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Referential transparency as the proper treatment for quantification*

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Abstract An important motivation for Montague's work on quantification (Montague 1973) was to achieve uniformity with respect to referential and quantificational subjects. This was attained by type raising all NPs to denote sets of sets (indeed there are claims that such a move is theoretically necessary) and by giving up a subject–predicate semantics where the verbal predicate predicates of the nominal argument. In this paper we argue for essentially the opposite move whereby all predication is genuine predication and involves arguments — *witnesses* of type *individual* or *set of individuals* (for plurals). We argue that such an approach is crucial if one is to capture a variety of fundamentally important phenomena involving anaphora, clarification interaction, and speech-gesture cross-references associated with the use of quantificational noun phrases in dialogue, and to explicate several recent key psycholinguistic results on quantifier processing — all features of an NP semantics which give rise to what we call "Referential Transparency". The discussion is couched in a new set-denotational framework for plural count nouns, namely *sets of ordered set bipartitions*. We argue that quantification happens entirely within the noun phrase and involves ref(erence)sets, comp(lement)sets, and max(imal)sets. As a corollary of this denotational foundation, the semantic conservativity universal is an immediate consequence and the range of quantifier denotations is significantly reduced. In addition to collecting empirical motivation for quantification from Referential Transparency Theory and to developing a count noun semantics, a theoretically grounded explanation for complement set anaphora is given.

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1 Motivation

One of the great achievements of generalized quantifier theory (GQT, Montague 1973, Barwise & Cooper 1981) is a uniform syntax–semantics interface. However, recent work in GQT has considered at least three topics, or obstacles, namely verb–noun predication, the type of quantified noun phrase (QNP) contents queried by clarification requests, and the (overly?) large logical space of quantifiers, which are reviewed in Sections 1.1, 1.2, and 1.3. Each of these obstacles seems to be solvable, but at the price of introducing additional machinery, which fixes the *technical* issue involved, but seems to lack further motivation. Given this, we propose a new theory of quantification for count nouns and collect supporting empirical motivation for the proposal. Sections 1 and 2 introduce sets of ordered set bipartitions as new denotations for nouns, and show how to derive witnesses therefrom. Witnesses are needed to explain the set status of quantified noun phrases as queried by clarification requests; Predication is then modelled as ordinary predication on (sets) of individuals. This kind of denotation gives rise to a significant reduction of the logical space of quantifiers. Sections 3 and 4 collect empirical evidence for the denotational theory from the first part. We first motivate a couple of semantic probes — summarized as *Referential Transparency* — that can be used to discover the structure and type of QNP contents. We then use Referential Transparency to refine the semantic representation of the content of QNPs. We argue that a QNP has to be represented as hosting a set triplet (a reference set, a complement set, and the union of both), where the reference set and the complement set can be straightforwardly construed in terms of the set bipartitions from the denotational framework in the first part. The semantic contribution of a quantifier word is represented in terms of a descriptive quantifier condition, a relation on the cardinalities of reference and complement set. We apply this "QNP anatomy" (Cooper 2013) to complement set anaphora and provide an explanation for its availability in terms of our new denotational framework (namely that complement set anaphora is only possible with QNPs whose quantifiers do not exclude the bipartition containing an empty reference set partition).

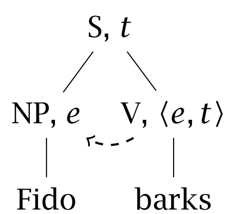
1.1 Obstacle 1: Predication and the syntax–semantics interface

Natural languages are efficient tools for attribution. In Latin, for instance, Caesar, with the first sentence of his *Bellum Gallicum*, attributes the property of being divided into three parts to the whole of Gaul: "Gallia est omnis divisa in partes tres […]" (*Gaul as a whole is divided into three parts*). This predicational structure is also reflected in grammar: the sentential head is the main verb and the verb phrase (VP) predicates of its subject noun phrase (NP). Virtually every formal grammar is set up in this way.[1]

Caesar continues: "Hi omnes lingua, institutis, legibus inter se differunt." (*These all differ in language, facilities and laws*, where *These all* refers to Belgians, Aquitaineans and Celts, the inhabitants of Gaul in those times.) Obviously, the property of differing in language, facilities and laws is predicated of the referent of *These all*, that is, of Belgians, Aquitaineans and Celts.[2] We would expect our grammar to reflect this. However, Caesar uses a quantifier word in forming his subject (*omnes*, 'all', nominative plural). It is not difficult to come up with logical representations for sentences containing quantified subjects. Consider *Fido barks* and *Every dog barks*. *Fido barks* is translated into the simple predication $\text{bark}'(f)$, and *Every dog barks* is represented by $\forall x[\text{dog}'(x) \implies \text{bark}'(x)]$. A problem with the latter formula is that there is no direct counterpart for the NP *every dog* within the logical form.

Using functional application in a Montagovian type theory, the two example sentences can be analysed as in (1) and (2), respectively.

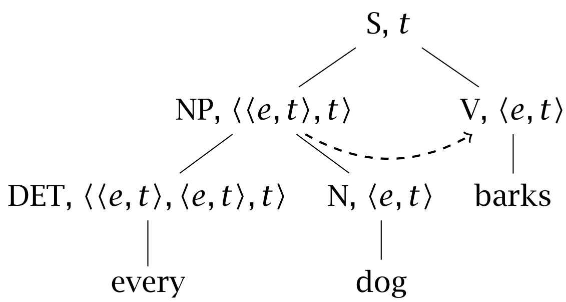
(1)



1 See Reboul 2001 on the motivating background for the reference–predication view.

2 Resolving demonstratives and accounting for sentence internal *different* have been issues of Latin semantics as much as they still are for contemporary Indo-European languages.

(2)

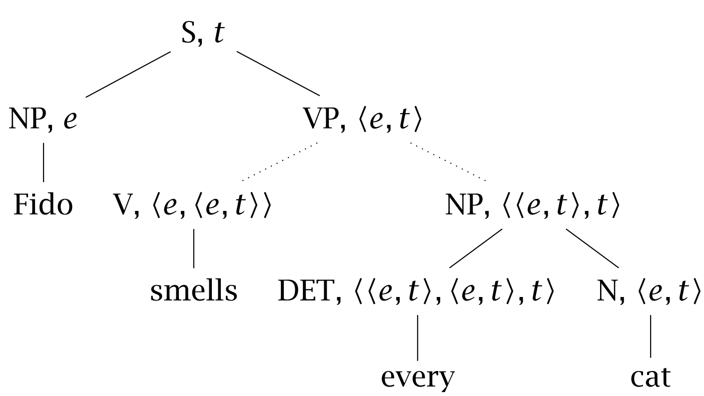


The dashed, bend arrows indicate the direction of predication: While (1) involves usual predication where the unsaturated verbal predicate applies to the saturated nominal argument, the direction of predication is reversed in (2). Hence, depending on the semantic type of subjects, there is a difference in the direction of functional application. An important achievement of Montague (1973) is to provide a uniform treatment of all NPs, proper names as in (1) or QNPs as in (2). In order to do so, all NPs are lifted to the type $\langle\langle e,t\rangle,t\rangle$: a *generalized quantifier* (GQ). All predication, then, follows the pattern in (2), where the NP (or determiner phrase, DP)[3] is the predicating expression, taking a VP as argument.

QNPs in object position induce a type-mismatch problem. Semantic composition (i.e., functional application) *via* the semantic types assigned to the constituents fails at the dotted edges: $\langle e,\langle e,t\rangle$ and $\langle\langle e,t\rangle,t\rangle$ are simply incompatible, in any direction of functional application:

(3) a. *Fido smells every cat*

b.



3 Recently, Salzmann (2020) argues again for a DP analysis. However, we think that there are good reasons for relying on NP constituents (e.g., languages without determiners like Serbo-Croatian or nominals without articles like pronouns) and couch our presentation in a grammar with nominal heads. For more motivation in favour of an NP analysis see Machicao y Priemer & Müller 2021. But none of the points we want to make hinge on this.

How to repair the type mismatch? One can distinguish *in situ* and *floating* (leaving v. moving quantifier phrases at a level of syntactic representation) approaches. *In situ* approaches repair the mismatch by postulating a type ambiguity for either NPs or VPs. Such accounts have been developed by, e.g., Partee & Rooth (1983).[4] This leads to a potential "type inflation",[5] though opinions differ on whether this is a problem or not.

Heim & Kratzer (1998) argue for a floating account: QNPs can move out of their *in situ* position in syntax into a fronted landing site in logical form, but leave a trace. Interpretation can then proceed in terms of already available rules of functional application.

There are also approaches that might be viewed as intermediate. Cooper (1975) enriches denotations so that they *store* QNP denotations and eventually these are *retrieved* to be composed with the initial non-quantificational nucleus. While syntactically *in situ*, arguably, movement is simulated in terms of the stacking of QNPs in storage.

Movement, however, raises issues with respect to psycholinguistic processing. Natural language meanings need to satisfy a constraint that is much more concrete than compositionality, namely *incrementality*: Natural language input is processed word by word (and indeed at a higher, sub-lexical latency). QNPs are no exception, at least when used in pragmatically supporting, comprehension-oriented contexts (Urbach, DeLong & Kutas 2015).[6] When sentences that contain quantificational arguments are presented as spoken input, quantifiers are interpreted in a fully incremental manner anyway (Freunberger & Nieuwland 2016), including the fact that they are interpreted *in situ* (i.e., at the position in the input string at which they occur). Quantifier raising, where a quantifier is moved out of its syntactic surface position into another position in logical form, seems to be a serious obstacle to this empirical fact.[7] Type ambiguities postulated by flexible types approaches fare

4 Another related strategy is to allow other modes of composition, e.g., using function composition (van Benthem 1990: 118).

5 Even more so if we look at properties that figure as arguments of other properties, as discussed by Chierchia (1985).

6 A "pragmatically supporting context" is established when instead of presenting stimulus sentences such as *Most/Few kids prefer sweets/vegetables* out of the blue, the experimental material is preceded by a preparatory context such as *Alex was an unusual toddler*. In a "comprehension-oriented" setting the subjects are asked to answer questions concerning the stimulus sentences. This contrasts with plausibility judgements. See Urbach, DeLong & Kutas (2015) for further details.

7 A reviewer for S&P suggests that there *is* evidence for quantifier raising deriving from studies on antecedent contained deletion in combination with QNPs in object position

better in this respect. Nonetheless, they trigger the question whether type ambiguities of the kind they posit induce the need for backtracking during parsing, comparable to garden path effects. We know of no study that has demonstrated such effects. Furthermore, any GQ account faces the question of what a mental representation of a set of sets of individuals could look like, a concern already formulated by Barwise & Cooper (1981). This eventually led to the notion of *witness sets*, which originated as an auxiliary notion for GQ processing and will be used in a much more central manner below.

1.2 Obstacle 2: Clarifying NP contents

Classical formal semantics, going back to Frege (1892), characterizes meanings in terms of (communicative) success conditions. For declarative clauses this involves *the proposition expressed*, for referential NPs *the referent of a given use*. A semantics intended for conversation is also required to explicate the resulting context in cases involving communicative *problems* since these result in the highly systematic process of *repair* (Schegloff, Jefferson & Sacks 1977) or *clarification interaction* (Purver, Ginzburg & Healey 2003), exemplified in (4).[8] Based on the communicative problem encountered, the addressee deduces a clarification question an answer to which will potentially resolve the problem.

(4) a. SARAH: Leon, Leon, sorry she's taken.
LEON: Who?
SARAH: Cath Long, she's spoken for.
(*The British National Corpus, version 2 (BNC World)* 2001, BNC file KPL, sentences 347–349)

(Koster-Moeller, Varvoutis & Hackl 2007). The study reports that sentences of the form *John talked to the student that Mary did before class* are more difficult to process (assessed in terms of reading time at word level) than sentences of the form *John talked to every student that Mary did before class*. That is, the resolution of the antecedent contained deletion (*did*) in both kinds of sentences seems to be easier with a QNP than with a definite NP. Such a difference is (according to the study's authors) only to be expected on a quantifier raising approach, not on an *in situ* one. However, there are alternative explanations for the observed effect, including an explanation that follows from our own account (see Section 4.7): An *every*-QNP in object position forces a distributive reading of the *verb* on its object argument (cf. also example (14) below). No quantifier raising is needed. Hence, there is a difference in the interpretation of the verb in the two kinds of stimuli sentences which, we would argue, is the reason for the observed reading time effect, implicating that distributive verb phrases are less complex than singular ones with a definite object.

8 We assume these two latter terms are synonymous, the former often used in the dialogue community, the latter among Conversation Analysis researchers.

b. CLARK: Did you ever engage in unauthorised briefings?
CUMMINGS: What do you mean by unauthorised briefings?
CLARK: Briefings that weren't authorised.
(https://twitter.com/IanDunt/status/1397497576526655492)

Although clarification interaction can address problems at various levels (attention, perception, discourse planning), we focus exclusively on clarification that concerns *intended meaning*. Hence, we introduce (in a consciously restricted sense) the notion of the *clarification potential* of an utterance u (from single words to sentences) — the set of possible clarification questions which u can trigger concerning its intended meaning on a given use.

Ginzburg & Cooper (2004), Purver, Ginzburg & Healey (2003), and Purver & Ginzburg (2004) argue in detail that the clarificational potential of an utterance u includes the question in (5), this can become the (maximal) question under discussion, and serve to resolve non-sentential clarification questions.[9]

(5) What did you mean as the content of u?

