Semantics & Pragmatics Volume 15, Article 6: 1–53, 2022
https://doi.org/10.3765/sp.15.6

Keep *only* strong*

Luis Alonso-Ovalle
McGill University

Aron Hirsch
Leibniz-Zentrum Allgemeine Sprachwissenschaft

Submitted 2018-11-15 / First decision 2019-02-05 / Revision received 2019-10-19 / Second decision 2019-10-22 / Revision received 2020-04-10 / Third decision 2020-04-16 / Revision received 2021-02-01 / Fourth decision 2021-02-03 / Revision received 2021-12-29 / Accepted 2022-06-17 / Published 2022-06-24 / Final typesetting 2024-02-21

Abstract While Horn (1969) proposed that ⟦only⟧(p) presupposes that the prejacent p is true, von Fintel & Iatridou (2007) showed that the expected prejacent inference is not observed when a necessity modal occurs in the scope of *only*: ⟦only⟧(□p) may convey that p is *possible*, rather than *necessary*. What is the mechanism behind the surprisingly weak inference? The approach in von Fintel & Iatridou 2007 is to revise the analysis of *only* itself to weaken its contribution. In this paper, however, we argue that Horn's *only* is correct after all, and introduce a source of weakening separate from *only*. In particular, in von Fintel & Iatridou's modal environment, a phonetically null operator (AT LEAST; Crnič 2011, Schwarz 2005) occurs in the scope of *only* to weaken the presupposed prejacent. Much recent attention has been paid to covert operators which *strengthen* meaning, in particular a covert EXH with a meaning similar to *only* (e.g. Chierchia 2006, Fox 2007,

* The authors' names are in alphabetical order. For their feedback, we thank Gennaro Chierchia, Luka Crnič, Kai von Fintel, Danny Fox, Martin Hackl, Irene Heim, Vincent Homer, Sabine Iatridou, Daniel Margulis, Paul Marty, Craige Roberts, Uli Sauerland, Bernhard Schwarz, Michael Wagner, participants in our Winter 2018 McGill seminar, audiences at AngelikaFest (UMass Amherst), ZAS Berlin, and SALT 28 (MIT), three reviewers for S&P, and the editor, Kjell Johan Sæbø. Thanks also to Jonathan Palucci for help with typesetting. The authors both received financial support from the Social Sciences and Humanities Research Council of Canada, through Insight Grants *Modality in the Nominal Domain* (435-2013-0103) and *Modality across Categories: Modal Indefinites and the Projection of Possibilities* (435-2018-0524) (Alonso-Ovalle, PI), and a postdoctoral fellowship to Hirsch. Hirsch received additional support from the Volkswagen Stiftung (VWZN3181) (Luka Crnič, PI). All errors are, of course, our own. This article builds on Alonso-Ovalle & Hirsch 2018, from which we depart: Section 3 presents more evidence against von Fintel and Iatridou's decompositional strategy, and Section 6 a new way of restricting unwanted LFs with AT LEAST.

Chierchia, Fox & Spector 2012). A key consequence of our analysis is that natural language incorporates a covert *weakening* operator, as well.

Keywords: *Only*, sufficiency modal construction, AT LEAST, grammatical weakening

1 Introduction

According to the lexical entry in (1), *only* applies to a proposition p (its 'prejacent'), presupposes that the prejacent is true, and asserts that all alternatives are false, except those already entailed by the prejacent. We indicate the alternative set (ALT) as a parameter. Our concern in this paper will be with *only*'s presupposition, which in (1) follows the classical proposal of Horn (1969).

(1) $[\![\text{only}]\!]^{\text{ALT}}(p) = \lambda w : p(w) \,.\, \forall p' \in \text{ALT}\ [p'(w) \rightarrow p \subseteq p']$

Horn's prejacent presupposition captures observed inferences in basic data. Example (2a), for instance, conveys that John visited the North End, and that is the prejacent of *only*, given the LF in (2b). With respect to the syntax, note that we assume that *only* attaches somewhere above the vP.[1]

(2) a. John only visited the North End.
b. $[_{\text{TP}}$ only $[_{v\text{P}}$ John visit [the North End]$_{\text{F}}]]$

While a prejacent presupposition is supported in (2a), it is observed in von Fintel & Iatridou 2007 (vF&I) that a problem arises in (3), where a necessity modal occurs in the scope of *only*. In this case, the prejacent presupposition seems too strong.

(3) To get good cheese, you only have to go to the North End.

Suppose (3) were assigned the LF in (4). For convenience, we exclude the purpose clause from the diagram. We assume that the purpose clause restricts the domain of quantification of the modal, a universal quantifier over possible worlds. In (4), $v\text{P}_2$ expresses the proposition that in all worlds where you get good cheese, you go to the North End — and that is the prejacent of *only*.

(4) $[_{\text{TP}}$ only $[_{v\text{P}_2}$ have $[_{v\text{P}_1}$ you go to [the North End]$_{\text{F}}]]]$

1 To streamline the structures, we show the subject reconstructed into its thematic position in spec-vP, and suppress elements not crucial for the interpretation of *only*, such as the T head.

If *only* is defined as in (1), (3) should thus presuppose a necessity claim. In actual fact, however, a weaker possibility inference is observed. (3) does not say that you *have* to go to the North End to get good cheese, but merely that you *can* go there. Abbreviating the proposition that you go to the North End as ϕ_{NE}, (5a) is predicted, but (5b) observed. vF&I refer to this as the *prejacent problem*, and name the configuration in (3) the *sufficiency modal construction*.

(5) *The prejacent problem*
   a. Predicted: $\Box\phi_{NE}$
   b. Observed: $\Diamond\phi_{NE}$

The intuition is in fact more nuanced still. In addition to conveying that going to the North End is a *way* to get good cheese, the sentence in (3) conveys that it is an *easy* place to go. If good cheese can be found in the North End of Boston or in Switzerland, and the cheese seeker is at MIT, (6), for instance, seems infelicitous, in contrast to (3). Going to Switzerland is difficult, not easy.

(6) #To get good cheese, you only have to go to Switzerland.

For now, we will ignore the easiness component, and focus on the prejacent problem, asking: what is the mechanism responsible for deriving the unexpectedly weak possibility inference in (3)?

1.1 Prior approach: weakening *only*

vF&I revise the analysis of *only* to weaken its contribution. Their proposal has two ingredients, one syntactic and one semantic. Syntactically, they deny that *only* is a simplex adverbial, and decompose it into two separate underlying morphemes: negation and an exceptive. The closest overt English parallel might be (7). While negation takes scope above the modal, the exceptive takes scope below. Semantically, the exceptive is the presupposition trigger, and introduces not the 'strong' prejacent presupposition in (1), but a 'weak' existential presupposition (in effect, that *some* alternative is true). As we will see, an existential presupposition triggered at the low scope site delivers an appropriately weak overall reading.

(7) To get good cheese, you do**n't have to** go anywhere **except** the North End.

