaction term (RESP ~ MODAL+STRUCTURE + (1 | SUBJECT) + (1 | ID)). The nested model comparison via log likelihood ratio tests revealed that the interaction term is justified ($\chi^2(2)=33.887$).

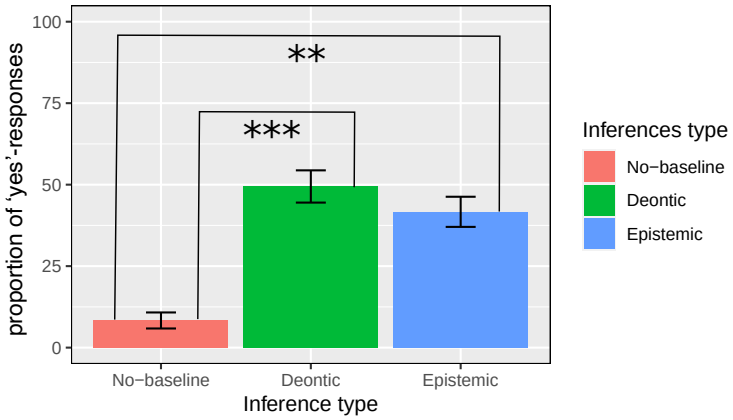

Figure 1 Proportion of "yes"-responses by inference type; error bars indicate the standard error of the mean. Black lines and stars indicate significant contrasts.

7 Specifically, we used a subset of the data containing "no"-baseline items, canonical structures for epistemics, and canonical structures for deontics and ran the maximally converging model: RESP ~ INFERENCE-TYPE + (1+INFERENCE-TYPE | SUBJECT) + (1 | ID). We used dummy-coding with "no"-baselines as the reference level.

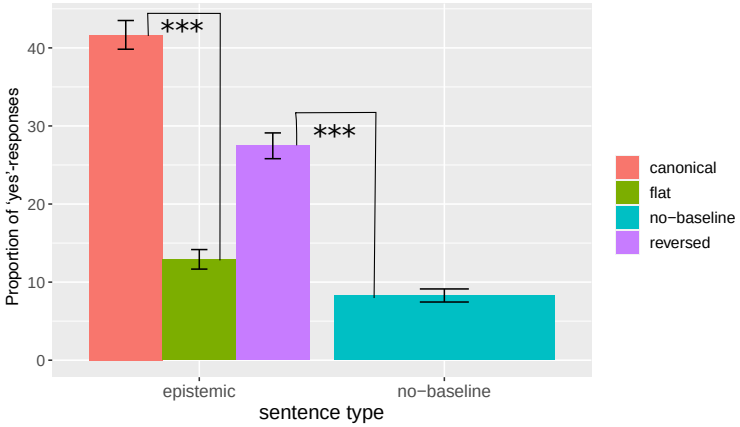

Figure 2 Proportion of "yes"-responses by STRUCTURE versus "no"-baselines for epistemics; error bars indicate the standard error of the mean. Black lines and stars show significant contrasts.

The interaction shows that structure affects epistemic and deontic modals differently. Zooming in on contrasts, we observe a significant difference between canonical versus flat structures for both modals, with the effect being more pronounced for epistemics. Canonical and reversed structures, however, do not differ significantly from each other for deontics. The contrast is only marginally significant for epistemics (Table 2). This result for epistemics is consistent with previous experimental results showing that the effect of order of premises is very subtle, and only visible with a very high number of participants (Mascarenhas & Koralus 2017). The presence of a significant interaction together with the difference in estimates, however, is suggestive of epistemics being affected by reverse versus canonical order of premises, whereas deontics are not.