Hence, *answers* to such questions provide indications as to intended content. For clarification questions triggered by proper names, as in (6) or deictic pronouns (4a), a resolving answer communicates an individual, in (6b) identified via its location:

(6) a. CHRISTOPHER: Could Simon come round tomorrow?
PHILLIP: Simon?
JANE: Mm mm. Simon Smith.
(BNC, KCH, 48–51, slightly modified)

b. DAVE: O'Connors again.
KEITH: O'Connors?
DAVE: Yeah

9 In fact, a second prominent clarification question, with the force of a confirmation question is also always available. One possible explication of this reading is given in (i) and exemplified in (ii); an alternative explication is discussed in Ginzburg 2012: pp. 195–198. The availability of the two clarification questions is what explains the ambiguity of *reprise fragments*, exemplified in (iii):

(i) Did you mean z as the content of u, for some potential content z.
(ii) A: Did Obama leave? B: Obama? A: Yeah.
(iii) George: you always had er er say every foot he had with a piece of spunyarn in the wire Anon: Spunyarn? George: Spunyarn, yes. Anon: What's spunyarn? George: Well that's like er tarred rope. (BNC, H5G)

KEITH: Where's that?
DAVE: [provides address]
KEITH: [repeats address]
(BNC, KCY, 1183–86)

Whereas for verbs the answers they elicit help specify a property, as in (7):

(7) a. A: Do you hate Bo?
B: hate?
A: Get very angry when you see him, be unable to even think of him.

b. AMY: Yes he was screaming.
ANN: Screaming?
RICHARD: Didn't wanna get up.
(BNC, KB8, 154–6)

This data from clarification questions and their answers accords with standard approaches that associate individuals as the content of proper names and deictic pronoun utterances, and properties with verb utterances.

What, then, for the clarificational potential of QNPs? Purver & Ginzburg (2004) show that answers to clarification questions (CQs) about QNPs communicate individuals and sets of individuals (as in (8a,b)), and even function denoting NPs. However, there is no evidence of *talk* about GQs (the contents associated with QNPs according to GQT).

(8) a. TERRY: Richard hit the ball on the car.
NICK: **What ball**? *[⇝ What ball do you mean by "the ball"?]*
TERRY: James [last name]'s football.
(BNC KR2, 862–866)

b. RICHARD: No I'll commute every day
ANON 6: **Every day?**
*[⇝ Is it **every** day you'll commute?]*
*[⇝ Is it every **day** you'll commute?]*
[⇝ Which days do you mean by 'every day'?]
RICHARD: as if, er Saturday and Sunday
ANON 6: And all holidays?
RICHARD: Yeah [*pause*]
(BNC KSV, 257–261)

In case of (8a), the exchange between Nick and Terry suggests that the CQ 'What ball?' targets the identity of an object — Nick requires information concerning the reference of *the ball*. Of course, this exchange could be recast in GQ terms. On such a view, the definite NP in (8a) denotes $\{X \subseteq D \mid \exists u \in D, [\![\text{ball}]\!] = \{u\} \text{ and } u \in X\}$, that is, the set of all sets containing a ball singleton (D the domain of quantification).[10] Given this semantic representation, there seem to be two dubious consequences. First, the *Wh*-question *What ball?* actually would have to be construed as targeting a higher order property ranging over sets (e.g., *what (distinctive) property does the set of sets containing a ball singleton have?*). Secondly, once the queried singleton is found within the set of sets we still have to move from the singleton to its element. The latter can be achieved, however, by making use of the "Montagovian individual" $I_a(A) \iff a \in A$ for a set A (Peters & Westerståhl 2013: p. 722).

Analogous argumentation applies to (8b), but to the effect that the reprise question *every day* queries a semantic value of type *Set*(*Ind*) instead of a function from pairs of sets of individuals to truth values. Given this, Purver & Ginzburg (2004) point out that the GQT view of what NPs denote is difficult to reconcile with what people are actually *talking about*. They argue for NP denotations construed as *witness sets* (Barwise & Cooper 1981), or *witness individuals*.

Now, as Cooper (2013: p. 2) points out, there is a standard reply to this argument, namely that the meanings assigned to non-sentential constituents are not intended to represent what people are talking about, but are mathematical means for deriving truth conditions for complete sentences. But then, as Cooper suggests, we are left with the puzzle of what people actually *are* talking about when using non-sentential expressions (which are pervasive in conversation, Fernández & Ginzburg 2002). It seems reasonable to demand from a semantic theory that it supplies an answer here. We should emphasize that this argument is orthogonal to the referential/quantificational distinction — there is no claim that QNPs are always or even frequently used referentially; merely that GQs are not the contents speakers intend for them. This point is further elaborated in Section 3.5.

10 One could of course impose a stronger uniqueness presupposition, but that would not change the general point being made.

1.3 (Potential) Obstacle 3: The logical space of quantifiers

Taking a relational perspective, a denotable type (1,1) quantifier $Q_M(A,B)$ is a relation between subsets A (from the NP) and B (from the VP) of a domain M, or equivalently a binary function from pairs $\langle A,B\rangle$ of subsets of M into $\{0,1\}$. If $|M| = n$, there are 2^n possible subsets of M (namely $|\wp(M)|$ many)[11] and hence $2^n \times 2^n$ possible pairs of subsets. Given these numbers, there are $2^{(2^n \times 2^n)}$ possible mappings of those pairs of subsets into $\{0,1\}$, which is equivalent to $2^{2^{2n}}$ and 2^{4^n}. For $n = 2$ this already yields 65,536 quantifiers in M (cf., e.g., Keenan 2002: p. 632). This is a dazzlingly large number. Accordingly, much work in GQT explores the formal properties of quantifiers and the expressive power of natural language quantification, partly in order to formulate constraints on the logical space of quantification (see, e.g., Barwise & Cooper 1981, Keenan & Stavi 1986). Complexity reduction has been desired for cognitive considerations (Barwise & Cooper 1981), which lead to the notion of *witnesses* as an auxiliary means for processing QNPs, and for empirical considerations, since natural language quantifiers do not seem to exhaust the logical space as modelled by GQT (Keenan & Stavi 1986). Recently, learnability considerations have been put forth (Steinert-Threlkeld & Szymanik 2019), namely that quantifiers exhibiting certain features (like monotonicity) are easier to learn than others.[12] Of course, starting with the most general possibility space and then formulating delimiting constraints is a methodologically sound approach. However, in particular cognitive considerations can suggest looking for a mathematical foundation of quantification which excludes quantifiers that do not seem to be denoted by any natural language expression from the outset. Let us briefly exemplify such a mathematical simplification in terms of one of the most important constraints on quantifiers, namely *conservativity*, a hypothesized semantic universal (Barwise & Cooper 1981, Keenan & Stavi 1986).[13] A quantifier Q_M is conservative iff for all A, B: $Q_M(A,B) \iff Q_M(A, A \cap B)$. Now, for any X such that $X \subseteq A \cap B$ it also trivially holds that $X \in \wp(A)$. Following this line, Klein (2012) treats quantifiers as *unary* functions (reducing their type from (1,1) to (1)) that apply only to restrictor sets R contributed by the noun, not to verb sets. The denotation of QNPs on this account are pairs $\langle R,W\rangle$, where the so-called witness set $W \subseteq \wp(R)$ is such that W satis-

11 We use the "Weierstrass p", '$\wp$', in order to denote a power set.

12 We thank an anonymous reviewer of S&P for pointing us to this reference.

13 Barwise & Cooper (1981) used the term "lives on" instead of conservativity.

fies the "descriptive quantifier condition" (as we call it in Section 4.2 below). For instance, the denotation of a noun phrase of the *every* kind, *every A*, is $\langle [\![A]\!], \{X \mid X \subseteq [\![A]\!] \wedge X = [\![A]\!]\}\rangle$, which in turn is $\langle [\![A]\!], \{[\![A]\!]\}\rangle$. This move leads to a reduction of the number of possible quantifiers precisely to the number of conservative quantifiers. For instance, for two elements in the domain, R can be one of $2^2 = 4$ possible subsets, $R_1, \ldots, R_4$, of R (namely empty set, one element (twice), both elements). In each case, the witness set W is a subset of the power set of the restrictor set. Thus, in general there are at most $|\wp(\wp(R_1))| \times |\wp(\wp(R_2))| \times |\wp(\wp(R_3))| \times |\wp(\wp(R_4))|$ possible quantifiers for $n = 2$; this is $2^{2^0} \times 2^{2^1} \times 2^{2^1} \times 2^{2^2} = 2 \times 4 \times 4 \times 16 = 512$ (which equals 2^{3^2}, the number of conservative quantifiers for $n = 2$). Assuming a fixed subset of R, we move on from quantifiers to GQs and observe that there are at most 16 possible functions from sets into truth values (namely that of R_4). Part of this complexity seems to derive from exclusively using power sets in the combinatorics. We employ another mathematical operation in Section 2.3, but also follow a unary or, as we prefer to say, NP-internal approach.[14]

1.4 The proposal in a nutshell

In (9a) the denotation of *every dog* as a GQ is visualized: the set of sets of which the set of dogs (represented by a hatched circle) is a subset (the illustration is adopted from Dowty, Wall & Peters 1981: p. 122 *via* Chierchia & McConnell-Ginet 2000: p. 503). A sentence like *Every dog barks* is true iff the set of barking things includes this set of dogs. On the envisaged NP-internal approach — sketched in (9b) — no such membership relation is required. The sentence is true iff (i) there is a *situation* or *event s* which involves *witnesses* of the extension of the plural type *dogs*,[15] (ii) the dog witnesses conform to the descriptive condition imposed by the quantifier word *every*, and (iii) the situation can be classified as a *barking* one (i.e., the dogs bark). The notion

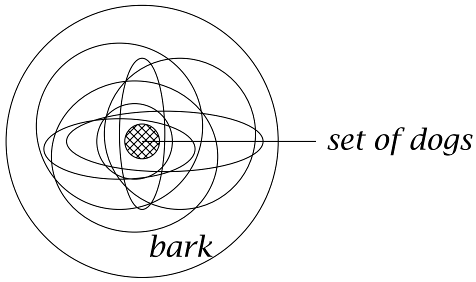
14 On both, Klein's and our approach, semantic composition of QNPs and verb phrases have to be adapted: Klein (2012: Section 4) assigns his DP denotation to semantic roles of verbs and changes verb denotations, we make use of plural types classifying situations involving witnesses and stick to a standard predicational approach (for QNPs in subject position see Section 2.2 and some non-trivial refinements are introduced in Section 4.5).

15 Actually, the quantifier word *every* is special in that it is syntactically singular but semantically plural. We show how to capture this in formal grammar terms in Section 4.7.

of *true in a model* is amended from set of set configurations to situational realisation.[16]

(9) *Every dog barks.*

a.



b. $\{\langle\{\cdot\},\emptyset\rangle,\langle\{\cdot\},\{\cdot\}\rangle,\ldots\}$ *dogs*

every (*via* descriptive quantifier condition)

$\{\langle\{\cdot\},\emptyset\rangle\}$ *every dog*

quantification

witnessing

s set of dogs barking

predication

We use a type-theoretical framework in order to develop the NP-internal approach, namely a *Type Theory with Records* (Cooper 2012, Cooper & Ginzburg 2015, Cooper 2023), though we hypothesize that this denotational foundation can be easily captured in other (denotational) frameworks. Within Type Theory with Records, nominal and verbal predicates (now construed as types) receive a denotational interpretation. The type of a situation is represented as a record type, which is true iff there is a situation (a record) of this type (see Section 2.1). The representational flavour we use in order to analyse the example sentence is indicated in (10), which shows a collection of labels (to the left of the colons) that label objects of a certain type (to the right of the colons), as will be explained in more detail in Section 2.1:

16 Situations also give rise to quantifier domain restrictions (Westerståhl 1985 employed contextually given sets of individuals to this end). With regard to (incomplete) definite descriptions, Barwise & Perry (1983) argued that they are implicitly evaluated against the *described* or *topic* situation, or an independently given *resource* situation. Cooper (1996) extended this approach to generalized quantifiers, showing that their domain can be restricted by the described situation or by different resource situations.

(10) $\begin{bmatrix} \text{x} & : \mathit{Set}(\mathit{Ind}) \\ \text{c0} & : \text{every(x)} \\ \text{c1} & : \text{dog(x)} \\ \text{c2} & : \text{bark(x)} \end{bmatrix}$ (Not a serious proposal, just a didactic indication of thrust!)

In order to make (10) into a well-behaved record type we have to spell out (i) what the *every*-condition c0 means, and (ii) how the predicate types in c1 and c2 apply to sets. This is done in Section 2, which includes a brief overview of the basic framework and plural predicate types (Sections 2.1 and 2.2, respectively). In order to provide an answer to (i), we introduce sets of ordered set bipartitions as new denotations of QNPs (Section 2.3). Predication on QNPs (ii) is modelled as predication on QNP witnesses (Section 2.4). Thereby all three obstacles discussed above are addressed: QNPs figure as arguments of predicational verb phrases (obstacle 1, Section 1.1), those arguments involve QNP witnesses of type *individual* or *set of individuals* (obstacle 2, Section 1.2), and the denotational underpinning in terms of sets of ordered set bipartitions lead to a significant reduction of the logical space of quantifiers and QNPs (obstacle 3, Section 1.3).

In the second part of the article, we provide further empirical justification for the theoretical set-up from the first part. We first motivate the semantic diagnoses we use to this end — summarized as “Referential Transparency” in Section 3.[17] Section 3.1 reviews the Reprise Content Hypothesis, a clarification request-based method for characterising the content of the fragments being reprised. Section 3.2 reviews the anaphoric potential of QNPs (i.e., the kinds of anaphora for which they provide antecedents). A multimodal variant of the anaphoric potential is reviewed in Section 3.3, where cross-references between speech and manual co-speech gestures are exemplified. These diagnoses are related to *addressability* (roughly, the contents identified by reprise questions, anaphora and speech–gesture cross-references need not only be available but also retrievable in context) and collected under the label *Referential Transparency* in Section 3.4. Some further background on reference and quantification, in particular the (non-) grounding mechanism of dialogue gameboard and quantificational parameters is provided in Section 3.5 (cf.

17 This use of the term “referentially transparent” is to be distinguished from that of Quine (1961: p. 142), and Whitehead & Russell (1963: Section C, p. 665) to denote contexts which allow for the *salva veritate* substitution of co-referential expressions and existential generalisation.

also the corresponding remarks in Section 1.2). This background is needed for deriving various quantificational or referential interpretations of QNP uses.