We, however, will discuss challenges for both revisions which vF&I propose. First, we show that *only* exhibits different properties from constructions which wear on their sleeve an overt decompositional syntax. Second, based on prior work, we show that weakening the presupposition to existential results in readings which are *too* weak in data involving conjunction and negation (e.g. van Rooj & Schulz 2007, Ippolito 2007). A prejacent inference is attested in those data, but not predicted if *only* is weakened. On this basis, we pursue a shift in perspective.

1.2 Proposal: weakening is from a separate operator

We take the challenges for vF&I to reveal that *only* is a simplex adverbial, which presupposes the truth of its prejacent, after all. But, then, a new explanation is needed for the weak inference in (3). If *only* is strong, weakening must come from a separate source. We propose that natural language makes available a covert weakening operator — AT LEAST (Crnič 2011, Schwarz 2005) — which may optionally be inserted into the scope of *only*. As a preview, the original LF in (4) updates to (8).

(8) $[_{TP}$ only $[_{vP_2}$ have [AT LEAST $[_{vP_1}$ you go to the [North End]$_F$]]]]

Despite surface appearances, the prejacent of *only* no longer expresses that, to get good cheese, you have to go to the North End. The contribution of AT LEAST is to introduce a disjunction below the modal, so the prejacent says that, in each cheese-world, you go to the North End *or somewhere further away*. The new, weakened prejacent is viably presupposed. By negating alternatives, *only* asserts that you are not *obliged* to go somewhere further — which, together with the presupposition, leads to the possibility inference that you *can* go to the North End.

Because *only* is a simplex adverbial, challenges for decomposition are resolved. Moreover, our proposal has the flexibility to reconcile vF&I's observation with the conjunction and negation data. Since the mechanism for weakening is separate from *only*, weakening will not necessarily arise in all environments where *only* occurs. We will suggest that insertion of AT LEAST is restricted on pragmatic grounds when conjunction and negation are present. Weakening is observed *sometimes*, but not *always*, due to factors restricting AT LEAST.

Recent work has widely argued for a covert operator, EXH, which has a meaning similar to *only* (e.g. Chierchia 2006, Fox 2007, Chierchia, Fox & Spec-

tor 2012). EXH explains how a sentence like (9b) can have a *stronger* meaning than the one expressed by the overt material. The overt material simply says that John visited the North End. Yet, in a context such as (9a), (9b) can be interpreted as saying that John visited the North End *and nowhere else*. This exhaustivity inference is an entailment of the LF in (10), where the overt material is the complement of the covert EXH.

(9) a. Where did John visit?
b. John visited the North End.

(10) $[_{TP}$ EXH $[_{vP}$ John visit [the North End]$_F$]]

From our perspective, vF&I's example has exactly the converse profile — the meaning is weaker than what we expect from the overt string — and the explanation is parallel. There exist not only hidden *strengthening* operators like EXH, but also hidden *weakening* operators like AT LEAST. Over the course of the paper, we aim to develop an extended argument for such an operator.

1.3 Roadmap for the paper

Discussion will proceed as follows. In Section 2, we present in detail von Fintel & Iatridou's means of capturing (3) by modifying *only* itself. In Sections 3–4, we present counterarguments. In Section 5, we put forward our own analysis with weakening sourced to AT LEAST and, in Section 6, show how that approach captures the problematic data. Finally, Section 7 concludes.

2 Prior approach: weakening *only*

We begin by presenting in detail vF&I's modification of *only*, which (a) syntactically decomposes *only* into negation and an exceptive, and (b) proposes that the exceptive triggers a weak existential presupposition. Our exposition will aim to justify why both (a) and (b) are required. First, in Section 2.1, we attempt to weaken the presupposition without decomposing *only*, and show that the meaning for (3) is still too strong. Decomposition is added in Section 2.2.

2.1 Step 1: an existential presupposition

A simplex *only* triggering an existential presupposition is stated in (11) (see Horn 1996 and Geurts & van der Sandt 2004 for the proposal to weaken the

presupposition of *only* along these lines). We refer to this operator as $only_{weak}$ and contrast it with the earlier one in (1), repeated in (12), which we now call $only_{strong}$. The two operators differ solely with respect to their presupposition. With respect to assertion, $only_{weak}$ like $only_{strong}$ negates non-weaker alternatives to the prejacent.

(11) $[\![\text{only}_{\text{weak}}\ ]\!]^{\text{ALT}}(p) = \lambda w : \exists p' \in \text{ALT}\ [p'(w)]\ .\ \forall p'' \in \text{ALT}\ [p''(w) \rightarrow p \subseteq p'']$

(12) $[\![\text{only}_{\text{strong}}]\!]^{\text{ALT}}(p) = \lambda w : p(w)\ .\qquad\qquad \forall p'\ \in \text{ALT}\ [p'(w) \rightarrow p \subseteq p']$

The transparent LF for (3) with $only_{weak}$ is provided in (13). To assess how *only* is interpreted, we must first state a value for the ALT parameter. Following Katzir 2007 and Fox & Katzir 2011, we assume that alternatives are constructed as syntactic objects by replacing the focused constituent with elements of equal or lesser structural complexity.[2] We will restrict our attention to three replacements for the North End: the North End itself, New York, and Switzerland. As a result, the alternatives *only* sees are the structures in (14). Abbreviating the proposition that you go to place x as ϕ_x, these structures express the propositions shown at the end of their line. ALT is the set of these three propositions, as in (15).

(13) $[_{\text{TP}}\ \text{only}_{\text{weak}}\ [_{vP_2}\ \text{have}\ [_{vP_1}\ \text{you go to [the North End]}_F]]]$

(14) a. $[_{vP_2}\ \text{have}\ [_{vP_1}\ \text{you go to [the North End]}_F]]$ $\Box\phi_{NE}$
b. $[_{vP_2}\ \text{have}\ [_{vP_1}\ \text{you go to [New York]}_F]]$ $\Box\phi_{NY}$
c. $[_{vP_2}\ \text{have}\ [_{vP_1}\ \text{you go to [Switzerland]}_F]]$ $\Box\phi_{S}$

(15) $\text{ALT} = \{\Box\phi_{NE}, \Box\phi_{NY}, \Box\phi_{S}\}$

Now, $only_{weak}$ introduces the presupposition that some element of (15) is true. The existential presupposition in (16a) is re-formulated as a disjunction in (16b). The presupposition in (16) is still too strong. It says that, to get good cheese, you must go to the North End or you must go to New York or you must go to Switzerland. Unlike (16), the sentence in (3) does not intuitively say that there is any particular place that you must go to get good cheese.

(16) a. P: $\lambda w\ .\ \exists p \in \{\Box\phi_{NE}, \Box\phi_{NY}, \Box\phi_{S}\}\ [p(w)] \Leftrightarrow$
b. $\Box\phi_{NE} \vee \Box\phi_{NY} \vee \Box\phi_{S}$

2 Throughout the paper, we will use the term 'alternative' in two different ways: to refer to the syntactic objects referred to above, and also to the propositions that they convey.