MODAL	Comparison	Est.	Std.Err	p-value	Corrected p-value
Deontics	Canon. vs. rev.	0.01	0.5	$p = 0.99$	$p = 0.99$
	Canon. vs. flat	1.5	0.5	$p < 0.01$	$p < 0.05$
Epistemics	Canon. vs. rev.	0.98	0.5	$p < 0.06$	$p = 0.11$
	Canon. vs. flat	2.8	0.5	$p < 0.0001$	$p < 0.001$

Table 2 Contrasts of interest for the model testing the interaction between structure and modal

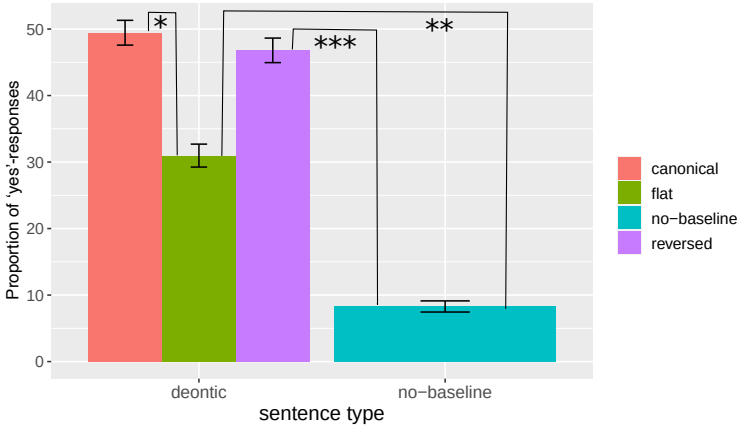

Figure 3 Proportion of "yes"-responses by STRUCTURE versus "no"-baselines for deontics; error bars indicate the standard error of the mean. Black lines and stars indicate significant contrasts.

Zooming in further, we looked at the contrast between "no"-baseline items and flat structure items, and "no"-baseline items versus reversed target items, for both modals separately.[8] We see that, for epistemics, flat structures do not differ significantly from "no"-baselines (Table 3). However, for deontics, they do (Table 4). Furthermore, there is a significant difference between reversed targets and "no"-baselines for both modals. The contrasts we find for each modal are displayed and highlighted in Figures 2 and 3.

Comparison	**Est.**	**Std.Err**	***p*-value**	**Corrected *p*-value**
Flat vs. no-baseline	0.5	2.2	$p = 0.83$	$p = 1$
Reversed vs. no-baseline	5.2	1.5	$p = 0.00053$	$p < 0.0001$

Table 3 Output of the model looking for the effect of structure (flat and reversed) on responses for epistemics ("no"-baselines as reference level)

8 We created a subset of data for each modal. Then, we used the same model as before: RESP ~ INFERENCE-TYPE + (1+INFERENCE-TYPE | SUBJECT) + (1 | ID)). We used dummy-coding with "no"-baselines as the reference level.

Comparison	Est.	Std.Err	p-value	Corrected p-value
Flat vs. no-baseline	5.0	1.2	$p < 0.0001$	$p < 0.001$
Reversed vs. no-baseline	6.5	1.2	$p < 0.0001$	$p < 0.001$

Table 4 Output of the model looking for the effect of structure (flat and reversed) on responses for epistemics ("no"-baselines as reference level)

We also investigated the contrast between modals for each structure. We summarize this contrast in Table 5. We observe that there are significantly more "yes"-responses for deontics than for epistemics for each structure.[9]

Comparison	STRUCTURE	Est.	Std.Err	p-value	Corrected p
Epist. vs. deont.	Canonical	0.6	0.14	$p < 0.0001$	$p < 0.01$
	Flat	1.9	0.2	$p < 0.0001$	$p < 0.001$
	Reversed	1.6	0.2	$p < 0.0001$	$p < 0.001$

Table 5 Contrasts between epistemic (reference level) and deontic modality for each structure.

2.4 Summary of experimental results

We replicated the findings for epistemics from Mascarenhas & Picat 2019. There are fewer fallacies for reversed and flat targets than there are for canonical targets. Furthermore, reversed and canonical targets differ from "no"-baselines, whereas flat targets do not.