The semantic diagnoses are applied to QNPs in Section 4. The so-called complement set is of particular interest, since from the denotational foundation in terms of sets of ordered set *bi*partitions it follows that quantification involves two NP-internal sets (and trivially a third one, namely the union of the former two). We interpret these sets in terms of a *reference set* (refset), a *complement set* (compset), and a *maximal set* (maxset). While the refset provides the actual QNP witness and is uncontroversial (as is the maxset), in Section 4.1 empirical evidence is collected that demonstrates also the fundamental nature of the the compset. Quantifier words operate on those sets "like sieves" (Barwise & Cooper 1981: Section 4.5) by means of a descriptive quantifier condition discussed in Section 4.2. Within the set of ordered set bipartitions there is one bipartition which is special, namely the one with an empty refset. In Section 4.3 it is argued that this bipartition explains (at least some data on) complement set anaphora. Following this rationale, Section 4.4 provides evidence from anaphora that singular is a special case of plural. Section 4.5 returns to the issue of predication and shows how refset and compset give rise to simultaneous, two-headed predication and *anti-predication*. Synthesizing these discussions into what can be called *Referential Transparency Theory* (RTT), Section 4.6 provides an explicit proposal as to the referentially transparent "quantified noun phrase anatomy": the didactic sketch from (10) is finally generalized and refined into the structure in (11):

(11) Quantified noun phrase anatomy which is argued for in this paper:

$$\text{QNP}_{\text{sem}} \coloneqq \begin{bmatrix} \text{q-params} : \begin{bmatrix} \text{maxset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c1} & : \overrightarrow{\mathit{PType}}(\text{maxset}) \\ \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c2} & : \text{union}(\text{refset},\text{compset},\text{maxset}) \end{bmatrix} \\ \text{q-cond} \quad : \mathit{Rel}(|\text{q-params.refset}|,\ |\text{q-params.compset}|) \\ \text{q-persp} \quad : \text{refset} = \emptyset \ / \ \text{refset} \neq \emptyset \ / \ \mathit{none} \end{bmatrix}$$

The feature q-persp can take one of three values, separated by slashes, including the empty value *none*, the vector notation indicates a plural property type.

Throughout the paper there are occasional references to the idiosyncratic behaviour exhibited by *every* (see, e.g., footnotes 7 and 15). For this reason, Section 4.7 briefly discusses *every* and offers a grammatical account of *every*-QNPs. We measure out the complexity of RTT in Section 4.8 and conclude in Section 5.

2 Formal framework

Within a TTR framework, an account of generalized quantifiers has already been developed. Motivated by considerations concerning the clarificational potential of quantified NPs reviewed in Section 1, Purver & Ginzburg (2004), Ginzburg & Purver (2012), and Ginzburg (2012) develop an NP-internal account of GQs by emphasizing the role of a witness set. In Cooper (2013) and Cooper & Ginzburg (2015), the witness approach is harmonized with more orthodox, Montagovian GQs in terms of an NP-internal definition of quantifier relations.[18]

2.1 Vanilla TTR

In a nutshell, TTR is a *rich type theory with records* — a cognitively construable formalism grounded in set theory. The TTR inventory consists among others of the following types (see Cooper 2012, Cooper & Ginzburg 2015, Cooper 2023 for expositions of TTR):

- *Basic types* (*BType*; 0-place; *Ind*, *Loc*, *Time*, …);
- *Predicate types* (*PType*; n-place; lion(x), carry(x,y), …), constructed out of a predicate and objects which are arguments of the predicate;
- *Set and list types* (*Set*(T) and *List*(T)). If $t_1 : T, \dots, t_n : T$, then $\{t_1, \dots, t_n\}$: *Set*(T) and $[\text{pos1} = t_1, \dots, \text{pos}n = t_n]$: *List*(T), for T being a type (a list is distinguished from a set by means of an inherent ordering index "pos");

18 In the 2013 paper Cooper accepts the need to revise the semantics of QNPs in order to capture their clarificational potential. He maintains a GQ analysis (in the sense of a denotation that projects the scope argument) primarily for the convenience it affords in providing a glue language for combining meanings. The paper contains two significant theoretical insights we draw on and discuss further below. First, he argues for the need to incorporate into the QNP anatomy the quantifier relation. Second, he puts forth the "addressability hypothesis" (see the short summary of Section 3 above). Cooper reconciles the RCH with the GQ approach by adding a *q-params* field into the architecture of signs, as an additional attribute to content. Cooper then provides a detailed and subtle empirical corroboration of his hypothesis.

- *Function types*. $(T_1 \mapsto T_2)$ is the type of functions from type T_1 to type T_2;
- *Records*: entities corresponding to situations, providing individuals, see (12) for an example;
- *Record types*: structured semantic representations classifying records, see (12) for an example;
- *Labels*: entities in records and record types are addressed by labels, see (12) for an example.

A key notion in TTR is a *judgement*, a classification that object o is of type T, notated as $o : T$. If the judgement is true, than the extension $[{}^{\vee}T]$ of T is non-empty, containing at least one witness, namely o. Judgements between records and record types, that is classifications such that a record r being of a record type RT, $r : RT$, give rise to witnessing between situations and situation types. For example, the record in (12a) is a witness for the record type in (12b) just in case the judgements in (12c) hold. The record type is built out of a basic type (*Ind*) and a predicate type *lion* applied to the value labelled "x").[19] The example in (12) also exemplifies the notational conventions we employ in order to represent records and record types.

(12) a. $\mathrm{r} = \begin{bmatrix} \mathrm{x} & = \mathrm{a} \\ \mathrm{c_{lion}} & = \mathrm{e1} \end{bmatrix}$

b. $\mathrm{T_{lion}} = \begin{bmatrix} \mathrm{x} & : \mathit{Ind} \\ \mathrm{c_{lion}} & : \mathrm{lion(x)} \end{bmatrix}$

c. $\mathrm{r} : \mathrm{T_{lion}}$ just in case a : *Ind* and e1 : *lion(a)*

Note that the labels are used as paths for addressing even nested values, in which case the corresponding labels are concatenated by periods. For instance, something of type individual is found at path "x" in (12b), and in (11) the path "q-params.refset" leads to a set of individuals.

2.2 Plural types

We represent a plurality in terms of a vector notation: If T is a one-place predicate which takes an individual as argument, then $\overrightarrow{T}$ is the corresponding

19 In the official set-up, predicate types give rise to properties, where a (singular) property is a function from records which host an individual to record types. For example, the property of being a lion is the function $\lambda r : [x : \mathit{Ind}].[e : \mathrm{lion}(r.x)]$.

plural predicate which applies to a set of individuals. For instance, if A : $Set(Ind)$, then $\overrightarrow{T}(A)$ is a plural predicate type:

(13) $\begin{bmatrix} \text{x} : \mathit{Set}(\mathit{Ind}) \\ \text{c} : \overrightarrow{\mathit{PType}}(\text{x}) \end{bmatrix}$

With two-place predicate types, that is, relations, we have to distinguish four classes, since the relata may be individuals or sets. The possible combinations are spelled out by the record types in (14), showing also the subscript notation on arrows indicating which argument is a set and receives a plural interpretation. For instance, "$\overrightarrow{\mathit{PType}}^{2}$" is the plural predicate whose second argument is a set (the first being of type *Ind*). The types, thus, can be part of representations of different kinds of situations, such as (in order of appearance): *Some dog chases some cat*, *Some dogs chase some cat*, *Some dog chases some cats*, and *Some dogs chase some cats*.

(14) $\begin{bmatrix} \text{x} : \mathit{Ind} \\ \text{y} : \mathit{Ind} \\ \text{c} : \mathit{PType}(\text{x, y}) \end{bmatrix}, \begin{bmatrix} \text{x} : \mathit{Set}(\mathit{Ind}) \\ \text{y} : \mathit{Ind} \\ \text{c} : \overrightarrow{\mathit{PType}}^{1}(\text{x, y}) \end{bmatrix}, \begin{bmatrix} \text{x} : \mathit{Ind} \\ \text{y} : \mathit{Set}(\mathit{Ind}) \\ \text{c} : \overrightarrow{\mathit{PType}}^{2}(\text{x, y}) \end{bmatrix}, \begin{bmatrix} \text{x} : \mathit{Set}(\mathit{Ind}) \\ \text{y} : \mathit{Set}(\mathit{Ind}) \\ \text{c} : \overrightarrow{\mathit{PType}}^{1,2}(\text{x, y}) \end{bmatrix}$

The types in (14) classify situations which involve individuals and, respectively, sets of individuals.[20] Both are also witnesses of NP denotations.

2.3 Quantified NPs as plural NPs

Our starting point towards a denotational type-theoretic approach to QNPs rests on a plural semantics. In plural semantics the extension of plural count nouns is modelled in terms of the power set (or an equivalent notion such as a join semi-lattice) of the domain of quantification (Link 1987). We propose *sets of ordered set bipartitions* as NP-internal QNP denotations.[21]

(15) **Ordered set bipartition.** An ordered set bipartition b of a set s is a pair of disjoint subsets of s including the empty set such that the union of these subsets is s.

20 We assume a plural type hierarchy rooted in $\overrightarrow{\mathit{PType}}$ which comprises distributive and collective subtypes which bring about fully distributive, fully collective, and intermediate cover (Scha 1984) readings. Distributivity is needed for *every*-QNPs (cf. Section 4.7).

21 Historically, the denotational foundation developed as a generalization from data collected from Referential Transparency (see Sections 3 and 4). However, we introduce the technical part first and in a top-down manner for the sake of accessibility.

(Examples follow shortly.) Ordered set bipartitions are computed in terms of the extensions of count nouns in the following way.

- Extension of a type: $[{}^{\vee}T] = \{a \mid a : T\}$.
- P-extension of a predicate (lemma): $[\downarrow P] = \{a \mid \exists e[e : P(a)]\}$ (adopted from the β-reduced property extension of Cooper 2023). Some explanation is required here. A predicate type in TTR is a complex type $P(a)$ which is constructed out of a predicate P and an argument a (usually of basic type *Ind*). The witness of a predicate type is a situation or event e that makes 'a is P' true. The P-extension of a predicate thus is the set of objects that figure in situations of P-ness.
- Q-extension of a plural predicate: $[\downarrow_Q \vec{P}] = p([\downarrow P])$ (where p is the operation bringing about the set of all ordered set bipartitions from its argument's P-extension. [22]
- S-extension of a singular predicate:
  $[\downarrow_S P] = \{a \in [\downarrow_Q \vec{P}].\text{first} \mid \exists e[e : P(a)]\}$, where the suffix ".first" (and ".second") denotes the first (respectively second) element of a pair.

A count noun like *bicycle* translates into a one-place predicate in semantics, *bicycle*(x). Now, there is a clear relation between one-place predicate types and zero-place basic types (Cooper 2023): $a : \mathit{Bicycle} \iff \exists e.e : \mathit{bicycle}(a)$. This equivalence will occasionally be exploited for notational convenience.

A simple example should illustrate how Q-extensions look:

(16) Let $[\downarrow \text{Bicycle}] = \{\textcolor{orange}{🚲}, \textcolor{blue}{🚲}, \textcolor{gray}{🚲}\}$. Then

$$
\begin{aligned}
p([\downarrow \text{Bicycle}]) = \{ & \langle \emptyset, \{\textcolor{orange}{🚲}, \textcolor{blue}{🚲}, \textcolor{gray}{🚲}\}\rangle,\\
& \langle \{\textcolor{orange}{🚲}\}, \{\textcolor{blue}{🚲}, \textcolor{gray}{🚲}\}\rangle,\\
& \langle \{\textcolor{blue}{🚲}\}, \{\textcolor{orange}{🚲}, \textcolor{gray}{🚲}\}\rangle,\\
& \langle \{\textcolor{gray}{🚲}\}, \{\textcolor{orange}{🚲}, \textcolor{blue}{🚲}\}\rangle,\\
& \langle \{\textcolor{orange}{🚲}, \textcolor{blue}{🚲}\}, \{\textcolor{gray}{🚲}\}\rangle,\\
& \langle \{\textcolor{orange}{🚲}, \textcolor{gray}{🚲}\}, \{\textcolor{blue}{🚲}\}\rangle,\\
& \langle \{\textcolor{blue}{🚲}, \textcolor{gray}{🚲}\}, \{\textcolor{orange}{🚲}\}\rangle,\\
& \langle \{\textcolor{orange}{🚲}, \textcolor{blue}{🚲}, \textcolor{gray}{🚲}\}, \emptyset\rangle\}
\end{aligned}
$$

22 An alternative would be to relativize plural extensions to situations, akin to the definition of P-extension. We use Q-extensions since they fully map out the logical space of plural extensions.

Each ordered set bipartition in the set of ordered bipartitions is structured in the form ⟨refset, compset⟩. The last ordered set bipartition in (16), the one with an empty compset, is the denotation of *every bicycle* in the sample universe. Note that it is just a pair of a set of bicycles and the empty set, rather than a set of bicycles which is a subset of all other sets, as assumed in GQT.

Sets of ordered set bipartitions provide a straightforward notion of *witness*:

(17) The witness of a QNP is the refset of an element of the set of ordered set bipartitions of the head noun N sifted out by the quantificational determiner Q.

Note that a QNP witness is a set of individuals. (Singular NPs, which are not the main focus of the present account, are briefly discussed in 4.4.)

We can now make the QNP part from the didactic representation in (10) more precise. The type in (18) represents the content of the QNP *every dog*:

(18)
$$\begin{bmatrix} \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c0} & : \overrightarrow{\text{dog}}(\text{refset}) \\ \text{c1=(compset=}\varnothing\text{)} & : \mathit{Rel}(|\text{compset}|,\ |\text{refset}|) \end{bmatrix}$$

The structure in (18) classifies a situation with a witness set consisting of dogs (plural type "$\overrightarrow{\text{dog}}$(refset)"). The quantifier *every* contributes the "sieve" that only those refsets from the head noun's Q-extension are witnesses which form a bipartition with the empty set (condition c1). Given the mismatch between syntactic and semantic number of *every*-QNPs, *deriving* them in grammar is a bit more complex; accordingly, we return to this issue in Section 4.7. Generalizing over *every*-QNPs, the basic template of QNP contents is given in (19):

(19) Basic template of QNP contents (provisional, refined in Section 4.6)
$$\begin{bmatrix} \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c0} & : \overrightarrow{\mathit{PType}}(\text{refset}) \\ \text{c1} & : \mathit{Rel}(|\text{compset}|,\ |\text{refset}|) \end{bmatrix}$$

The head noun contributes a plural property which is distributed over the members of the refset, the quantifier word contributes a quantificational relation on the cardinalities of refset and/or compset. The basic QNP template in (19) will only be slightly refined according to Referential Transparency below.