The problem is compounded when presupposition and assertion are considered together. Of the three alternatives in ALT, just one is entailed by the prejacent of $only_{weak}$, which is the prejacent itself ($\Box\phi_{NE}$). $Only_{weak}$ negates that you have to go to New York ($\Box\phi_{NY}$) and that you have to go to Switzerland ($\Box\phi_{S}$). This, together with the existential presupposition, entails that you have to go to the North End. The very same too strong necessity inference is thus derived with $only_{weak}$ as with $only_{strong}$ — the prejacent problem returns.

2.2 Step 2: decomposition + weakening

Since weakening the presupposition is not enough, vF&I propose to combine the revised presupposition with a further syntactic revision. Their starting point is the observation that an exclusive meaning similar to what *only* conveys may be expressed with two overt ingredients co-occurring: negation and an exceptive phrase. We illustrated above with the English example in (17).

(17) To get good cheese, you **don't** have to go anywhere **except** the North End.

Although (17) seems rather marked, counterpart data are fully natural in other languages. The Spanish data in (18) and (19) illustrate. The sentence in (18) features the counterpart of *only*, the exclusive adverb *solo*, closely matching vF&I's example in English. Example (19) conveys the same meaning with two expressions. The first is *no*, which is sentential negation. The second is *más que XP* ('more than XP'), which vF&I characterize as an exceptive phrase. Similar constructions occur productively in many other languages, as well.

(18) Para conseguir buen queso, **solo** tienes que ir al
to get:INF good cheese, only have:2s that go:INF to-the
North End.
North End
'To get good cheese, you only have to go to the North End.'

(19) Para conseguir buen queso, **no** tienes que ir **más que**
to get:INF good cheese, not have:2s that go:INF more than
al North End.
to-the North End
'To get good cheese, you only have to go to the North End.'

While *only* in English appears to be a single morpheme, vF&I propose that the surface phonology is not representative of the underlying syntax. Rather, they decompose *only* into the same two elements that Spanish wears on its sleeve in (19): negation and an exceptive. To consider their analysis in detail, we begin with the basic sentence in (20a), which is assigned the LF in (20b).[3]

(20) a. John only visited the North End.
b. [$_{TP}$ NEG [[EXC [the North End]$_F$] λ1 [$_{vP}$ John visit t_1]]]

NEG adjoins on the clausal spine, while EXC forms a constituent with the focus which, at least under one plausible focus structure, is the DP *the North End*. Overt pre-*v*P *only* occurs at the site of NEG in (20a), while EXC is abstract. EXC composes with the focused DP to form a quantifier, which undergoes Quantifier Raising (QR) to a propositional node below NEG. vF&I interpret EXC as in (21).

(21) $[\![\text{EXC}]\!] = \lambda x.\ \lambda f_{\langle e,st\rangle}\ .\ \lambda w : \exists y\ [f(y)(w)]\ .\ \exists z\ [z \neq x \wedge f(z)(w)]$

Applied to the North End and a property f, EXC presupposes that some entity satisfies f, and asserts that some entity other than the North End satisfies f. In (20b), f is the property of being an entity that John visited. Hence, the triggered presupposition in (20b) is that John visited somewhere, as in (22a), and the assertion below negation is that John visited somewhere other than the North End, as in (22b).

(22) a. P: $\phi_{NE} \vee \phi_{NY} \vee \phi_S$
b. A: $\phi_{NY} \vee \phi_S$

The presupposition projects over NEG, while the assertion is negated, yielding the overall meaning in (23): it is presupposed that John visited somewhere, and asserted that he didn't visit anywhere other than the North End. Combined, it follows that John visited the North End. For a basic case, vF&I's analysis yields the same prejacent inference that $only_{strong}$ would yield.

(23) a. P: $\phi_{NE} \vee \phi_{NY} \vee \phi_S$
b. A: $\neg(\phi_{NY} \vee \phi_S) \Leftrightarrow \neg\phi_{NY} \wedge \neg\phi_S$

3 Throughout the paper, we will use ϕ_x interchangeably to abbreviate the propositions that {John, you} {visited, went to} place x.

The new means of presupposition triggering, however, has impact in vF&I's core example. The element triggering the existential presupposition is now not at the site of overt *only*, but rather is abstract and takes covert scope with the object DP through QR. That makes available the LF in (25) for (24).

(24) To get good cheese, you only have to go to the North End.

(25) $[_{\text{TP}}$ NEG $[_{vP_2}$ have [[EXC [the North End]$_{\text{F}}$] $\lambda 1$ $[_{vP_1}$ you go to t_1]]]]

While NEG scopes above *have* at the site of overt *only*, the exceptive phrase moves to a position below *have*. The presupposition is thus triggered at a non-transparent low scope site, and a very weak presupposition results. The triggered presupposition is (26a) — that you go somewhere — analogous to (22a) above. Assuming that the presupposition projects universally through the modal, the projected presupposition will say that in all cheese-worlds, you go somewhere, as in (26b). In turn, (26b) projects over negation to become a global presupposition.

(26) a. P: $\phi_{NE} \vee \phi_{NY} \vee \phi_S$ (triggered)
b. P: $\Box(\phi_{NE} \vee \phi_{NY} \vee \phi_S)$ (projected)

The presupposition is clearly weaker than that of $only_{strong}$, and is weaker than the one predicted in Section 2.1 with simplex $only_{weak}$, as well. The presupposition from Section 2.1 is repeated below for comparison:

(27) P: $\Box\phi_{NE} \vee \Box\phi_{NY} \vee \Box\phi_S$

The difference between the two presuppositions is one of scope. In Section 2.1, an existential presupposition was triggered by overt *only* above the modal and, accordingly, the existential (indicated with disjunction) takes wide scope over the modal in (27). Low triggering at the site of EXC results in a scope reversal within the presupposition: in (26b), the modal takes wide scope over the existential. The scope difference impacts whether the place may vary from cheese-world to cheese-world. As noted earlier, the presupposition in (27) requires that you go to some particular place across all cheese-worlds. The presupposition in (26b) requires that you go somewhere in every cheese-world, but is compatible with you going to all different places from world to world. The latter is a weaker requirement than the former, and is compatible with the intuited meaning of vF&I's example.

In fact, given the weaker presupposition, the target possibility inference derives in combination with the assertion. Referring back to the LF in (25), EXC asserts at the edge of vP_1 that you go somewhere other than the North End. The modal and negation integrate in turn to derive the overall assertion in (28): that you do not have to go anywhere other than the North End.

(28) A: $\neg\Box(\phi_{NY} \vee \phi_S)$

The presupposition in (26b) and the assertion in (28) are conjoined in (29). Together, they say that you go somewhere in every cheese-world, but not always somewhere other than the North End. In other words, in *some* cheese-worlds, you go to the North End (and to no other place). The possibility inference in (29a) is derived, while the necessity inference in (29b) is not — solving the prejacent problem.