We observed a significant rate of fallacies with deontics as well, as evidenced by a significantly higher rate of "yes"-responses to canonical targets than "no"-baselines (main effect). However, the rate is significantly higher

9 We also checked for a possible effect of ORDER OF CONJUNCTS (Was the left or right conjunct of premise 1 mentioned in premise 2/conclusion?) by comparing a model RESP ~ MODAL*ORDER*STRUCTURE + (1+MODAL*ORDER | subjectId)+ (1+ORDER | id) to MODAL+ORDER+STRUCTURE + (1+MODAL*ORDER|subjectId)+ (1+ORDER|id). We found the interaction term to be justified ($\chi^2(6)=13.683$). Since we had no specific predictions regarding the effect of ORDER of conjuncts, we refrain from offering *post-hoc* speculative explanations. The presence of an interaction, however, suggests that following the linear order of presentation of the conjunctions in premise/conclusion is not (one of) the main sources of the overall effect.

than for epistemics and they are much less sensitive to structure (interaction). Specifically, we found no difference between reversed and canonical targets with deontics, and we found that deontic flat structures produce a fallacy.

3 General Discussion

At first glance, our results appear to challenge the conjecture that the fallacies of interest are intrinsically connected with inquisitive/Hamblin alternatives: we found what look like the epistemic illusory inferences in deontics. This is surprising because, unlike in the case of epistemics, no extant theories of deontics propose that they generate the relevant kinds of alternatives.

However, standard possible-worlds accounts see epistemic *might* and deontic *allowed to* on par with one another and the common core semantics traditionally proposed for both cases, and indeed for possibility modals in general, is existential in nature: *There is* some (properly restricted) accessible world where the prejacent proposition is true. If this existential quantifier in the truth conditions for possibility modals were interpreted within inquisitive/Hamblin semantics, it would give rise to alternatives, roughly corresponding to the question "Which accessible possible world is such that the prejacent is true in it?" This idea can be developed into an account of illusory inferences with epistemics quite naturally.

Assuming the relevant ordering between possible worlds is well founded (the limit assumption), Kratzer's (1991) semantics for modality predicts that a sentence *might*(ϕ) will be true just in case there is a ϕ-world among the best-ranked worlds. Along with a few assumptions, there is a strategy for accounting for epistemics in a relational semantics such as Kratzer's.

(14) a. **Assertion as truth in the actual world**
When asserting a proposition ϕ, a speaker communicates (their belief) that ϕ holds in the actual world.

b. **Reflexivity**
The epistemic modal base is reflexive, that is $w \in f(w)$.

c. **Inquisitiveness of existentials and erotetics**
The existential quantifier that occurs *in the truth conditions* of a *might*-sentence is inquisitive. Human reasoning is erotetic, and in particular inquisitive existential quantifiers raise questions that are resolved along the lines outlined in Section 1.3 for illusory inferences with indefinites.

Under these assumptions, the first premise of epistemic fallacies *might*($a \wedge b$) provides some information about the existence of a possible world, and draws attention to the question "Which is this best-ranked $a \wedge b$-world we're discussing?" Asserting the second premise then states that the actual world is an a-world. Now erotetic mechanisms kick in: The actual world is an a-world, and this points in the direction of answering the question at hand with "The actual world is the best-ranked $a \wedge b$-world in question." From here, b follows immediately.

Extending this account to deontics in order to assimilate them to epistemics is in principle possible and constitutes an important theoretical avenue to explore. The main trouble is that the assumption of reflexivity in (14b), entirely standard for epistemic modality, is generally unwarranted in deontic modality. Indeed, reflexivity in a deontic frame guarantees that everything that is in fact the case is ipso facto permitted. This is highly implausible as an assumption that experimental participants should make when reasoning about deontic problems, but it remains to be seen whether there are other ways to leverage the shared *existential* semantics of epistemics and deontics and extend the relational account of epistemics we just sketched to a general theory that predicts illusory inferences for *all* possibility modals. In the remainder of this discussion section, however, we pursue a very different approach.