To summarize: In any NP-internal approach the quantificational relation contributed by a quantificational expression is defined without reference to a scope set (cf. Section 1.3). On our approach the quantificational relation obtains between refset and compset.[23] Quantifiers act as *sieves* on sets of ordered set bipartitions. The contribution of a quantifier word receives an explicit semantic representation in terms of the *descriptive quantifier condition*.

2.4 Blueprint of predication on subject QNPs

Two plural types are required to describe the derivation of the content of a simple sentence involving a subject QNP such as *Every dog barks*: the first distributes the property of being a dog onto the members of a refset, the second distributes the property of barking onto the same refset. The corresponding compositional structure is shown in Figure 1, ignoring tense ("NP.refset" in condition c2 of the VP indicates that the refset argument is found in the NP constituent).

The record type representing the content of the S node is true if there exists a situation that provides a set of dogs from a witnessing refset of the set of ordered set bipartitions sifted out by the quantificational determiner, and the members of that witnessing refset bark. Since the quantificational determiner *every* lets only one ordered set bipartition pass — namely the one with an empty compset — it follows (if true) that there are no non-barking dogs in the described situation. The basic predicational pattern shown in (19) will be refined along this line, leading to two-headed predication in Section 4.5.

For the purposes of the present article, the simple treatment of transitive predicates as relations is sufficient. We note that in order to capture so-called narrow scope readings, relations have to be complemented with dependent functions (on functional NP uses see Jacobson 2000, Ginzburg 2012, Steedman 2012; further remarks are given in the conclusions in Section 5.)

23 Contextual interpretations of QNPs, however, also involve a contextually given standard of comparison, see Section 4.2.

S

$$\begin{bmatrix} \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c0} & : \overrightarrow{\text{dog}}(\text{refset}) \\ \text{c1}{=}(\text{compset}{=}\varnothing) & : \mathit{Rel}(|\text{compset}|,\ |\text{refset}|) \\ \text{c2} & : \overrightarrow{\text{bark}}(\text{refset}) \end{bmatrix}$$

NP

$$\begin{bmatrix} \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c0} & : \overrightarrow{\text{dog}}(\text{refset}) \\ \text{c1}{=}(\text{compset}{=}\varnothing) & : \mathit{Rel}(|\text{compset}|,\ |\text{refset}|) \end{bmatrix}$$

VP

$$\begin{bmatrix} \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c2} & : \overrightarrow{\text{bark}}(\text{NP.refset}) \end{bmatrix}$$

barks

DET

$$\begin{bmatrix} \text{c1} = (\text{compset}{=}\varnothing) : \mathit{Rel}(|\text{compset}|,\ |\text{refset}|) \end{bmatrix}$$

every

N

$$\begin{bmatrix} \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c0} & : \overrightarrow{\text{dog}}(\text{refset}) \end{bmatrix}$$

dog

Figure 1 Compositional derivation of *Every dogs barks.*

3 Referential transparency

In the preceding section we have shown how witness-based quantification within our denotational theory addresses the obstacles collected in Section 1. Here we argue that further motivation for our theoretical set-up is gained from observing QNPs *in vivo*. To this end, we collect *semantic probes* that let us delimit the anatomy of QNPs ("what's in a QNP", Nouwen 2010). We consider three kinds of probes: clarification requests (Section 3.1), anaphora (Section 3.2), and co-speech gesture cross-references (Section 3.3). Since all of these semantic probes are related to the (discourse-)referential content of QNPs we term the methodology principle *Referential Transparency*. With the addition of a final refinement, *addressability*, Referential Transparency is systematized in Section 3.4. In this respect, the basic template of QNP contents in (19) receives two modifications: motivated by so-called maxset anaphora, we add the union of refset and compset to the QNP structure (Section 3.2), and the resulting set triplet is connected to the mechanism of grounding and quantifying away (Section 3.5). Referential Transparency is then used in Section 4 as a collection of *desiderata for the semantic representations of NPs* that go beyond their role in computing truth conditions.

3.1 Reprised contents

As discussed in Section 1.2, Purver & Ginzburg (2004) argue that the content of the utterance of a constituent can be queried by clarification requests. They distinguish different kinds of reprise fragments, including *intended meaning* requests, that is, reprise fragments that follow the template "A: …u_1… B: u_1?"; for examples see footnote 9 ex. (iii), and (6) and (7) above. Purver & Ginzburg (2004) show further that reprise fragments of the *intended meaning* type, at least when they address a non-sentential constituent, do not query pragmatically inferred material but are restricted to direct semantic content. On the basis of this they posit the *Reprise Content Hypothesis* whose strong version is given in (20):[24]

(20) **Reprise Content Hypothesis:** A reprise fragment question queries exactly the standard semantic content of the fragment being reprised.

Hence, looking at clarification data, in particular reprise fragments, provides a semantic probe for the meaning associated with the queried constituent.

It should be emphasized that the Reprise Content Hypothesis (RCH) provides a significantly stronger constraint on meanings than Fregean compositionality (Purver & Ginzburg 2004, Ginzburg & Purver 2012). The latter merely requires a means of decomposing the meaning associated with a complex phrase Φ into sub-meanings, each sub-meaning being the meaning associated with a constituent. The only constraint on the sub-meaning is that they compose *somehow* into Φ. In contrast, clarification potential requires that in addition to composing into the complex meaning, each sub-meaning itself satisfies the requirements enforced by the clarification potential for that constituent.

Consider, for instance, the made-up exchange in (21):

(21) A: Did you drink each yogurt container?
B: Drink? (What do you mean 'drink'?)

Processing A's initial question involves combining the verb *drink* with its object *every yogurt container* into the verb phrase *drink every yogurt con-*

24 The weak version replaces "queries exactly" with "queries a part of".

tainer. The denotations of *drink* and *drink every yogurt container* obviously differ, but once combined the former is "merged" with the latter. The meaning of the verb can nonetheless be queried by the reprise fragment *Drink?*; An answer, however, cannot be computed from the composed sentential or VP meaning, but needs solely the bare verbal meaning.

The Reprise Content Hypothesis constraint on compositionality goes hand in hand with a representational problem: since the contents of the fragments being reprised have been "absorbed" *via* semantic composition into the content of the larger constituent of which they are a fragment, how can they be identified and retrieved for clarification? We address this problem in terms of *addressability* which we discuss in Section 3.4.

3.2 Antecedent contents and anaphora

Anaphoric expressions are particularly suited for detecting contents, since their minimal descriptive content makes them strongly contextually dependent, drawing on either an earlier utterance (anaphoric uses) or the perceived audio-visual situation (exophoric uses). As is widely accepted, the antecedent contents allow for two kinds of witnesses, a so-called *maximal set* and a *reference set*. Both are exemplified in (22), where the plural pronoun in (22a) refers back to environmentalists that actually took part in the rally (the *reference set*, or *refset*), and the plural pronoun in (22b) picks up an antecedent which denotes the totality of environmentalists that could have come (the *maximal set*, or *maxset*).

(22) Only seventy environmentalists came to the rally …

  a. … but they raised their placards defiantly.
  b. … although they had all received an invitation.

Even *No*-type QNPs allow for refset anaphora in certain circumstances. Examples illustrating incremental understanding, modelled on similar examples in Ginzburg et al. (2019), are given in (23): in (23a) A modifies her utterance based on the perceived visual situation and uses the 'discarded' QNP as antecedent for a pronoun; in (23b) a clarification request by B occurs immediately after A has uttered the subject NP of a yet to be completed utterance:

(23) a. A: [*enters class*] No students … Oh, they're hiding.
  b. A: Everyone … B: Who?

When the antecedent NP involves a downward monotone, proportional quantifier even a further witness can be picked out (Nouwen 2003):

(24) Few environmentalists came to the rally. They went to a football game instead.

The plural pronoun from the second sentence in (24) refers back to those environmentalists that stayed away from the rally. Accordingly, (24) is an instance of *complement set anaphora*, or *compset* anaphora.

Just as denotations can be used to delimit the clarification potential of (Q)NPs, maxset, refset and compset stake out their anaphoric potential. Accordingly, we slightly adjust our initial pair of sets by adding their union labelled as "maxset":[25]

(25)
$$\begin{bmatrix} \text{maxset} & : \mathit{Set}(\mathit{Ind}) \\ \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c0} & : \overrightarrow{\mathit{PType}}(\text{maxset}) \\ \text{c1} & : \text{union}(\text{refset},\text{compset},\text{maxset}) \\ \text{c2} & : \mathit{Rel}(|\text{compset}|,\ |\text{refset}|) \end{bmatrix}$$

Although not every set, especially the compset, is always available as antecedent — we formulate constraints on compset availability in Section 4.3 — this set triplet can be potentially picked up by anaphoric pronouns. And this is indeed the case, as studies on QNP processing show. Using electroencephalogram (EEG), Filik et al. (2011) examined the event-related brain potential (ERP) of subjects interpreting the plural possessive pronoun *their* following sentences which contained either positive (e.g., *many*) or negative (e.g., *not many*) quantifiers. The predicational part of the pronoun sentences used as stimuli were further designed so that it is clear whether a compset or a refset reference is made. Disentangling the interaction of quantifier word and anaphoric reference reveals that (i) compset reference following a positive quantifier evoked a larger N400[26] than refset reference; (ii) refset ref-

25 It is worth emphasizing that a QNP content representation as in (25) is not just a matter of bookkeeping, as had been suggested by an anonymous reviewer for S&P. On the one hand, the set triplet is firmly grounded in our denotational framework (Section 2.3). However, positing bookkeeping labels requires us also to offer them an interpretation. And yet, the QNP contents we argue for have (semantic and cognitive) repercussions for anaphora and predication, as discussed in Sections 4.3 and 4.5, respectively.

26 N400 is a negatively deflected ERP which occurs with a latency of about 400 ms to the triggering event. If the triggering events are words, a common explanation of this is that an N400 indicates a difficulty to integrate those words into context, though there exist com-

erence following a negative quantifier evoked a larger N400 than compset reference. Note that finding (ii) is not compatible with the assumptions that the refset is the default antecedent and compset reference is a fall-back option (as claimed, e.g., by Nouwen 2003). Rather, a negative quantifier makes the compset the *expected* antecedent, which can be accommodated by the presupposition-denial where the so-called *shortfall* is the mechanism for compset reference (e.g. Moxey 2006). On a presupposition-denial account the complement set is available or even expected as antecedent when the difference (i.e., the shortfall) between the amount conveyed by a quantifier word and a large presupposed amount is focused. Compset reference is also immediately available if negativity is not expressed quantitatively but emotionally (e.g., "The judge was *happy/angry* about the number of people who turned up for jury duty", Ingram & Ferguson 2018: p. 148). However, this study also found that "the N400 component was more negatively-oriented after a compset reference than a refset reference, regardless of the prior emotion word, suggesting that integration of the compset was generally more difficult" (Ingram & Ferguson 2018: p. 153). Semantic probes and psycholinguistic studies in sum suggest that the compset plays a systematic role in the interpretation of QNPs, especially in negative contexts.

3.3 Co-speech gesture cross-references

Anaphoric potential extends into the non-verbal domain. From studies on speech-gesture integration it is known that manual gestures are usually bound to verbal expressions in terms of discourse referent (DR) identity (Rieser 2008), and that such gestures cannot introduce DRs on their own, that is, manual gestures cannot introduce DRs that do not relate to a (explicit or implicit) DR introduced in speech (Lascarides & Stone 2009: p. 19). From this it follows directly that gestures, like pronouns, pick up DRs already introduced by the accompanying speech. This quasi-anaphoric analysis seems to be sufficient even in case of gestures co-occurring with plural NPs, as illustrated in Figure 2 (taken from the SaGA corpus, Lücking et al. 2010). The speaker talks about a fountain which looks like it is made up of two chalices. According to standard dynamic semantics (Kamp & Reyle 1993), the plural NP *two chalices* introduces a plural DR. The open hand, fingers bent, palm-up gesture also produced by the speaker can be bound to the plural DR: the shape information associated with the gesture is interpreted distributively,

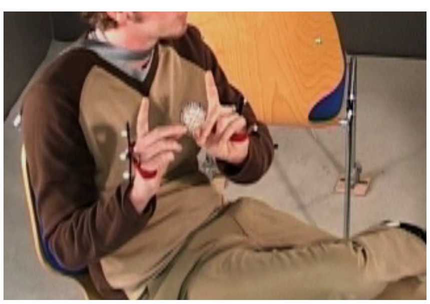
peting accounts in terms of lexical access and hybrid accounts (Delogu, Brouwer & Crocker 2019).

[*talking about a fountain*]
"und besteht aus zwei Kelchen"
and consists of two chalices

In compliance with the received view on plurals, the plural NP *two chalices* introduces a plural DR to which the gesture is bound. The gesture then can be interpreted distributively.

Figure 2 Two chalices (SaGA dialogue V24, time stamp 11:10).

amounting to the interpretation that every object the plural DR stands for has a chalice-like shape. However, in case of numerically modified plural NPs as in Figure 3, speech–gesture cross-reference requires more than a plural DR accounted for so far. The speaker here talks about a church with two church towers. Simultaneously, he raises the index fingers of both hands. The obvious interpretation of the gesture is that each finger represents one church tower. But this interpretation cannot be expressed, since, on standard accounts, there are no DRs for the individual church towers available; all we get is a plural DR from the plural noun phrase, like in the example in Figure 2. The numerical seems to make a DR available for each single object within the plural DR, DRs which have been termed *pointers* or *pointer objects* and are part of the construction of *complex reference objects* (Eschenbach et al. 1989). We briefly return to this in Section 4.4.



"die rechte Kirche die hat zwei spitze Türme"
the church to the right it has two pointed towers

The interpretation of the gesture is that each hand/pointing finger models one of the two towers talked about. However, given that a plural semantics introduces just a plural DR, there is no way of addressing the single towers.