(29) $(P \wedge A) \Leftrightarrow \Box(\phi_{NE} \vee \phi_{NY} \vee \phi_S) \wedge \neg\Box(\phi_{NY} \vee \phi_S)$

a. $\Rightarrow \Diamond\phi_{NE}$

b. $\not\Rightarrow \Box\phi_{NE}$

3 Evidence against decomposition

We turn now to assessing vF&I's revisions to *only*. We first evaluate the claim that apparently simplex adverbs such as *only* decompose into negation and an exceptive, parallel to overt NEG + EXC constructions cross-linguistically. Our investigation will zero in on Spanish. Since an apparently simplex adverb (*solo*) and overt NEG + EXC (*no más que*) productively co-exist in Spanish, it is possible to directly compare them, and determine whether the former reduces to the latter. We will consider, in particular, properties of sentential negation. We will see that *no más que* inherits properties of its component negation, while *solo* patterns differently. Our results pose a challenge for decomposing *solo* into NEG + EXC.

3.1 Polarity licensing

The first test involves licensing of polarity-sensitive items. Consider negative concord items ('n-words', Laka 1990). N-words are licensed by sentential negation, as is most clear for post-verbal n-words, as in (30). *Ningún* ('n-one') is acceptable when *no* is present, but the string is ungrammatical if *no* is removed.

(30) Juan *(no) visitó ningún barrio con María.
Juan *(not) visited:3s N-one neighbourhood with María
'Juan didn't visit any neighbourhood with María.'

As with sentential negation by itself, n-words are licensed by the negation in *no más que*, as in (31). But, what about with *solo*? If *solo* were decomposed into NEG + EXC, we might expect n-words to be licensed in the scope of *solo*, as well, since the n-word would then be in the scope of the negation which *solo* contributes. Yet, n-words are *not* in fact licensed with *solo*, as shown in (32).

(31) Juan no visitó ningún barrio más que con María.
Juan not visited:3s N-one neighborhood more than with María
'Juan only visited any neighbourhood with María.'

(32) *Juan solo visitó ningún barrio con María.
Juan only visited:3s N-one neighborhood with María
'Juan only visited any neighbourhood with María.' (Intended)

To maintain a decompositional analysis of *solo*, *solo* would have to differ from *no más que* in some way which prevented its component negation from licensing n-words. Zeijlstra (2004, 2008) proposed that n-words must enter into an Agree relation with a higher negation for a [NEG] feature in order to be licensed. From that perspective, one possibility is that not all negation morphemes have the same feature specification. While *no* bears the feature targeted for Agree, suppose that the negation in a decomposed *solo* did not. That negation, unlike *no*, would then not be a valid licenser.[4] At least in Spanish, however, it appears that any negation *can* in general license n-words, regardless of its phonological realization. Consider (33), with a pre-verbal n-word. Since Spanish is a 'non-strict' negative concord language, there is no overt negation in (33), but a prominent line of research holds that the n-word is licensed by a *covert* negation (see Zeijlstra 2016 for an overview of approaches). The object in (33) is another n-word, again licensed by the covert negation. The negation that *solo* introduced would have to differ from *both* overt *no* and from the covert negation in (33) in its feature specification.

4 N-words might also have a semantic component to their licensing condition, analogous to proposals for strong NPIs. If so, another potential route could be to define the exceptive to prevent the semantic condition from being met with *solo*. For further discussion, see fn. 7.

(33) Ningún estudiante visitó ningún barrio con María.
N-one student visited:3s N-one neighbourhood con María
'No student visited any neighbourhood with María.'

If *solo* is simplex, on the other hand, the data fall naturally into place. Any sentential negation in Spanish can license n-words, and no such negation morpheme is present with *solo*. Assuming that only negation can bear an interpretable [NEG] feature, the deviance of (32) is then predicted in Zeijlstra's approach.

The paradigm can be extended. Another property of sentential negation is that it licenses NPIs — including strong NPIs — and anti-licenses PPIs. Again, *no más que* exhibits these properties, while *solo* is divergent. Temporal *hasta* ('until') is a strong NPI. When combined with a telic event description, *hasta*, like its English counterpart, is licensed by negation (Bosque 1980). The negation in *no más que* licenses *hasta* as well in (34a), but *solo* does not in (34b).

(34) a. Juan *(no) apareció hasta las nueve más que una vez.
Juan not showed-up:3s until the nine more than one time
'Juan only showed up at nine once.'
b. *Juan solo apareció hasta las nueve una vez.
Juan only showed-up:3s until the nine one time

Parallel contrasts obtain with further strong NPIs, including *tampoco* ('either') in (35), and post-nominal *alguno* (≈ 'any') in (36) (López 2000, Cepeda 2015). In each case, the NPI is licensed with *no más que*, but not *solo*.

(35) a. Juan *(no) habló más que con Juana tampoco.
Juan not talked:3s more than with Juana either
'Juan didn't talk to anybody other than Juana either.'
b. *Juan solo habló con Juana tampoco.
Juan only talked:3s with Juana either

(36) a. Juan *(no) logró acuerdo alguno más que con Juana.
Juan not got:3s agreement ALGUNO more than with Juana
'Juan only reached any agreement with Juana.'
b. *Juan solo logró acuerdo alguno con Juana.
Juan only got:3s agreement ALGUNO with Juana

While post-nominal *alguno* is an NPI, pre-nominal *algún* is a PPI, as established in the baseline in (37), where the existential cannot take scope under

negation. In turn, *algún* cannot take narrow scope in the *no más que* example in (38). By contrast, *algún* is eligible to take narrow scope under *solo* in (39).

(37) Juan no presentó algún estudiante a Juana.
Juan not introduced:3s ALGÚN student to Juana
'There is some student that Juan didn't introduce to Juana.'
a. ✓'There is some student that Juan didn't introduce to Juana.' (∃ > *not*)
b. *'Juan didn't introduce any student to Juana.' (*not* > ∃)

(38) Juan no presentó algún estudiante más que a Juana.
Juan not introduced:3s ALGÚN student more than to Juana
a. ✓'There is some student that Juan only introduced to Juana.' (∃ > *only*)
b. *'Juan only introduced any student to Juana.' (*only* > ∃)

(39) Juan solo presentó algún estudiante a Juana.
Juan only introduced:3s ALGÚN student to Juana
a. ✓'There is some student that Juan only introduced to Juana.' (∃ > *only*)
b. ✓'Juan only introduced any student to Juana.' (*only* > ∃)

All things equal, if *solo* were decomposed, its component negation should license strong NPIs and anti-license PPIs, like the negation in *no más que*.[5,6,7]

5 A reviewer asks whether *no más que* and *solo* could be identical at LF, but differ in their *surface* syntax: with *solo*, NEG might originate with EXC, and move at LF, as in (i) for (34b). Since NPIs must in general be licensed in the surface syntax, *hasta* ('until') would be anti-licensed before NEG moves. PPIs pose one problem, since the PPI effect only emerges at LF: PPIs can scope under *solo*, as in (39), indicating that *solo* does not pattern with *no más que* even at LF.