Further experimental and theoretical research is needed that would lie outside the scope of this reply, but we submit that there are two orders of reasons to suspect that the fallacies we found with deontics are in fact of a different kind than the ones of interest, found with disjunctions, indefinites, and epistemics. In other words, the deontic fallacies are generated by a different mechanism (expanded on below) than the ones with epistemics, and specifically deontic fallacies do not originate from the same alternative-generating and alternative-handling mechanisms as epistemic fallacies. Below, we furthermore propose a novel operationalization of the inference-making phenomenon of interest that defines a more complex signature for these fallacies, going beyond simply looking at the rate of acceptance of canonical cases. Our proposed definition of "illusory inferences with alternatives" puts together previously observed properties of the clear and experimentally robust cases with disjunctions and indefinites. We submit that the picture that emerges allows us to distinguish epistemics from deontics, thereby rehabilitating Mascarenhas & Picat's (2019) hypothesis that illusory inferences of this kind are inextricably connected to inquisitive/Hamblin al-

ternatives as studied by semanticists, and without pursuing the radical and unlikely hypothesis that all possibility modals should give rise to illusory inferences.

3.1 Another theory of deontic "fallacies"

We found that about 50% of the time our participants drew the fallacious conclusion in (15).

(15) **Premise 1**: *allowed to* $(a \wedge b)$
Premise 2: a
Fallacious conclusion: b

There is an independently observed property of deontics that would produce behavior as in (15) for deontics, without making such predictions for epistemics. Witness first that the inference in (16a) does not necessarily go through for the sentence in (16). This effect has been dubbed a *package deal* by the literature on deontics (Merin 1992, van Rooy 2000).

(16) John is allowed to steal from the rich and give to the poor.
a. $\nrightarrow$ John is allowed to steal from the rich.

The effect can be described as a sentence 'allowed to a and b' only giving permission to do both or neither a or b, which van Rooy (2000) calls *biconditional permission*.[10] For reasons of space we cannot go into the details of the suggested analyses. Still, we would like to propose that the *package deal* reading could play a role in the rate of fallacies we observe, despite the fact that it does not make the inference that John stole from the rich a valid one: Given that there is no reflexivity with deontic modals, it is not granted that John only did what he was allowed to do. However, the first premise suggests that he partly did what he was allowed to do, making it more plausible that he is following the rules. This, in turn, means that he likely respects the package deal, and wouldn't perform one action without also performing the

10 The effect seems to get stronger the more related the two conjuncts are, as is the case in the example given. For that reason, we varied the degree to which it was possible to create a link between the two conjuncts in our items; see the sentence material in the appendix. For the analysis, we put items in different groups according to the degree of relatedness and looked at them separately in the analysis. We saw no differences between groups. To see whether this speaks for or against an analysis based on *package deals*, we need independent evidence that this reading indeed exists and a clear idea of what factors might influence its generation.

other, making the observed conclusion a very attractive one. Crucially, this attractiveness stems from a mechanism entirely independent from alternative generation or question-answer dynamics.

This line of explanation clearly requires more investigation as the reasons for drawing the fallacious conclusion could be multi-factorial, with *package deal* effects only being one contributing factor.

3.2 Anatomy of illusory inferences from alternatives

The most conspicuous property of the illusions of interest is of course drawing the target fallacious conclusion. Yet, the well-studied data pertaining to disjunctions and indefinites strongly suggest that these inferences have a more complex signature. We propose that the conjunction of four points is required to diagnose the illusory inferences of interest.

A "Fallacy" More fallacious conclusions than for invalid controls. The hallmark of illusory inferences from alternatives is a fallacious conclusion that picks one of the available possibilities (alternatives) raised by the first premise on the basis of merely partial overlap with the information in the second premise.

B "Order of premises" Fewer fallacious conclusions with reverse order of premises. The erotetic theory of reasoning explains illusory inferences from alternatives in terms of a question-answer dynamic. Reasoners consider the disjunctive or indefinite first premise as raising a question, to which the second premise provides a hint at an answer. It should be harder for participants to engage in question-based reasoning if the order of the two premises is reversed. Indeed, illusory inferences with disjunction and with indefinites both show a drop in acceptance of about 10 percentage points when the order of the premises is reversed (Mascarenhas & Koralus 2017, Koralus & Mascarenhas 2018). Crucially, the same experimental studies did not find a drop in acceptance in control inferences with premises of comparable syntactic complexity.