Figure 3 Two towers (SaGA dialogue V24, time stamp 6:25).

3.4 The principle of referential transparency

As we have suggested, building on much past work, QNPs have more duties than merely contributing to truth conditions: QNPs act as antecedents for anaphoric expressions, they supply verbal affiliates of co-speech gestures, and they are objects of discourse dynamics which becomes apparent in terms of acceptance or clarification requests (we restrict attention here to nominals, but the conditions generalize cross-categorially):

(26) **Referential Transparency**: a semantic representation for an NP is *referentially transparent* if

a. it provides antecedents for pronominal anaphora;

b. it provides the semantic type required by a clarification request;

c. it provides an attachment site for co-verbal gestures;

d. its content parts can be identified and addressed.

Recall from Section 1.2 that the Reprise Content Hypothesis provides a stronger claim than Fregean compositionality: more complex contents are not just systematically combined from their parts, but the contributions from the parts have to be traceable within the complex content. To this end, clause (26d) connects referential transparency to *addressability* following Cooper 2013: p. 16: "what can be addressed by a clarification in response to a clarification request are paths within the type corresponding to the content of the clarification request". Referential transparency in combination with addressability provides a methodological principle that guides discovering the "anatomy" of quantified noun phrases pursued below: *linguistic theorising has to come up with denotations in such a way that they are truth-conditionally apt* and *exhibit the property of being referentially transparent. The latter includes the recursive requirement that these denotations have to be retrievable (identifiable and addressable) from semantic representations.*

3.5 Referentiality, non-referentiality, and intensionality

Our emphasis on "referentiality" might suggest that we are missing the point. After all, the whole point of quantification is that it enables us *not* to refer, but simply to *describe*. In fact, every (Q)NP can be used in two ways, either picking out an entity from common ground (*via* the visual situation or *via* shared knowledge), or introducing a discourse referent as a means of talking about it. The universally quantified NP in (27), for instance, can be used to refer to a particular person when the interlocutors know that the description applies only to that person (Gómez-Torrente 2015: Ex. 3).

(27) Well, everyone taking my seminar came to the party.

Furthermore, as noted by Ludlow & Neale (1991: p. 177), prefixing *look* is a productive "deictic operator" (though they use an example with a different wording):

(28) a. Look! A man wearing big boots is stealing our lemons.
b. Look! The man wearing big boots is stealing our lemons.
c. Look! Many men wearing big boots are stealing our lemons.
d. Look! Men wearing big boots are stealing our lemons.

The reference relation in the discerning sense (relating the semantic value of a referring expression to a perceptually or mentally known entity) and in the conversational sense (providing a means to talk about something) can become manifest in different ways for different interlocutors. Suppose the speaker uses the possessive *my mother*, then it is very likely that she has a particular individual in mind and is able to discern it. But this does not necessarily hold for the addressee, who simply might not know the speaker's parents. Does the possessive refer in this case? We don't think there is an unequivocal answer. For the speaker it denotes a particular individual, so it refers in the sense of concrete identification (reference by knowledge, assuming the mother is not around in the visual situation). For the addressee, it potentially provides a sufficient means for talking about some particular individual. So, relative to certain discourse goals, it refers in the sense of enabling successful communication. But, this is not invariably the case: if the speaker had said "Go find my mother.", clarification or information requesting interaction would probably ensue (e.g., "What does your mother look like?"[27]). In light of this, we can say that an utterance typically gives rise to referential instantiations of certain labels in the participants' information states. For some meaning-bearing sub-utterances there is an explicit expectation that this will happen, whereas for others this expectation is not present, which leads to their being in effect existentially quantified away. In certain versions of HPSG (Ginzburg & Purver 2012, Ginzburg 2012, Cooper 2013) this has been handled via a distinction between *dialogue gameboard parameters* (*dgb-params*) and *quantificational parameters* (*q-params*).[28] And this plays a

27 We owe this example to an anonymous reviewer.

28 dgb-params are a generalization of the Montague/Kaplan notion "contextual parameters", referred to in standard HPSG as "c-params".

significant role in the treatment of an account of the two main branches that can follow an utterance, namely *grounding* and *clarification interaction*.[29] The distinction into *dgb-params* and *q-params* implements "referential management" of nominal expressions in dialogue: the labels corresponding to the *dgb-params* elements are intended to be instantiated, whereas the asserted proposition has the force of existentially quantifying over the *q-params* element.[30]

Given this set-up, a schematic meaning for the NPs from the sentence *A thief stole my iPod* is in (29a) and a possible instantiation in context is in (29b). In (29), *q-params* is a sub-record type of the content. In what follows, a notational simplification we adopt is to factor out *q-params* from the descriptive content, as in (29c) (the path prefix "./" represents a path starting at the root level of a record type; we will omit "./" where confusion cannot arise).

(29) a.

$$\begin{bmatrix} \text{dgb-params} : \begin{bmatrix} \text{spkr} : \mathit{Ind} \\ \text{addr} : \mathit{Ind} \\ \text{z} : \mathit{Ind} \\ \text{c1} : \text{possess(spkr,z)} \wedge \text{ipod(z)} \\ \text{so} : \mathit{Sit} \end{bmatrix} \\ \text{cont} = \text{Assert(spkr,addr,} \\ \qquad \begin{bmatrix} \text{sit} = \text{so} \\ \text{sit-type} = \begin{bmatrix} \text{q-params} : \begin{bmatrix} \text{x} : \mathit{Ind} \\ \text{r2} : \text{thief(x)} \end{bmatrix} \\ \text{nucl} : \text{steal(q-params.x,./dgb-params.z)} \end{bmatrix} \end{bmatrix} \text{)} : \mathit{IllocProp} \end{bmatrix}$$

b.

$$\begin{bmatrix} \text{dgb-params} = \begin{bmatrix} \text{spkr} = \text{A} \\ \text{addr} = \text{B} \\ \text{z} = \text{j1} \\ \text{c1} = \text{p1} \\ \text{so} = \text{sit0} \end{bmatrix} \\ \text{cont} = \text{Assert(spkr,addr,} \begin{bmatrix} \text{sit} = \text{so} \\ \text{sit-type} = \begin{bmatrix} \text{q-params} : \begin{bmatrix} \text{x} : \mathit{Ind} \\ \text{r2} : \text{thief(x)} \end{bmatrix} \\ \text{nucl} : \text{steal(q-params.x,j1)} \end{bmatrix} \end{bmatrix} \text{)} \end{bmatrix}$$

29 For detailed discussion see Ginzburg 2012: Sections 5.2, 6.4–6.6, 8.5, and for a briefer discussion see Ginzburg & Purver 2012.

30 For a more detailed discussion of a similar example, see Ginzburg 2012: pp. 331–333.

c. $$\left[\begin{array}{l} \text{dgb-params} : \left[\begin{array}{ll} \text{spkr} & : \textit{Ind} \\ \text{addr} & : \textit{Ind} \\ \text{z} & : \textit{Ind} \\ \text{c1} & : \text{possess(spkr,z)} \wedge \text{ipod(z)} \\ \text{s0} & : \textit{Sit} \end{array}\right] \\ \text{q-params} : \left[\begin{array}{ll} \text{x} & : \textit{Ind} \\ \text{r2} & : \text{thief(x)} \end{array}\right] \\ \text{cont} = \text{Assert(spkr,addr,} \\ \qquad \left[\begin{array}{l} \text{sit} = \text{s0} \\ \text{sit-type} = \left[\text{nucl : steal(./q-params.x,./dgb-params.z)}\right] \end{array}\right]\text{)} : \textit{IllocProp} \end{array}\right]$$

Indeed, consideration of dialogue data should change one's perspective on the referential/descriptive divide since what have often been taken to be intrinsically referential terms like proper names can fail to be referential for an addressee. This can either trigger clarification interaction or lead to existential quantification, when there is no need to resolve the reference. Both possibilities are exemplified in (30).

(30) A: Did Jo phone? B: Who's Jo? / No one phoned.

The converse case has been much discussed under the guise of such notions as "specific indefinites" (Fodor & Sag 1982), exemplified in (31).

(31) A: Who did it? B: A friend of ours. A: Oh I see.

The current framework allows a straightforward definition of operations effecting the permutation of content labels. One such operation from *dgb-params* to *q-params* is sketched in (32):

(32) a. Input: $$\left[\begin{array}{l} \text{dgb-params} : \left[\begin{array}{l} \text{x} : \textit{Ind} \\ \text{r} : \text{named(x,“Jo”)} \end{array}\right] \\ \text{q-params} = [\,] : \textit{RecType} \\ \text{cont} : \text{arrive(dgb-params.x)} \end{array}\right]$$

b. Output: $$\left[\begin{array}{l} \text{dgb-params} = [\,] : \textit{RecType} \\ \text{q-params} : \left[\begin{array}{l} \text{x} : \textit{Ind} \\ \text{r} : \text{named(x,“Jo”)} \end{array}\right] \\ \text{cont} : \text{arrive(q-params.x)} \end{array}\right]$$

Given this referential management system, in the following we just use either dgb-params or q-params for notating QNP contents. That is, QNP content representations receive the following sample grouping:

(33)
$$\begin{bmatrix} \text{q-params} : & \begin{bmatrix} \text{maxset} & : \mathit{Set}(\mathit{Ind}) \\ \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c0} & : \overrightarrow{\mathit{PType}}(\text{maxset}) \\ \text{c1} & : \text{union(refset,compset,maxset)} \end{bmatrix} \\ \text{c2} & : \mathit{Rel}(|\text{q-params.refset}|,\ |\text{q-params.compset}|) \end{bmatrix}$$

Using the permutation rule from (32), the q-params in (33) can be moved to dgb-params. But we do not need to keep all sets in the same parameter space: any distribution of refset, maxset, and compset onto q-params and dgb-params is possible — regimented by evidence for grounding or quantifying away the corresponding parameter. Depending on which element goes there, referential and quantificational/describing uses are distinguished on a fine-grained level. The "classic" QNP readings are characterized by the following witnessing conditions, where a is an ordered set bipartition from the set of ordered set bipartitions of the head noun in question.

- *quantificational*: refset is part of *q-params*.

  Example: *The thieves (whoever they are) escaped with the loot.*

  $$\text{a} : \begin{bmatrix} \text{q-params} : & \begin{bmatrix} \text{maxset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c1} & : \vec{P}(\text{maxset}) \\ \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \end{bmatrix} \\ \text{q-cond} : & \text{Rel}(|\text{q-params.refset}|, |\text{q-params.compset}|) \end{bmatrix}$$

  iff $a \in p([\downarrow P]) \land \text{Rel}(|a.\text{first}|, |a.\text{second}|) = 1$

- *plural reference*: refset is part of *dgb-params*.

  Example: *Look! Many men wearing big boots are stealing our lemons.*

  $$\text{a} : \begin{bmatrix} \text{dgb-params} : & \begin{bmatrix} \text{maxset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c1} & : \vec{P}(\text{maxset}) \\ \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \end{bmatrix} \\ \text{q-cond} : & \text{Rel}(|\text{dgb-params.refset}|, |\text{dgb-params.compset}|) \end{bmatrix}$$

  iff $a = \iota x[x \in p([\downarrow P]) \land \text{Rel}(|x.\text{first}|, |x.\text{second}|) = 1$
  $\land\ x \in \text{common-ground}(\text{spkr}, \text{addr})]$

- *indefinite*: refind is part of *q-params*.

  Example: *Can anybody find me somebody to love?* (Queen)

$$\text{a} : \begin{bmatrix} \text{q-params} : \begin{bmatrix} \text{maxset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c1} & : \vec{P}(\text{maxset}) \\ \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{refind} & : \mathit{Ind} \\ \text{c2} & : \text{in(refind,refset)} \end{bmatrix} \end{bmatrix},$$

  iff $a \in p([\downarrow P]) \wedge \exists x[x \in a.\text{first}] \wedge \text{ refind } = x$

- *singular reference*: refind is part of *dgb-params*.

  Example: *The current world chess champion is Magnus Carlsen.*

$$\text{a} : \begin{bmatrix} \text{dgb-params} : \begin{bmatrix} \text{maxset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c1} & : \vec{P}(\text{maxset}) \\ \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{refind} & : \mathit{Ind} \\ \text{c2} & : \text{in(refind,refset)} \end{bmatrix} \end{bmatrix},$$

  iff $a \in p([\downarrow P]) \wedge \iota x[x \in a.\text{first}] \wedge \text{refind } = x$
  $\wedge\ x \in \text{common-ground}(\text{spkr}, \text{addr})$

Besides the "classic" readings distinguished above, our referential/quantificational mechanism captures further, more finegrained, possibilities. For instance, detective Hercule Poirot (a figure of the crime stories of Agatha Christie) often finds himself in a situation where he knows the refset (i.e., the group of suspects, which is part of Poirot's dgb-params), but the actual culprit still has to be convicted, that is, the refind initially is part of q-params. The tension in such *Whodunit* crime novels consists in the detective transferring the refind from q-params to dgb-params.[31] In *Spectre*, James Bond soon learns that Franz Oberhauser is a member of a criminal organisation (the eponymic secret society *Spectre*), but is still unaware of who else belongs to it. In this case, the refset (i.e., *Spectre* members) is part of Bond's q-params, while refind Oberhauser is already grounded in dgb-params. One can also conceive of cases where the compset is part of dgb-params, while the refset

31 It might actually turn out that there are two refinds, as in *Death on the Nile*, or even that the whole refset is guilty, as in — Caution! Spoiler alert! — *Murder on the Orient Express*.

is part of q-params. This configuration is exemplified by John F. Kennedy's question "If not us, who?".[32]

These examples illustrate the range of, and the need for, a cognitively oriented referentiality/non-referentiality mechanism which interacts with quantification, a mechanism of the kind developed here. Reference is accounted for in terms of common ground membership, which is compatible with various approaches. One such approach which seems to be particularly well suited is the discourse-based definite description interpretation theory of Poesio (1993) (with its slight revision in Poesio 1994), according to which semantic values of definites are located within a topic of conversation (there can be several, since a discourse usually is "about" more than one topic). On this account the interpretation of a definite noun phrase is additionally constrained by a familiarity presupposition as argued by Heim (1982) (see also Roberts 2003). We do not develop such an approach further here nor do we say anything about what it means that an object or a set of objects is part of the common ground (which is usually assumed to be constituted out of propositions). We rely on these notions to be intuitively clear enough.