(i) $[_{TP}$ Juan$_1$ [NEG $[_{vP}$ t$_1$ aparecer hasta las nueve [~~NEG~~ [EXC [una vez]$_F$]]]]]

6 A reviewer asks whether decomposition of *solo* might be viable in face of the NPI data if NPIs were not licensed in the complement of EXC and (34b) were obligatorily parsed as in (i), where EXC attaches not to the focused DP, but higher at the *v*P, with *hasta* in its complement. There are reasons to be skeptical, however. As we will discuss with English, weak NPIs *are* licensed under *only*, and so EXC would have to block strong, but not weak NPIs, unlike what happens with a visible *but*-exceptive, as shown in (ii) (based on observations due to Vostrikova 2021).

(i) $[_{TP}$ Juan$_1$ [NEG [EXC $[_{vP}$ t$_1$ aparecer hasta las nueve [una vez]$_F$]]]]

(ii) a. *John didn't do anything but arrive until 9pm once. *(*strong NPI)*
b. *John didn't do anything but read any phonology books. *(*weak NPI)*

To the extent testable, the results replicate with English *only*. As English does not make productive use of a visible NEG + EXC construction, it is difficult to create overt decompositional baselines. However, in the Spanish data discussed, *no más que* patterned with simple *no* in how it licenses polarity items, and negation baselines can, of course, be created in English. Like in Spanish, sentential negation in English licenses NPIs, both weak NPIs such as *any*, and strong NPIs such as *until* and *either*. While von Fintel (1999) observed that *only* can license weak NPIs, Giannakidou (2006) and Gajewski (2011) noted that *only* does not license strong NPIs, as in (40). PPI facts converge. Whereas PPI *someone* is anti-licensed under *not* in (41a), *someone* can scope under *only* in (41b). As with *solo*, these contrasts challenge a decomposition of *only* based on sentential negation.

(40) a. Mary only read any phonology books once.
b. *Mary only arrived until 9pm once.
c. *Mary only left once either.

(41) a. Peter didn't introduce someone to John. ($\exists > not$, $*not > \exists$)
b. Peter only introduced someone to John. ($\exists > only$, $only > \exists$)

Still, vF&I note certain properties which they take as evidence that *only* does introduce sentential negation. Notably, they observe that only modals which can scope beneath negation participate in sufficiency readings. In the familiar (42a), the modal is *have to*, which takes scope beneath *not* in (42b):

7 Depending on how NPIs are licensed, there might be other ways to try to reconcile the data with decomposition. Traditionally, NPIs are taken to be licensed by certain operators (e.g. Linebarger 1987, Guerzoni 2006). Strong NPIs are licensed by operators which denote functions that are downward monotonic and, moreover, anti-additive (Zwarts 1996). Since negation is such an operator, anti-licensing with *solo* would straightforwardly support a simplex analysis. Recent work, however, argues that licensing is sensitive (at least in part) to the semantic properties of the broader environment where the NPI occurs. A complication arises, first, with *no más que*, where strong NPIs are licensed. While *no* by itself would create an anti-additive environment, *más que* is also relevant for environment-based licensing. If *más que* triggers an existential presupposition, that would not be downward monotonic, and should interrupt licensing (Gajewski 2011, Homer 2011, 2021). We cannot provide a full analysis of *no más que* here, but the NPI data might suggest that *más que* is *not* presuppositional (see Crnič 2021 on a related construction). In turn, strong NPIs would be anti-licensed with *solo* so long as its parse contains some element which *does* carry a non-downward monotonic presupposition. In principle, *solo* could be split into NEG and a presuppositional EXC, differing from *más que*. So, if licensing is environment-based, the data challenge decomposition only in so far as *más que* is semantically representative of exceptives generally. We thank Uli Sauerland for discussion.

(42b) conveys that you are not obliged to go to the North End, rather than that you are obliged to not go there. By contrast, a sufficiency reading is unavailable, for instance, with *must* in (43a), and *must* takes scope over negation in (43b). If sufficiency readings with *only* require the modal to occur in the immediate scope of an underlying NEG, the contrast between *have to* and *must* is expected from the baseline data with *not*.

(42) a. To get good cheese, you only have to go to the North End.
b. You don't have to go to the North End. ($\neg > \Box$)

(43) a. To get good cheese, you (only) must (only) go to the North End.
b. You must not go to the North End. ($\Box > \neg$)

We leave to future work to understand which modals can participate in sufficiency readings. Yet, since *only* and its cross-linguistic kin in general fail to pattern with negation in which elements can occur in their scope, we suspect that contrasts like the one between (42a) and (43a) do not diagnose negation.

3.2 Double negation

Turning back to Spanish, a further property of the negation *no* is that it cannot occur with a second instance of *no* within the same clause. Example (44) is sharply ungrammatical. Double negation effects can be observed even when the two instances of *no* are not adjacent in the linear string. Rivero (1970) observed, for instance, that sentential *no* is also prohibited from co-occurring with complex negative quantifiers like *no muchos* ('not many') in subject position, as in (45).

(44) *Juan no no visitó el North End.
Juan not not visited:3s the North End
'Juan didn't not visit the North End.' (Intended)

(45) *No muchos estudiantes no saben francés.
not many students not know:3pl French
'Not many students don't know French.'

Now, let us compare *no más que* and *solo*. Just like bare *no*, *no más que* exhibits double negation effects. *No más que* cannot occur with an additional sentential *no* in (46), or a subject negative quantifier in (47).

(46) *Juan no no visitó más que el North End.
Juan not not visited:3s more than the North End
'Juan didn't only visit the North End.' (Intended)

(47) *No muchos estudiantes no saben más que francés.
not many students not know:3pl more than French
'Not many students only know French.' (Intended)

Solo, on the other hand, can freely occur together with sentential *no*, and take scope above or below that negation, as in (48). *Solo* is able to occur with a subject negative quantifier in (49), as well. Again, the data present a challenge for decomposition. If *solo* contributed a sentential negation, that negation would create double negation configurations in (48) and (49), which, all things being equal, would be expected to be deviant, like their counterparts with *no más que*.

(48) Juan (solo) no (solo) visitó el North End.
Juan only not only visited:3s the North End
'Juan only didn't visit the NE.' / 'Juan didn't only visit the NE.'

(49) No muchos estudiantes solo saben francés.
not many students only know:3pl French
'Not many students only know French.'

As before, one response might be that only certain negation morphemes are sensitive to the phenomenon. Consider, though, the case in (50). This example contains a subject n-word which, as discussed, has been proposed to be licensed by a covert negation. An overt negation occurs in addition.