C "Dynamics" No fallacious conclusions without dynamics. A much more radical mitigation of the phenomenon occurs if the dynamic structure of the stimulus is altogether destroyed (as in the flat structures in this article and its

precursor). This is a novel criterion we propose due to the findings presented in this paper and Mascarenhas & Picat 2019. When the order of the premises is reversed and combined into a single premise with a conjunction, we see no illusory inferences from alternatives.

D "Rate" Rate of fallacious conclusions depends on available mechanisms. As explained in the introduction, we expect there to be a lower rate of fallacies if the mechanism behind arriving at the conclusion is solely based on erotetic reasoning. Illusory inferences with disjunctions have alternative accounts in terms of scalar implicature, while illusory inferences with indefinites lack such alternative accounts, explaining their diverging acceptance rates.

In view of these criteria, the fallacies with *might* constitute an instance of the illusory inferences of interest, for they satisfy all points A-D, as do the paradigmatic cases of disjunctions and indefinites. Yet, the fallacies we found with deontics do not satisfy all of these points. Specifically, they fail B and C, and have a doubtful status for D. Accordingly, we cautiously propose that the fallacies with deontics are not an argument in favor of their inquisitive/Hamblin-alternative potential, but the product of altogether different mechanism(s).

4 Conclusions

We extended experimental work on reasoning with alternatives from epistemic modals to deontic modals. Our goal was to investigate the hypothesis that all and only linguistic elements that generate inquisitive/Hamblin alternatives produce the illusory inferences the literature has found with disjunctions and indefinites. We found fallacies with deontics reminiscent of the ones with epistemics, but we failed to find other characteristic properties of the illusory inferences with disjunctions and indefinites. Our results are compatible with two main interpretations. The first is that deontics produce alternatives just like epistemics. From a theoretical standpoint, it would be most natural to leverage the shared existential semantics of all possibility modals in relational semantics and give a unified account of the alternative-generating potential of epistemics and deontics. The central challenge in this connection will be to discover plausible assumptions about the accessibility relation for deontics that would derive this result. From an experimental per-

spective, the challenge will be to explain the sharp differences we found between epistemics and deontics once one zooms out from the main fallacious structure of interest to consider well-established properties of these fallacies with disjunctions and indefinites, the paradigmatic cases of alternative generation and illusory inferences.

The second possibility is that only epistemics generate alternatives and participate in the illusory inferences of interest. The fallacies we observed with deontics would be generated by an entirely unrelated mechanism. We proposed that the notion of package deals, independently proposed in the literature on deontics, would predict the inference-making behavior we observed without postulating alternatives. Additionally, this analysis would explain the fact that fallacies with deontics have a different broader signature than fallacies with epistemics, since they would be generated by entirely independent mechanisms.

5 Appendix

Targets

John might have stolen from the rich and given to the poor.
John stole from the rich.
John gave to the poor.

Daniel was allowed to skip a day of work and go to a march in Washington.
Daniel skipped a day of work.
Daniel went to a march in Washington.

George might have skipped afternoon classes and attended team practice.
George skipped afternoon classes.
George attended team practice.

Audrey was allowed to go to the mall and buy a book.
Audrey went to the mall.
Audrey bought a book.

Linda might have left the base and called her parents.
Linda left the base.
Linda called her parents.

Riley was allowed to take Mary's car and go grocery shopping.
Riley took Mary's car.
Riley went grocery shopping.

Alexander might have had his friends over and played video games.
Alexander had his friends over.
Alexander played video games.

Bob was allowed to go to a bar and talk to a friend.
Bob went to a bar.
Bob talked to a friend.

Sam might have skipped school and visited his grandmother in the hospital.
Sam skipped school
Sam visited his grandmother in the hospital.