4 Anatomy of quantified noun phrases

In this section, semantic probes from Referential Transparency are applied to the QNP content type in (33). Since the compset is probably the most controversial component of QNP contents, further compset evidence in addition to compset anaphora (see Section 3.2) are collected in Section 4.1. The "quantifier sieve" receives a systematic place within QNP contents in terms of the descriptive quantifier condition in Section 4.2, where we also show how a QNP is interpreted against a contextually given standard of comparison. The main contribution of this section is an explanation of compset anaphora in Section 4.3, which is grounded in the theroretical framework from Section 2. This is applied to explicate the different anaphoric potentials of *few* and *a few*. Based on the anaphoric potential of singular NPs, they are analysed as special cases of QNPs in Section 4.4. We return to predication in Section 4.5 where we introduce an extended notion of predication and anti-predication. The resulting "anatomy" of QNPs is summarized in Section 4.6. Only two modifications are finally effected on the structure in (33): condition c2 is in-

32 The original quotation continues "If not now, when?". This saying is probably inspired by the Talmudic "If I am not for myself, then who will be for me?" (*Pirkei Avot (Sayings of the Fathers)*, attributed to Hillel the elder).

corporated as "q-cond", and a feature "q-persp" is added. Q-persp's feature value is triggered by an empty refset from the set of ordered set bipartitions and allows for compset anaphora (cf. (47) below). Due to its highly idiosyncratic behaviour, the quantifier *every* is discussed in Section 4.7.

4.1 Complement sets

Apart from anaphora, the following sections collect some independent evidence that the compset has a systematic role to play within QNPs.

4.1.1 Compset enumeration

Despite lacking compset reference, an internal threefold partitioning even in the case of *most* is evinced by the fact that the semantic content of the refset can be clarified in terms of the compset, see (34).

(34) a. A: Most students came to the party.
b. B: Most students?
c. A: Yes, all but Tristan and Isolde. [→ compset enumeration]

"Few" shows the mirror image explication behaviour, as is illustrated in (35).

(35) a. A: Few students came to the party.
b. B: Few students?
c. A: Yes, just Tristan and Isolde. [→ refset enumeration]

Although it is perfectly possible to clarify the meaning of a *most*-QNP in terms of its refset and the meaning of a *few*-QNP in terms of its compset, it seems more natural, that is, easier, to enumerate the reference of both of the reprised fragments in the shortest manner — in case of *most* this is usually the compset, in case of *few* this is usually the refset.[33]

33 That people actually discuss the force of *most* (and similar issues) is evinced in the comments section here: https://www.theguardian.com/education/2022/apr/02/dear-nadhim-zahawi-great-big-bag-ideas-feels-empty-michael-rosen#comment-155720844 (lastly accessed 11th April 2022). A sample extract:
A: The last 50 years go back to 1972, and as Labour were in government for 18 of those years, (1974 – 1979, 1997 – 2010), some 36 % of the time, the conservatives were not in power for "most" of the last 50 years.
B: Sooooo, by your calculations the Conservatives were in power for 64 % of the time. Most of the time then.

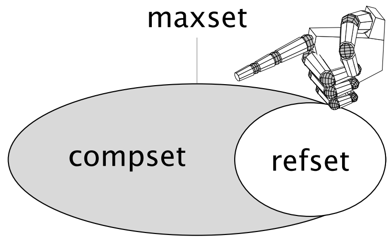


Figure 4 Iconic plural loci in ASL. Depicting maxset and refset automatically makes compset (shaded area) available.

4.1.2 Sign language

American Sign Language (ASL) developed an iconic strategy for realising anaphoric reference to plural antecedents. This strategy consists in drawing a large elliptical area representing the maxset in gesture space and a smaller refset area into the right part of the first one — see Figure 4. Crucially, drawing maxset and refset also the region associated with compset comes into being[34] and is available for pronominal reference by pointing at it (Schlenker, Lamberton & Santoro 2013).

4.1.3 *As many X as not*

The QNP anatomy involving a refset–compset pair is further supported by the "bipartition construction" *as many X as not*.[35] For instance, (36) roughly says that a certain method is helpful for about half of its users:

(36) These methods work for just as many people as not.

If we regard *as many X as not* as a quantificational NP, its meaning has to be spelled out in terms of non-empty refset and compset, and a descriptive quantifier condition that carries the information that both sets are of (roughly) equal size.

A: I disagree. 76–80 % would be most of the time period.
C: Most means majority i.e. anything over 50 %. Are you pulling our legs here?

34 Such processes are known as "closure under constraints" of diagrammatic representations (Perry & Macken 1996) or "transitive closure" (Lücking 2013: p. 77).

35 http://languagelog.ldc.upenn.edu/nll/?p=33321, accessed 19th June 2017.

4.2 Descriptive quantifier conditions

The descriptive quantifier condition itself can be the object of a clarification request:

(37) A: Few students left. B: What do you mean by "few"?

Therefore, the quantifier condition should also be the value of a particular path within QNPs (*addressability*) — we use "q-cond" (*quantifier condition*) for this purpose.

An appropriate answer to B's clarification request in (37) could be (38a) referring to the condition expressed in *q-cond*. Also an answer in terms of a cardinal quantity is possible, as in (38b). Obviously, this is not an answer to a question relating to q-cond. It clarifies a contextually provided standard of comparison, which we address by θ within the dialogue-gameboard parameters (dgb-params; cf. Section 3.5).

(38) a. Less than half. / Well, fewer students left than didn't.
b. Just two, I think.

The examples in (38) indicate two differing notions of "few": the refset can be *few* in comparison to the compset, or in comparison to a contextual norm "dgb-params.θ". With regard to the first sense, any number would count as "few", as long as the refset is smaller than the compset. With regard to the second sense, θ establishes a third point of comparison to the effect that "few" can be numerically explicated with reference to that standard.

A contextual parameter is also at work in *many*-QNPs. In this regard, Lappin (2000) shows how the various readings ascribed to "many" can be deduced from a contextually underspecified meaning. We notate this latter entity as a threshold contextual parameter θ, analogous to notions needed to capture the meaning of scalar adjectives like "big". For instance, the intensional meaning of "many" is given in (39), where the cardinality is evaluated against θ's value in context instead of the cardinality of the compset:

(39)
$$\begin{bmatrix} \text{dgb-params} : \begin{bmatrix} \theta : \mathbb{N} \end{bmatrix} \\ \text{q-params} : \begin{bmatrix} \text{maxset} : \mathit{Set}(\mathit{Ind}) \\ \text{refset} : \mathit{Set}(\mathit{Ind}) \\ \text{compset} : \mathit{Set}(\mathit{Ind}) \\ \text{c2} : \text{union(refset,compset,maxset)} \end{bmatrix} \\ \text{q-cond} : |\text{refset}| > \theta \end{bmatrix}$$

The semantic structure in (39), in particular θ, provides the semantic bit that is requested by B in (40) (obviously, A thinks that θ is instantiated by "5", which will be shared knowledge between A and B after their clarification exchange).

(40) a. A: I ate many apples yesterday.

b. B: Many?

c. A: Yes, more than five.

The contextual standard of comparison is also involved in the expectancy semantics of evaluative expressions. For instance, when used as a degree modifier as in (41), *surprisingly* shifts the standard of comparison according to which a quantity is "many" (Nouwen 2005).[36]

(41) I ate surprisingly many apples yesterday.

Applying clause (b), clarification potential, and (d), addressability, from the Referential Transparency principle (26) requires us to explicitly incorporate the descriptive meaning of the quantificational expression. We use the reserved label "q-cond" to this end. Accordingly, the anatomy of QNPs at this stage looks as follows:

(42 [to be revised])
$$\begin{bmatrix} \text{q-params} : \begin{bmatrix} \text{maxset} & : \mathit{Set}(\mathit{Ind}) \\ \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c0} & : \overrightarrow{\mathit{PType}}(\text{maxset}) \\ \text{c1} & : \text{union}(\text{refset},\text{compset},\text{maxset}) \end{bmatrix} \\ \text{q-cond} \quad : \mathit{Rel}(|\text{q-params.refset}|, |\text{q-params.compset}|) \end{bmatrix}$$

4.3 Quantifier perspective, anaphoric accessibility, and *a few*

A common view, due to Nouwen (2003), is that complement anaphora is licensed only with downward monotone proportional quantifiers, as exemplified in (43). Downward monotonicity is violated in (43b), proportionality in

36 Nouwen (2005) claims further that *surprisingly*, in particular used ad-sententially as in *Surprisingly, Megan runs quickly*, is downward monotone and hence the surprise relation expressed towards a proposition p' also obtains for any proposition p that entails p' (e.g., *Surprisingly, Megan runs*). However, assuming that I know that Megan is a frequent but slow runner, I still may be surprised about the former but not the latter. This example shows that evaluative expressions such as *surprisingly* seem to be driven by expectancies as much as by — or maybe even instead of — entailments.

(43d) (in all cases, *They* = music lovers that do *not* admire Reger, i.e., the complement set).

(43) a. *Few music lovers* admire Reger. *They* prefer Mozart.
b. *Many music lovers* admire Reger. #*They* prefer Mozart.
c. *Fewer than 20% of music lovers* admire Reger. *They* prefer Mozart.
d. *Fewer than 100 music lovers* admire Reger. #*They* prefer Mozart.

Now *few* and its sibling *a few* are at first glance related since they share the same quantifier condition, namely that the refset is (much) smaller than the compset (i.e., $|\text{refset}| < |\text{compset}|$, or $|\text{refset}| \ll |\text{compset}|$). Hence we could expect the latter to give rise to compset anaphora like the former does, which is, however, not the case:

(44) *A few music lovers* admire Reger. #*They* [= music lovers that do not admire Reger] prefer Mozart (instead).

GQT offers the possibility of explaining why there is no compset available as antecedent in (44) because *a few* is upward monotone, not downward monotone. For this reason, *few* and *a few* constitute a kind of minimal pair. However, this leads to the follow-up issues of why this is so and how to represent it in grammar/the lexicon. GQT does not seem to offer a good explanation here.

Since *a few* seems to include the indefinite article, the question arises whether *few* can be a quantificational determiner itself (likewise for *many*). While such quantificational expressions are often treated as determiners in the semantics literature, their distribution casts doubts: they pattern with determiners in just one of several uses. Solt (2015: p. 222) gives the following distributional data (we simplified (45d) to just one *many/few* pair, though):

(45) a. Many/few students attended the lecture. **quantificational**
b. John's friends are many/few. **predicational**
c. The many/few students who attended enjoyed the lecture. **attributive**
d. Many/few more than 100 students attended the lecture. **differential**

Obviously, *few* and *many* are only used as determiners in (45a). Based on their distribution, both Solt (2015) and Rett (2018) suggest to assign them into a class of their own, termed, respectively, *Q-adjectives* and *quantity words*.

Haspelmath (1997) provides further evidence from language change. He observes that languages which have a free-choice indefinite pronoun (an expression corresponding to current English *any*) develop into two directions: to *some* and to *every*.[37]

The paradigmatic adjective pattern of *few* and *many* observed in (45) is not complete, however: the indefinite article combines with a singular noun, but *a few* combines with a plural noun. *Many* is not compatible with the indefinite article. *A few* indeed seems to be derived from a combination — no longer productive — of the indefinite article and the adjective *few*. In this respect it is like its German counterpart *ein paar*, which is a lexicalized phrase consisting of the German indefinite article and the quantity word *paar*. However, unlike English *few*, German *paar* cannot be used on its own. Hence, there are reasons to decompose *a few* into a combination of *few* with indefinite *a*. In the next subsection the notion of "refind" is introduced, this is an individual selected by the indefinite article from the refset of its head noun. Combining the refind mechanism with the q-cond of *few* we receive the following structure for *a few*:

(46)
$$\begin{bmatrix} \text{phon} & : /\text{a few}/ \\ \text{q-params} & : \begin{bmatrix} \text{maxset} & : \mathit{Set}(\mathit{Ind}) \\ \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c2} & : \text{union(refset,compset,maxset)} \\ \text{refind} & : \mathit{Ind} \\ \text{c3} & : \text{in(refind,refset)} \end{bmatrix} \\ \text{q-cond} & : |\text{q-params.refset}| \ll |\text{q-params.compset}| \end{bmatrix}$$

In a grammar framework (46) can either be lexicalized — reflecting its somewhat frozen status — or derived in a strictly compositional manner — accounting for its apparent composite structure. The q-cond of *a few* in (46) is the same as that of *few*. What *a* adds is the refind and condition c3 in q-params. There is an immediate semantic effect: the refset sifted out by *few* must be such that it provides a refind. This in turn is only guaranteed if the refset has at least one element: the refind condition excludes the empty set. But why should this detail have an effect on the anaphoric potential of *a few* in comparison to *few*?

37 Haspelmath (1997: p. 156) only found two exceptions to this diachronic pattern, namely Hebrew *kol* 'every, any' and Turkish *herhangi* 'any', which contains *her* 'every'.

To address this issue, it is instructive to consider the psycholinguistic work of Moxey and Sanford and colleagues (Sanford, Dawydiak & Moxey 2007, Moxey 2006) shows that QNPs exhibit a number of context-dependent features, including expectancy-sensitive effects.[38] To this end, such works introduce the notion of quantifier *perspective* (or directivity, or polarity).[39] A negative quantifier like *few* or *not many* brings the compset into focus, while a positive quantifier like *a few* or *many* maintains focus on the refset. By this means, the perspectivity of a quantifier provides an interface for its anaphoric potential. There is substantiation for the focusing metaphor in terms of the denotational set-up spelled out in Section 2.3: compset anaphora is only licensed when the denotation of the QNP in question includes the ordered set bipartition with an empty refset. Pronouns, we argue, suffer from *horror vacui*: they avoid empty antecedent denotations. In fact, (possible) *emptiness* of an antecedent denotation has been claimed to be a factor in the optimality-theoretic account to plural pronoun interpretation of Hendriks & de Hoop (2001: p. 21). This view seems to be the reverse conjecture for an explanation of compset anaphora than that of Nouwen, which, among others, involve to "guarantee the non-emptiness of the compset" (Nouwen 2010). Since the compset is non-empty in all but one ordered set bipartition, what pronouns really do not like instead is a potentially empty refset. We notate this possibility in terms of the feature labelled "q-persp". The perspective feature "q-persp" comes in two manifestations: "q-persp: refset $= \varnothing$" and "q-persp: refset $\neq \varnothing$". The former feature value signals that the empty refset is included in a QNP's denotation, allowing for compset anaphora. The latter value excludes an empty refset, preventing the compset to act as an antecedent. Now the difference between *few* and *a few* is that the former carries the condition "q-persp: refset $= \varnothing$" while the latter the condition "q-persp: refset $\neq \varnothing$".