(50) ?*Ningún estudiante no visitó el North End.
N-one student not visited:3s the North End
'No student didn't visited the North End.' (Intended)

Laka (1990: p. 119) reports that sentences like (50) are acceptable under a double negation reading. Espinal et al. (2016), however, show experimentally that such data are rejected with a default prosody, and only accepted when the n-word is realized with a marked contour. Our informants likewise reject (50), at least with a default prosody. While double negation effects may be less crisp with the covert negation in (50), they can still be observed nonethe-

less.[8,9] *Solo*, on the other hand, does not induce double negation effects at all. As further evidence, minimal variants of (50) are shown in (51). In (51a), the subject n-word is replaced by *solo* and its associate, which co-occur with overt negation. In (51b), *solo* replaces the overt negation, and co-occurs with the n-word and, thus, covert negation. Both are acceptable, and neither require a special prosodic contour.

(51) a. Solo Juan no visitó el North End.
only Juan not visited:3s the North End
'Only Juan didn't visit the North End.'
b. Ningún estudiante solo visitó el North End.
N-one student only visited:3s the North End
'No student only visited the North End.'

If *solo* is simplex, its behavior is directly predicted. With a simplex *solo*, there is just one negation in the preceding cases — overt *no* in (48), (49), and (51a), and the covert NEG with the n-word in (51b) — and, therefore, these data are expected to be acceptable, with no double negation effects.

8 Zeijlstra (2008: fn.1) and Chierchia (2013: p. 229) also report that double negation readings are generally not possible in similar data in Italian. The former notes that they depend on marked prosody.

9 Rivero (1970) observed that *pocos* ('few') can occur with *no* in (i). As she notes, if *pocos* were decomposed into a covert NEG and a 'many' component, covert negation would seem to be immune to double negation effects in this case. Yet, in a range of data, *pocos* fails to pattern as if it involves a covert negation. Unlike with a subject n-word, when *pocos* occurs in subject position, an object n-word is not clearly licensed, as in (ii), nor are strong NPIs, as in (ii) and (iii) (see Cepeda 2016 for data with postnominal *alguno*). Since the analysis of *pocos* requires further study, we have relied on subject n-words to construct a baseline with covert negation here.

(i) Pocos aviones no se estrellan.
few planes not SELF crash:3PL
'Few planes don't crash.'

(ii) ?*Pocos médicos resolvieron {ningún problema, problema alguno}.
few doctors solve:3PL n-one problem problem ALGUNO
'Few doctors solved any problem.' (Intended)

(iii) ?*Pocos médicos llegaron hasta las siete.
few doctors arrive:3PL until the seven
'Few doctors arrived until seven.' (Intended)

4 Too weak readings

So far, we have cast doubt on a decomposition of exclusive adverbials into negation and an exceptive parallel to *no más que*. We now turn to the other ingredient of vF&I's proposal: weakening the presupposition. Prior works, such as van Rooj & Schulz 2007 and Ippolito 2007, argue against encoding just an existential presupposition in *only*. They show that readings which are *too* weak are predicted in data with conjunction and negation. In these cases, a strong prejacent inference is observed, but not derived. While the exposition in these works is based on $only_{weak}$, we will centre our discussion on vF&I's system with NEG and existential EXC. vF&I's analysis rightly predicts no prejacent inference in their example, but it seems to go *too far* in bleeding prejacent inferences.

4.1 Association with conjunction

To start, consider the example in (52), where *only* associates with a conjunction (van Rooj & Schulz 2007, p. 212, Ippolito 2007, p. 57). The intuitive inferences of (52) are that John visited the North End and New York, as in (52a), and that John did not visit anywhere else, as in (52b).

(52) John only visited [the North End and New York]$_F$.

a. $\rightsquigarrow \phi_{NE} \wedge \phi_{NY}$

b. $\rightsquigarrow \neg\phi_S$

An analysis with strong *only* would straightforwardly capture both inferences. To make the interpretation clear, the LF for (52) is provided in (53).

(53) [$_{TP}$ only$_{strong}$ [$_{vP}$ John visit [the North End and New York]$_F$]]

The inference in (52a) follows from *only*'s prejacent presupposition, since the *v*P expresses the proposition that John visited both the North End and New York. The assertive component of *only* yields (52b). As noted earlier, we assume that alternatives are computed by replacing the focus with other elements of equal or lesser structural complexity. The result is that the alternatives that *only* sees express the propositions in (54). All of the alternatives

entailing ϕ_S are non-weaker than the prejacent and *only* negates them to derive the $\neg\phi_S$ inference in (52b).[10]

(54) $\{\phi_{NE}, \phi_{NY}, \phi_S, \phi_{NE} \wedge \phi_{NY}, \phi_{NY} \wedge \phi_S, \phi_{NE} \wedge \phi_S\}$

Now, suppose the LF in (53) was replaced with the vF&I style LF in (55), with EXC contributing an existential presupposition. EXC attaches to the conjunction, and, as defined above, requires an entity as its first argument. In this case, *and* is analyzed as the operator and_{sum}, defined in (56), after Link (1983). And_{sum} conjoins two referential DPs and outputs the mereological sum of their referents — the plural entity made up of the North End and New York in (55).

(55) $[_{TP}$ NEG [[EXC [the North End and New York]$_F$] $\lambda 1$ [$_{vP}$ John visit t_1]]]

(56) $[\![\text{and}_{\text{sum}}]\!] = \lambda x_e \,.\, \lambda y_e \,.\, x \oplus y$

Some initial housekeeping is in order with the assertive component of EXC. By the earlier definition, repeated in (57), EXC(x)(f) asserts that some entity non-identical to x is an f. Negating that, in turn, says that *no* entity non-identical to x is an f. In (55), x is the sum NE⊕NY, and f is the property of being an entity that John visited. One entity non-identical to NE⊕NY is Switzerland and, thus, the entailment $\neg\phi_S$ is predicted. There is, however, a problem: because NE⊕NY is not identical to either of its atomic members, (56) should further entail $\neg\phi_{NE}$ and $\neg\phi_{NY}$. If so, (55) would assert that John visited *nowhere*.

(57) $[\![\text{EXC}]\!] = \lambda x \,.\, \lambda f_{\langle e,st\rangle} \,.\, \lambda w : \exists y\, [f(y)(w)] \,.\, \exists z\, [z \neq x \wedge f(z)(w)]$

To correct the assertion, we entertain the revised entry for EXC in (58). EXC is no longer sensitive to non-identity with x, but instead is defined in terms of the Overlap relation in (59). EXC(x)(f) asserts that some entity *non-overlapping* with x is an f, where two entities are non-overlapping if they have no part in

10 We leave open how *and* is analyzed in a parse with $only_{strong}$. *And* could be interpreted as logical conjunction, and compose with the two DPs, each lifted to quantifier type (or with propositional constituents, obscured by ellipsis). A second possibility is that *and* denotes a sum formation operator, with the conjunction forming a plurality (see (56) below). On a plural parse, when $only_{strong}$ negates the conjunctive alternatives, note that a homogeneity effect might arise. With homogeneity, negating ($\phi_{NE} \wedge \phi_S$) would yield an additional inference of $\neg\phi_{NE}$, and negating ($\phi_{NY} \wedge \phi_S$) would likewise yield $\neg\phi_{NY}$, contradicting the prejacent. However, *only* can be re-defined based on Fox (2007) to avoid such contradictions, in which case just the atomic ϕ_S alternative would be negated.

common. Thomas (2011) and Hirsch (2016) analyzed exceptive *other* and *but* in terms of Overlap, and vF&I suggest a parallel refinement to EXC (in their fn. 22).