June was allowed to visit her boyfriend and do her homework.
June visited her boyfriend.
June did her homework.

Paul might have taken his parents' car and picked up dinner.
Paul took his parents' car.
Paul picked up dinner.

Jeremy was allowed to leave school and get a job.
Jeremy left school.
Jeremy got a job.

John might have stayed at school till late and used the library.
John stayed at school till late.
John used the library.

Thomas was allowed to take a painkiller and call a friend.
Thomas took a painkiller.
Thomas called a friend.

Bill might have gone fishing and cooked dinner.
Bill went fishing.
Bill cooked dinner.

James was allowed to join the army and study medicine.
James joined the army.
James studied medicine.

Peter might have called Mary and talked about church.
Peter called Mary.
Peter talked about church.

Heather was allowed to buy a lottery ticket and sell her car.
Heather bought a lottery ticket.
Heather sold her car.

Laura might have gone to a concert and drunk beer.
Laura went to a concert.
Laura drank beer.

Charlotte was allowed to take the bus and go to the dentist.
Charlotte took the bus.
Charlotte went to the dentist.

Brittany might have gone to France in summer and sold her guitar.
Brittany went to France in the summer.
Brittany sold her guitar.

Owen was allowed to go to a baseball game and apply for a job.
Owen went to a baseball game.
Owen applied for a job.

Jean might have run a marathon and bought tickets for the super bowl.
Jean ran a marathon.
Jean bought tickets for the super bowl.

Nathan was allowed to open a bank account and get a SIM card.
Nathan opened a bank account.
Nathan got a SIM card.

Controls

If yesterday was Wednesday, Carol went to the theater.
Yesterday was Wednesday.
Carol went to the theater.

If Arthur's favorite team won the game, he partied all night long.
Arthur's favorite team won the game.
Arthur partied all night long.

If George dyed his hair, Mary was delighted .
George dyed his hair.
Mary was delighted.

Daniel ate an apple or a pear.
Daniel did not eat an apple.
Daniel ate a pear.

Kit or Rose learned Latin at school.
Rose did not learn Latin at school.
Kit learned Latin at school.

Sally or Norman came to the party.
Sally did not come to the party.
Norman came to the party.

If Brian was brave, he asked Lydia out.
Brian got a cat.
Brian asked Lydia out.

If Sam was hungry, he ate three cheeseburgers.
Sam bought a new computer.
Sam ate three cheeseburgers.

If Bruce went to Tokyo, he took a lot of pictures.
Bruce spent the weekend with his cousin.
Bruce took a lot of pictures.

Selina or Robin came early this morning.
Robin did not come early.
Selina got engaged.

Lois knows how to juggle with 4 balls or how to do a back flip.
Lois does not know how to juggle with 4 balls.
Lois knows how to breathe fire.

Clint or Wanda ate the whole cake.
Clint did not eat the whole cake.
Wanda loves chamber music.

Baselines

Luke moved to New York and bought a new phone.
Luke moved to New York.

Joan visited her mother and bought new shoes.
Joan bought new shoes.

Diego went to a basketball game and stopped at the gas station.
Diego went to a basketball game.

Roberta picked out a wedding present and paid her bills.
Roberta paid her bills.

Kim made coffee and greeted a colleague.
Kim made coffee.

Janine played chess with her brother and went grocery shopping.
Janine went grocery shopping.

Hugo watched a play with a friend.
Hugo watched a play with a friend and went to a dinner party.

Sina handed out flyers for a restaurant.
Sina handed out flyers for a restaurant and picked up a book from the library.

Carlos picked up his daughter from school.
Carlos picked up his daughter from school and mowed the lawn.

Estelle signed a lease and called her landlord.
Estelle signed a lease and went for a run.

Vaughn booked a vacation and changed the oil in his car.
Vaughn went on a date and changed the oil in his car.

Fiona went to Starbucks and registered for classes.
Fiona met her neighbor and went to Starbucks.

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