We can now formulate the constraint on anaphoric accessibility:

(47) **Anaphoric accessibility**

a. Maxset and refset are, other things being equal, available as antecedents for anaphoric expressions.

38 Expectancies have also been invoked in order to substantiate the provenance of contextual norms that figure in intensional interpretations of *many* by Fernando & Kamp (1996).

39 Psycholinguistics has extended such a view to the so-called *supposition-denial account* of the processing of sentences with a quantified subject (Sanford, Dawydiak & Moxey 2007). According to this account, a QNP is interpreted in terms of the difference between asserted and expected quantity (the Δ being the so-called *shortfall*).

b. Compset is available as an antecedent just in case q-persp has the value "refset = ∅".

The value "refset = ∅" of q-persp is not to be confused with an eponymous quantifier condition. While the latter (q-cond) says that the empty refset *is* the denotation of the QNP (that is, it is a QNP of the *no* type), the former (q-persp) just says that the empty refset is *included* in the denotation of the QNP, triggering the *horror vacui* of pronouns.

Clause (a), anaphoric potential, of the Referential Transparency principle (26), in addition to insights from psycholinguistic work on quantifier processing, lets us introduce a new feature, *q-persp*, which assists in regimenting anaphoric accessibility as detailed in (47). The QNP anatomy now looks as follows:

(48)
$$\begin{bmatrix} \text{q-params} : \begin{bmatrix} \text{maxset} & : \mathit{Set}(\mathit{Ind}) \\ \text{refset} & : \mathit{Set}(\mathit{Ind}) \\ \text{compset} & : \mathit{Set}(\mathit{Ind}) \\ \text{c0} & : \overrightarrow{\text{PType}}(\text{maxset}) \\ \text{c1} & : \text{union}(\text{refset},\text{compset},\text{maxset}) \end{bmatrix} \\ \text{q-cond} \quad : \mathit{Rel}(|\text{q-params.refset}|, |\text{q-params.compset}|) \\ \text{q-persp} \quad : \text{refset} = \emptyset \vee \text{refset} \neq \emptyset \vee \text{none} \end{bmatrix}$$

4.4 Singular and pointer objects

Singular as well as plural NPs behave strikingly similarly in the scope of negation. The minimal pair in (49) shows that universal QNPs modified by *not* make a compset available:

(49) a. *All music lovers* admire Reger. #*They* [= music lovers that do not admire Reger] love Mozart (instead).

b. *Not all music lovers* admire Reger. *They* [= music lovers that do not admire Reger] love Mozart (instead).

Also negated singular NPs allow for compset anaphora. It is worth emphasizing that comp*set* anaphora is the correct naming: it is sets of individuals that act as antecedents:

(50) *Not a single music lover* admires Reger. *They* all [= music lovers that do not admire Reger] love Mozart instead.

This also works with objects of negated verb phrases, as shown in (51):

(51) A: Go get a bike from the vélib station. B: Oh, but I don't see *any bike* that works there.

a. *It* is probably rented out.

b. *They* are probably rented out.

The singular pronoun in (51a) picks out a refind antecedent, the plural pronoun in (51b), however, seems to be ambiguous between a refset or a maxset anaphora. A specification from a maxset to a subset thereof (that is the inverse of domain widening) often happens in clarification exchanges:

(52) A: Go get a bike from the vélib station.
B: Any bike?
A: No, a working one.

B's clarification question targets a free choice from the maxset in the given situation, A's response constrains the refset by giving further descriptive information. Hence, there is evidence that singular NPs seem to recognize the maxset–refset–compset triplets but add an individual (which we term *ref ind*) to the quantificational parameters, as indicated in (53):

(53) **Shortcut singular NP**

$$\left[\text{q-params} : \begin{bmatrix} \text{refset} : \textit{Set}(\textit{Ind}) \\ \text{refind} : \textit{Ind} \\ \text{c3} \quad : \text{in(refind,refset)} \end{bmatrix}\right]$$

If there is a record which is of the type in (53), then that record has to provide an individual (refind) from a set of individuals (refset). In other words, the membership relation 'in' in condition c3 is existentially quantified. Thus, singular NP semantics according to (53) is equivalent to a choice function analysis on the refset (Reinhart 1997).[40]

Contributing individuals to contents is also required in multimodal dialogue. Recall that part of our motivating data stems from speech–gesture integration, see in particular Figure 3 in Section 1. The anaphoric potential of co-speech gestures includes so-called pointer objects (Eschenbach et al. 1989), at least with regard to verbal affiliates denoting dyadic structures. Groups of size two also have a special status in unimodal discourse. They

40 "A function f is a choice function (CH(f)) if it applies to any non-empty set and yields a member of that set." (Reinhart 1997: p. 372). This is exactly what 'in(refind,refset)' does.

may involve a contrast relation which can be exploited by anaphoric reference:

(54) a. *A couple* was walking by.
b. *He* was wearing glasses, *she* was wearing a hat.

These data can be accounted for by associating pointer objects with dyadic contents such as couples and the cardinal number *two*:[41]

4.5 Predication and "anti-predication"

In order to demonstrate that our system can fulfil the aim of predicational uniformity across NPs mentioned in Section 1.1, we need to embed our account in a formal grammar. For reasons of framework consistency we use a TTR-based variant of *Head-driven Phrase Structure Grammar* (HPSG; Sag, Wasow & Bender 2003), HPSG_{TTR}, which has been developed and motivated in Cooper (2008) and Ginzburg (2012). An example for an HPSG_{TTR} structure is given in Figure 5, more details can be found in the references just mentioned, in particular Ginzburg (2012: p. 326). A verb phrase, a plural predicate $\overrightarrow{PType}$, predicates of the refset of its syntactic subject (feature "nucl") *and exerts an "anti-predication" on the compset* ("anti-nucl"). Postulating multidimensional denotations is not uncommon in semantics, for instance *Alternative Semantics* (Rooth 1992) argues for a related move.

The **subj** constituent from the head–subject rule in Figure 5 is an output of the plural determiner–noun rule given in Figure 6. Among others, the NP rule connects the *cont* values of the subject to refset and compset. These sets can be part of dgb-params or q-params, as discussed in Section 3.5. Dgb-/q-params switches can be embedded in grammar by a family of coercion rules that license moving refset, maxset or compset to the different parameter sets.

41 That this is not the full story is obvious from examples involving triples such as: *A dog–female–male-threesome was walking by. He was smoking, she was talking on the phone and it was barking wildly.* Note, however, that the example draws on a group whose members are introduced by mutually distinct descriptions that are taken up by the minimal descriptive information bound up with the subsequent pronouns. Replacing "dog–female–male-threesome" by just "group" does not work.

$$\mathbf{S} \coloneqq \left[\begin{array}{l} \text{cat} : \left[\text{head} : \left[\text{pos=s} : \textit{PoS}\right]\right] \\ \text{dgb-params:} \left[\begin{array}{l} s_0 : \textit{Rec} \\ \textbf{subj}.\text{dgb-params} : \text{DP1} \\ \textbf{head}.\text{dgb-params} : \text{DP2} \end{array}\right] \\ \text{cont} = \left[\begin{array}{l} \text{sit} = s_0 \\ \text{sit-type} = \left[\begin{array}{ll} \textbf{subj}.\text{q-params} & : \text{QP1} \\ \textbf{head}.\text{q-params} & : \text{QP2} \\ \text{nucl} & : \text{hd-dtr.cont}(\textbf{subj}.\text{cont.x}) \\ \text{anti-nucl} & : \neg\text{hd-dtr.cont}(\textbf{subj}.\text{cont.y}) \end{array}\right] \end{array}\right] : \textit{Prop} \end{array}\right]$$

$$\textbf{subj} \coloneqq \left[\begin{array}{l} \text{cat} : \left[\text{head} : \left[\text{pos=np} : \textit{PoS}\right]\right] \\ \text{q-params} = \text{QP1} : \textit{RecType} \\ \text{dgb-params} = \text{DP1} : \textit{RecType} \\ \text{cont} : \left[\begin{array}{l} \text{x} : \textit{Set}(\textit{Ind}) \\ \text{y} : \textit{Set}(\textit{Ind}) \end{array}\right] \end{array}\right] \qquad \textbf{head} \coloneqq \left[\begin{array}{l} \text{cat} : \left[\text{head} : \left[\text{pos=vp} : \textit{PoS}\right]\right] \\ \text{q-params} = \text{QP2} : \textit{RecType} \\ \text{dgb-params} = \text{DP2} : \textit{RecType} \\ \text{cont} : \overrightarrow{\text{IV}} \end{array}\right]$$

Figure 5 Declarative plural head–subject rule (where $\overrightarrow{\text{IV}}$ labels the type of a plural intransitive verb and QPx and DPx the q-params respectively dgb-params values that get inherited to the mother node). The set labels x and y within the subject NP's content feature (cont) resolve to refset respectively compset from the subject's dgb- or q-params, according to the plural NP-forming rule in Figure 6.

$$\textbf{np} \coloneqq \left[\begin{array}{ll} \text{phon} & : \textit{List}(\textbf{det}.\text{phon}, \textbf{n}.\text{phon}) \\ \text{cat} & : \left[\text{head} : \left[\text{pos=np} : \textit{PoS}\right]\right] \\ \text{dgb-params} & : \left[\begin{array}{l} \text{dgb1} = \textbf{n}.\text{dgb-params} \\ \text{dgb2} = \textbf{det}.\text{cont.q-cond} \end{array}\right] \\ \text{cont} & : \left[\begin{array}{l} \text{x} = \textbf{n}.\text{cont.x} \\ \text{y} = \textbf{n}.\text{cont.y} \end{array}\right] \end{array}\right]$$

$$\textbf{det} \coloneqq \left[\begin{array}{ll} \text{phon} & : \textit{Phon} \\ \text{cat} & : \left[\text{head} : \left[\text{pos=det} : \textit{PoS}\right]\right] \\ \text{cont} & : \left[\begin{array}{l} \text{q-cond} : \textit{Rel}(|\textbf{n}.\text{dgb-params.refset}|, \\ \qquad\qquad |\textbf{n}.\text{dgb-params.compset}|) \end{array}\right] \end{array}\right] \qquad \textbf{n} \coloneqq \left[\begin{array}{ll} \text{phon} & : \textit{Phon} \\ \text{cat} & : \left[\text{head} : \left[\begin{array}{l} \text{pos=n} : \textit{PoS} \\ \text{num=pl} : \textit{Num} \end{array}\right]\right] \\ \text{dgb-params} & : \left[\begin{array}{ll} \text{maxset} & : \textit{Set}(\textit{Ind}) \\ \text{refset} & : \textit{Set}(\textit{Ind}) \\ \text{compset} & : \textit{Set}(\textit{Ind}) \\ \text{c1} & : \overrightarrow{\textit{PType}}(\text{maxset}) \end{array}\right] \\ \text{cont} & : \left[\begin{array}{l} \text{x=dgb-params.refset} : \textit{Set}(\textit{Ind}) \\ \text{y=dgb-params.compset} : \textit{Set}(\textit{Ind}) \end{array}\right] \end{array}\right]$$

Figure 6 Plural determiner–noun rule.

4.6 Stock-taking: the anatomy of quantified noun phrases

In short, we propose to analyse the conservative reading of the example sentence in (55a) as in (55b):

(55) a. Few students left.

b.
$$\left[\begin{array}{l}
\text{sit} = \text{s1} : \textit{Rec} \\
\text{sit-type} = \left[\begin{array}{ll}
\text{q-params} : & \left[\begin{array}{ll}
\text{maxset} & : \textit{Set}(\textit{Ind}) \\
\text{co} & : \overrightarrow{\text{student}}(\text{maxset}) \\
\text{refset} & : \textit{Set}(\textit{Ind}) \\
\text{compset} & : \textit{Set}(\textit{Ind}) \\
\text{c1} & : \text{union}(\text{refset,compset,maxset})
\end{array}\right] \\
\text{q-cond} & : |\text{q-params.refset}| \ll |\text{q-params.compset}| \\
\text{nucl} & : \overrightarrow{\text{left}}(\text{q-params.refset}) \\
\text{anti-nucl} & : \neg\overrightarrow{\text{left}}(\text{q-params.compset}) \\
\text{q-persp} & : \text{refset} = \varnothing
\end{array}\right] : \textit{RecType}
\end{array}\right]$$

The record type in (55b) is referentially transparent since it provides discourse referents for refset and maxset anaphora. Since it also hosts a compset, it can act for compset anaphora — licensed by q-persp's feature value "refset = ∅" (cf. Section 4.3). By means of negative predication on the compset (label "anti-nucl"), (55) expresses that the students from the complement set did not leave. The descriptive quantifier condition or "sieve" (*q-cond*) is part of the content of the NP, since it can be the object of clarification (following Section 4.2). All content constituents are addressable *via* their path names.