(58) $[\![\text{EXC}]\!] = \lambda x_e \,.\, \lambda f_{\langle e,st\rangle} \,.\, \lambda w : \exists y\, [f(y)(w)] \,.\, \exists z\, [\neg \text{Overlap}(z,x) \wedge f(z)(w)]$

(59) Overlap(x, y) iff $\exists z\, [z \leq x \wedge z \leq y]$

EXC now correctly predicts the assertion in (60). The LF in (55) asserts that John visited no entity non-overlapping with NE⊕NY. That sum overlaps with each of the North End and New York, as well as any sum with either as an atom, but it does not overlap with Switzerland. The assertion is just (60), as with $only_{strong}$.

(60) A: $\neg \phi_S$

With matters of assertion squared away, we can isolate the critical feature of vF&I's weak semantics: the existential presupposition. In (55), EXC triggers the presupposition that John visited somewhere, as in (61), and that projects globally, over negation. The resultant meaning is too weak. Combined with the assertion that John did not visit Switzerland, the presupposition results in a disjunctive inference that John visited the North End *or* New York. The conjunctive inference in (52a) — that John visited the North End *and* New York — is not derived.

(61) P: $\phi_{NE} \vee \phi_{NY} \vee \phi_S$

vF&I recognize the issue (see again their fn. 22), and suggest that the derived disjunctive meaning may be pragmatically strengthened to the conjunctive inference as an implicature. We note, however, that the conjunctive inference does not pattern like a pragmatic enrichment. In particular, it is not defeasible. The continuations in (62a) and (62b) are incompatible with the conjunctive inference, and are deviant. We conclude that the semantics must output the conjunctive inference.

(62) John only visited the North End and New York.

a. # …And, in fact, he didn't visit the North End.
b. # …But I'm not sure which he visited.

It bears note that (52) does not itself present a problem for decomposing *only* into negation and an exceptive. The sentence in (63) is a counterpart to

(52) in Spanish constructed with overt *no más que*. Like (52), (63) licenses the conjunctive inference that Juan visited both the North End and New York. *Más que* must itself yield a conjunctive inference. In turn, to capture (52), *only* could be a simplex adverbial, or decomposed — so long as a conjunctive prejacent inference is derived.

(63) Juan no visitó más que el North End y Nueva York.
Juan not visited:3s more than the North End and Nueva York
'Juan only visited the North End and New York.'

4.2 Negation (*not* > *only*)

We now turn to cases where *only* co-occurs with negation, beginning with (64), where negation appears at a wide scope site above *only* (Ippolito 2007, p. 57, see also Geurts & van der Sandt 2004, p. 26). Intuitively, (64) gives rise to a transparent prejacent inference that John visited the North End, as in (64a), and a further inference that he additionally visited somewhere else, as in (64b).

(64) John didn't only visit [the North End]$_F$.
a. $\rightsquigarrow \phi_{NE}$
b. $\rightsquigarrow \phi_{NY} \vee \phi_S$

Only$_{strong}$ captures both inferences in (64). In the LF in (65), *only*$_{strong}$ triggers the prejacent presupposition that John visited the North End, and that projects over negation, yielding (65a). *Only* locally asserts that John visited neither New York nor Switzerland and, in turn, negation derives (65b).

(65) [$_{TP}$ not [only$_{strong}$ [$_{vP}$ John visit [the North End]$_F$]]]
a. P: ϕ_{NE}
b. A: $\neg(\neg\phi_{NY} \wedge \neg\phi_S) \Leftrightarrow \phi_{NY} \vee \phi_S$

The approach in vF&I cannot match the prediction. Their analysis would assign (64) the LF in (66). Two negations are adjacent in the structure and — as far as interpretation is concerned — they cancel out one another, so (66) expresses an equivalent meaning to the fragment in (67). Given vF&I's weak semantics for EXC, the LF in (67) has the presupposition in (68a), that John visited somewhere, and the assertion in (68b), that he visited somewhere other than the North End.

(66) $[_{TP}$ not [NEG [[EXC [the North End]$_F$] $\lambda 1$ $[_{vP}$ John visit t_1]]]]

(67) [[EXC [the North End]$_F$] $\lambda 1$ $[_{vP}$ John visit t_1]]

(68) a. P: $\phi_{NE} \vee \phi_{NY} \vee \phi_S$
b. A: $\phi_{NY} \vee \phi_S$

The assertion is unchanged from the analysis with *only*$_{strong}$, so the inference in (64b) is still captured. (64a), however, is not. The existential presupposition does not itself entail that John visited the North End, and it does not entail that John visited the North End in combination with the assertion either. Since (68b) asymmetrically entails (68a), the conjunction of the presupposition and assertion is equivalent to the assertion, as shown in (69). The overall predicted inference is just that John visited somewhere other than the North End, leaving the ϕ_{NE} inference unaccounted for. Again, a prejacent inference is observed, but not predicted.[11]

(69) $(P \wedge A) \Leftrightarrow (\phi_{NE} \vee \phi_{NY} \vee \phi_S) \wedge (\phi_{NY} \vee \phi_S) \Leftrightarrow \phi_{NY} \vee \phi_S$

4.3 More negation (*only* > *not*)

In considering vF&I's proposal, we also discuss example (70), where *only* appears with negation in its scope. Here, too, there is a prejacent inference, now the negative inference that John did *not* visit the North End, as in (70a). In addition, there is the inference that he did visit everywhere else, as in (70b).

(70) John only didn't visit [the North End]$_F$.
a. $\rightsquigarrow \neg\phi_{NE}$
b. $\rightsquigarrow \phi_{NY} \wedge \phi_S$

Once again, an LF with strong *only* would deliver the correct result. The prejacent presupposition, shown in (71a), is precisely (70a). With regard to the assertive component, the alternatives *only* operates over express the negative propositions in the set in (71c). The propositions $\neg\phi_{NY}$ and $\neg\phi_S$ are not entailed by the prejacent, and each gets negated to derive (71b), capturing (70b).

11 We assume that EXC must remain in the scope of the NEG component to be licensed. If the exceptive phrase could QR higher than NEG, there would be a second LF where EXC scopes between *not* and NEG. A similar LF is entertained for a different example in the next subsection. As we will see, it would also fail to derive the observed reading for (64).

(71) $[_{TP}$ $only_{strong}$ [not $[_{vP}$ John visit [the North End]$_F$]]]

a. P: $\neg\phi_{NE}$
b. A: $\neg\neg\phi_{NY} \wedge \neg\neg\phi_S \Leftrightarrow \phi_{NY} \wedge \phi_S$
c. ALT $= \{\neg\phi_{NE}, \neg\phi_{NY}, \neg\phi_S\}$

For (70), vF&I's approach does furnish a possible LF which yields the observed reading. That LF is (72). The high negation is the one contributed by *only*, the lower negation is the overt *not* separate from *only*, and the exceptive phrase takes scope between the two negations. With these scope relations, the negative and exceptive components of *only* both take scope above the separate negation.