The general anatomy of QNPs is given in (56):

(56) **Quantified noun phrase anatomy (final version):**

$$\text{QNP}_{\text{sem}} \coloneqq \left[\begin{array}{ll}
\text{q-params} : & \left[\begin{array}{ll}
\text{maxset} & : \textit{Set}(\textit{Ind}) \\
\text{co} & : \overrightarrow{\textit{PType}}(\text{maxset}) \\
\text{refset} & : \textit{Set}(\textit{Ind}) \\
\text{compset} & : \textit{Set}(\textit{Ind}) \\
\text{c1} & : \text{union}(\text{refset,compset,maxset})
\end{array}\right] \\
\text{q-cond} & : \textit{Rel}(|\text{q-params.refset}|,\ |\text{q-params.compset}|) \\
\text{q-persp} & : \text{refset} = \varnothing \ /\ \text{refset} \neq \varnothing \ /\ \text{none}
\end{array}\right]$$

The noun phrase anatomy in (56) implements the argument part of a predicate-argument semantics as outlined in Sections 1.1 and 4.5.

4.7 The maverick *every* (and *each*)

The quantifier words, *every*, *all*, and *each* are usually interpreted identically in terms of logical ∀. However, in contrast to *all*, *every* and *each* are special in being syntactically singular but, on the view developed here, semantically plural. We show how this can be accommodated in the grammar. That *every* exhibits a distributive nature has been emphasized by Vendler (1962); it is also illustrated by Beghelli & Stowell (1997: p. 88) by means of the following pair of sentences:

(57) a. The Pope looked at all members of his flock.

b. The Pope looked at every member of his flock.

While the phrasing in (57a) suggests that the Pope watched his people in a single looking-event, (57b) prompts a distributive interpretation, saying that there are as many looking-events as flock members (Schein 1986) — the verb phrase in (57b) is not only a plural predicate type, but also receives a distributive interpretation (on the second, the grammatical object argument; cf. also Winter 2000, Schein 1986, Tunstall 1998). The role of *every* is to signal distributivity already on the lexical level and to require a distributive predicate to combine with. In terms of a feature-based grammar framework this can be implemented by passing a distributivity feature that triggers the plural type interpretation (cf. also Beghelli & Stowell 1997: pp. 103 ff.). This linguistic knowledge is encoded in the following lexical entry for *every* within HPSG_{TTR} (Cooper 2008, Ginzburg 2012).

(58)
$$\begin{bmatrix}
\text{phon} & : \text{/every/} \\
\text{cat} & : \begin{bmatrix}
\text{head} : \begin{bmatrix}
\text{pos=det} & : \textit{PoS} \\
\text{agr} & : \begin{bmatrix} \text{num=sg} : \textit{Num} \end{bmatrix} \\
\text{count=+} & : \textit{Binary}
\end{bmatrix} \\
\text{spec} : \left\langle \begin{bmatrix} \text{cat} : \begin{bmatrix} \text{head} : \begin{bmatrix}
\text{pos=n} & : \textit{PoS} \\
\text{distr=+} & : \textit{Binary}
\end{bmatrix} \end{bmatrix} \end{bmatrix} \right\rangle
\end{bmatrix} \\
\text{q-params} & : \begin{bmatrix}
\text{maxset} & : \textit{Set}(\textit{Ind}) \\
\text{refset} & : \textit{Set}(\textit{Ind}) \\
\text{compset=}\varnothing & : \textit{Set}(\textit{Ind}) \\
\text{c1} & : \text{union(refset,compset,maxset)}
\end{bmatrix} \\
\text{cont} & : \begin{bmatrix} \text{q-cond} : |\text{q-params.refset}| = |\text{q-params.maxset}| \end{bmatrix}
\end{bmatrix}$$

The head feature of *every* contains the information that it is a count quantifier — count nouns are distinguished by selecting for determiners that have the "count= +" feature (Sag, Wasow & Bender 2003: 112–113). The specifier (*spec*) of the quantifier has to be a noun (*pos*=*n*), which contributes the maxset–refset–compset triplet and also carries the distributivity feature (*distr*= +). When combining *every* with a noun, this feature is passed on to the NP level where it is visible to the verb phrase. Since this feature requires a plural type, it is possible to apply the head–subject rule from Section 4.5 despite the NP being syntactically singular.

With respect to *each* it is known that it is fully distributive — exemplified in (59), taken from Tunstall (1998: p. 99) — and that it is order-sensitive — exemplified in (60), taken from (Vendler 1962: p. 150):

(59) a. Ricky weighed every apple from the basket, but not individually.
b. *Ricky weighed each apple from the basket, but not individually.

(60) a. Each deputy rose as his name was called.
b. ?Every deputy rose as his name was called.

Both properties seem to be captured by assuming that *each*-QNPs operate on list types instead of set types. Accordingly, the q-params/dgp-params of *each*-QNPs are spelled out in terms of lists (where set union is replaced by list appendage):

(61)
$$\begin{bmatrix} \text{phon} & : \text{/each/} \\ \text{q-params} : & \begin{bmatrix} \text{maxset} & : \textit{List}(\textit{Ind}) \\ \text{refset} & : \textit{List}(\textit{Ind}) \\ \text{compset} & : \textit{List}(\textit{Ind}) \\ \text{c1} & : \text{append(refset,compset,maxset)} \end{bmatrix} \\ \text{q-cond} & : |\text{q-params.refset}| = |\text{q-params.maxset}| \end{bmatrix}$$

The focus of *each* on individuals, which becomes apparent in examples like (59) and (60), and is captured in our (61), is also buttressed in recent studies on QNP processing by Knowlton et al. (2021).

4.8 Complexity

We saw how RTT (*Referential Transparency Theory*) achieves predication and compositionality for quantified arguments, satisfies the *Reprise Con-*

tent Hypothesis, provides an explanation for (the unavailability of) compset anaphora, and distinguishes between the universal quantifiers, among others. Here, we briefly want to return to the issue of complexity mentioned in Section 1.3. *Inter alia* for cognitive reasons, constraining the logical space of quantification can be worth striving for. In Section 1.3, a logical space-constraining mathematical set-up has been exemplified in terms of an NP-internal account of quantification, which coincides with the conservativity universal. RTT also employs NP-internal quantification, so one has to ask how it comes off with respect to complexity.

For $|U| = 2$ there are four ordered set bipartitions: $p(\{a,b\}) = \{\langle\varnothing, \{a,b\}\rangle, \langle\{a\},\{b\}\rangle, \langle\{b\},\{a\}\rangle, \langle\{a,b\},\varnothing\rangle\}$. We restrict attention, as is common practice, to quantitative quantifiers (that is, in essence, quantifiers which are insensitive to any permutations of the objects from the denotation of their head noun), hence, the middle two (those without empty sets) are indistinguishable for quantifiers. Making the quantifier conditions sensitive only to cardinalities of sets captures the GQT constraint of being *quantitative* (see Section 2.3).[42] Virtually collapsing the two middle bipartitions, there are seven combinatorically possible QNP denotations which can potentially be sifted out by a quantifier, namely the following ones:

i. $\{\langle\varnothing,\{a,b\}\rangle\}$

ii. $\{\langle\{a\},\{b\}\rangle, \langle\{b\},\{a\}\rangle\}$

iii. $\{\langle\{a,b\},\varnothing\rangle\}$

iv. $\{\langle\varnothing,\{a,b\}\rangle, \langle\{a\},\{b\}\rangle, \langle\{b\},\{a\}\rangle\}$

v. $\{\langle\{a\},\{b\}\rangle, \langle\{b\},\{a\}\rangle, \langle\{a,b\},\varnothing\rangle\}$

vi. $\{\langle\varnothing,\{a,b\}\rangle, \langle\{a,b\},\varnothing\rangle\}$

vii. $\{\langle\varnothing,\{a,b\}\rangle, \langle\{a\},\{b\}\rangle, \langle\{b\},\{a\}\rangle, \langle\{a,b\},\varnothing\rangle\}$

Thus, for two objects there are seven possible QNP denotations. This is a significant reduction even compared to Klein's (2012) already reduced witness set approach, where at most 16 GQs can be denoted (cf. Section 1.3).

What is the general quantificational complexity of a quantifier operating on a set $p([\downarrow P])$ of ordered set bipartitions of a plural count noun P? Both refset and compset correspond to the power set of the denotation set of the underlying head noun. They are linked in reversed order so that each set from the power set is paired with its corresponding complement from the compset. This implies that the cardinality of the set of ordered set bipartitions is equal to the cardinality of the power set. Thus, for k elements in

42 We are grateful to an anonymous reviewer for pointing this out.

the denotation of the head noun there are 2^k ordered set bipartitions. However, these 2^k ordered set bipartitions belong to $k+1$ cardinally different bipartition types: for k elements, the largest refset partition has k members. The second largest refset partition has $k-1$ members. The third largest refset partition has $k-2$ members. And so on. The smallest refset partition finally, the one with counter $k+1$ (the "$k+1$th largest" refset partition) has $k-k$ members (the empty refset). Thus, for a Q-extension with k elements there are $k+1$ numerically distinct ordered set bipartitions. Now, the empty set is already built into the bipartitions. Subtracting the empty set we get $2^{k+1}-1$ combinatorically possible QNP denotations for a quantifier sieving its head noun's set of ordered set bipartitions with k elements. To take up the example above: the two individuals a and b give rise to $2^{2+1}-1=7$ QNP denotations, namely the ones enumerated above.

In order to assess the complexity of quantifiers, the number of QNP denotations has to be applied against the number of possible nominal Q-extensions (i.e., sets of ordered set bipartitions). For a domain of $n=2$ elements there are $2^2=4$ possible sets of ordered set bipartitions, the power set of the set of individuals (see above). The cardinalities of the Q-extensions' refsets are 0 (empty set), 1 (first element), 1 (second element), and 2 (both elements) (conversely for the compset). Putting the refset cardinalities into the "QNP denotation coefficient", it follows that there are at most $(2^{0+1}-1)\times(2^{1+1}-1)\times(2^{1+1}-1)\times(2^{2+1}-1)=1\times3\times3\times7=63$ quantifiers for $|M|=2$. As a comparison, recall that there are $2^{16}=65{,}536$ possible unconstrained quantifiers according to GQT (Keenan 2002: p. 632) and 512 conservative ones (Klein 2012). Ordered set bipartitions provide a considerable simplification of quantificational complexities and are therefore, in our view, preferable in terms of processing and comprehension.

Note finally, that RTT gives rise to a straightforward notion of the content of quantifier words. The denotation of a quantificational determiner on our account is just the set of sets of ordered set bipartitions sifted out according to the quantifier condition. Let us suppose that the domain of quantification consists of the elements of the three predicates *bicycle*, *person* and *ball*: {🚲, 🚲, 🚲, 👤, 👤, ⚽, ⚽}. Then there are three sets of ordered set bipartitions (one for each type). The denotation of *all* will be the set of ordered set bipartitions with an empty compset, namely: $\{\langle\{$🚲, 🚲, 🚲$\}, \varnothing\rangle, \langle\{$👤, 👤$\}, \varnothing\rangle, \langle\{$⚽, ⚽$\}, \varnothing\rangle\}$. The denotation of *most* will be the set of ordered set bipartitions where the refset is larger than the compset: $\{\langle\{$🚲, 🚲, 🚲$\}, \varnothing\rangle$, $\langle\{$🚲, 🚲$\}, \{$🚲$\}\rangle$, $\langle\{$🚲, 🚲$\}, \{$🚲$\}\rangle$, $\langle\{$🚲, 🚲$\}, \{$🚲$\}\rangle$, $\langle\{$👤, 👤$\}, \varnothing\rangle$, $\langle\{$⚽, ⚽$\}, \varnothing\rangle\}$. Likewise for other quantifiers. Thus, the denotational content of a quantifier word is just the application of the quantifier's q-cond to the union of the Q-extensions of the *PType*s of a given model.

5 Conclusions and further work

In this paper we have proposed RTT (*Referential Transparency Theory*), a new approach to the meaning of QNPs. This involves both a denotational component and an account of the anatomy of QNP meaning. The denotational foundation of QNPs is given in terms of sets of ordered set bipartitions. This not only entails the conservativity universal, but also provides considerable improvements with respect to quantificational complexity and explanations for compset anaphora. Quantifier words contribute a descriptive quantifier condition which acts as a sieve on the set bipartitions. Unlike generalized quantifiers from GQT, in our account QNPs follow a "naïve" predicational semantics where the (non type-raised) VP applies to its (non type-raised) subject argument.

Following earlier work on the anatomy of quantified noun phrases — most notably Purver & Ginzburg (2004) — we argue for a general QNP anatomy in terms of maxset, refset and compset. This anatomy is motivated by Referential Transparency, a collection of semantic desiderata that incorporate clarification potential, anaphoric potential, speech–gesture integration, and addressability.

While our theory accounts for the incremental interpretation of QNPs, it needs to be shown how it deals with the ambiguity imposed by multiple QNPs as in *Every dog chased a cat* (is there one cat in total, or (at least) one cat for each dog?). Traditionally, these ambiguities have been modelled by means of quantifier scope relations obtained by quantifier raising/dislocation at a level of logical form or by semantic analogues in terms of stores. As pointed out in Section 1, quantifier raising seems to be at odds with incremental quantifier processing, which does not exhibit delayed interpretations. However, there are other means to analyse so-called inverse scope readings, namely as dependent functions (on functional NP uses see Jacobson 2000, Ginzburg 2012, Steedman 2012). On the relational reading of *Every dog chased a cat*, the main verb *chased* contributes a plural relation between a set of dogs and a (specific) cat. On the functional reading, the verb phrase *chased a cat* contributes a function whose domain is the set of dogs. Both semantic interpretations can be derived in an incremental fashion. We leave a detailed spelling out of such an account to future work.

The quantificational determiners *every*, *all*, *each*, while often treated uniformly as universal quantifiers, receive a different referentially transparent semantics each: briefly, *every*-QNPs require a distributive predicate type, *each*-QNPs an interpretation on lists on top of that, and *all*-QNPs remain neutral (cf. Section 4.7). Note that this individual treatment of the universal quantifiers is in accordance with psycholinguistic findings which reveal that there is no mutual priming among them (Feiman & Snedeker 2016).

Referentially transparent QNP semantics includes the view that the meaning of complex quantifiers is derived in a compositional manner on q-conds. Following this direction, *not* has been analysed as a noun phrase negation operator by Lücking & Ginzburg (2019) in such a way that the meaning of complex QNPs like *most people but not Bill* can be fully decomposed. The scope of the strict compositional stance is to be widened in future work, in particular incorporating mass nouns and quantification over times and events.

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