(72) $[_{TP}$ NEG [[EXC [the North End]$_F$] $\lambda 1$ [not $[_{vP}$ John visit t_1]]]]

Due to the negation below EXC, the existential presupposition that EXC triggers is that there is some place that John did not visit, as in (73a). That projects globally, over NEG. With respect to assertion, EXC here says that there is some place other than the North End that John did not visit, and negating that delivers the overall assertion in (73b): that John did visit everywhere other than the North End.

(73) a. P: $\neg\phi_{NE} \vee \neg\phi_{NY} \vee \neg\phi_S$
b. A: $\neg(\neg\phi_{NY} \vee \neg\phi_S) \Leftrightarrow \phi_{NY} \wedge \phi_S$

The assertion continues to capture the inference in (70b). While the presupposition alone does not capture (70a), that inference does derive in combination with the assertion. If there is somewhere John did not visit, but he did visit everywhere other than the North End, it follows that he didn't visit the North End.

So far so good — but a complication still arises. Since the exceptive phrase can take scope at different sites, vF&I predict a structural ambiguity in (70). In addition to (72), the LF in (74) should be available, too. In (74), the exceptive scopes not between the two negations, but beneath the lowest negation.

(74) $[_{TP}$ NEG [not [[EXC [the North End]$_F$] $\lambda 1$ $[_{vP}$ John visit t_1]]]]

This LF is equivalent to the problematic LF in (66) from the preceding subsection: it presupposes that John visited somewhere, and asserts that he visited somewhere other than the North End. The conjunction of presupposition and assertion is equivalent to the assertion. As an analysis of the example at

hand, the interpretation is too weak in two ways. First, the $\neg\phi_{NE}$ prejacent inference in (70a) is not derived. Second, the predicted assertion just says that John visited *some* place other than the North End, not that he visited *every* other place, so (70b) is not derived either.

Combining this and the previous subsection, we conclude that vF&I's analysis runs into challenges when *not* overtly scopes over *only* and when *only* overtly scopes over *not*. The problem has a different character in the two cases, however. When *not* scopes over *only*, one LF is predicted, and it yields a too weak reading. When *only* scopes over *not*, two LFs are predicted, one of which yields the target reading, and the other of which yields a too weak reading. The former is an *under-generation* problem. The latter is an *over-generation* problem.

Over-generation is in general less severe than under-generation, as it can in principle be solved by introducing additional constraints. In this case, though, it is not straightforward to block the LF in (74). First, (74) has the same basic skeleton as the LF that vF&I propose for their core case. The two LFs are shown together.

(75) a. To get good cheese, you only have to go to the North End.
b. $[_{TP}$ NEG $[_{vP_2}$ have [[EXC [the North End]$_F$] $\lambda 1$ $[_{vP_1}$ you go to t_1]]]]
= (25)

(76) a. John only didn't visit the North End.
b. $[_{TP}$ NEG [not [[EXC [the North End]$_F$] $\lambda 1$ $[_{vP}$ John visit t_1]]]]
= (74)

In each LF, the NEG component of *only* scopes where *only* is overtly realized, while EXC scopes below some intervening operator, a modal in the first LF, and negation in the second. Both structures instantiate the skeleton in (77). Since (25) is vF&I's central contribution, they commit to the skeleton being well-formed.

(77) [NEG … [Op … [[EXC …] …]]]

To block (74), there would have to be some constraint which specifically penalizes an intervening negation. One possibility might be that EXC is an NPI and is only licensed when the *global* environment in which it occurs is downward monotonic. In (74), the lower negation does create a downward monotonic environment locally, but the higher negation cancels it, so EXC would be anti-licensed. Yet, it would not be viable to impose such a strong licens-

ing condition on EXC. Consider (78a), where *only* occurs in the antecedent of a conditional. If *only* were decomposed, (78a) would have the LF in (78b). Here, NEG creates a downward monotonic environment, but the global environment is not downward monotonic, since conditional antecedents (presumably) reverse entailment. The same considerations apply to cases where *only* is in the restrictor of a universal quantifier, as in (79).

(78) a. If John only visited the North End, he will complain.
b. [[if [NEG [[EXC [the North End]$_F$] λ1 [John visit t$_1$]]]] …]

(79) a. Every tourist who only visited the North End complained.
b. [[every tourist [λ1 [NEG [[EXC [the NE]$_F$] λ2 [t$_1$ visit t$_2$]]]]] …]

Since we are uncertain how to block the too weak parse in (74) in a principled way, we will take the over-generation problem seriously as a reason to question vF&I's semantic revision to weaken *only*.

4.4 A shift in perspective

To solve the prejacent problem in vF&I's modal example, it appears that *only* must be decomposed into negation and an exceptive, and that the exceptive must trigger a weak existential presupposition. Yet, we saw that both revisions face a range of challenges. We take the totality of results in Sections 3 and 4 to reveal that *only* is simplex and strong. Our first task, then, is to provide a solution for the prejacent problem consistent with *only*$_{strong}$.

5 Proposal: a separate weakening operator

If *only* carries a strong prejacent presupposition, the solution to the prejacent problem must come from a re-analysis of the prejacent in (81a), the transparent LF for (80) with strong *only*. Despite appearances, vP_2 must not necessarily express that you go to the North End in all cheese-worlds. We propose that natural language makes available a covert operator, AT LEAST, which optionally occurs in the scope of *only*, weakening the prejacent. The sentence in (80) can, then, be parsed as (81b).[12]

12 Our proposal shares a key ingredient with Franke 2006, which likewise suggests that, in the sufficiency construction, the prejacent of the modal has a scalar 'at least' interpretation. We elaborate on how that comes about compositionally. Coppock & Beaver (2014) propose on independent grounds to encode an 'at least' component in *only* itself. In the LF for vF&I's

(80) To get good cheese, you only have to go to the North End.

(81) a. [$_{TP}$ only [$_{vP_2}$ have [$_{vP_1}$ you go to [the North End]$_F$]]]
b. [$_{TP}$ only [$_{vP_2}$ have [AT LEAST [$_{vP_1}$ you go to [the North End]$_F$]]]]

Our approach will reach a similar overall meaning to vF&I's, but maintains the crucial difference that the source of weakening is an optional operator separate from *only*. We unpack the LF in steps, and then offer a comparison with vF&I's derivation afterwards. In addition to addressing the prejacent problem, we will also account now for the second feature of vF&I's example, suppressed so far: we will derive an inference that the North End is a (relatively) *easy* place to go.

5.1 Step 1: interpreting *only*'s prejacent

We define AT LEAST as the focus-sensitive scalar operator in (82). The definition is slightly modified from Crnič 2011, which took AT LEAST to occur in the scope of an *even* operator in certain languages (see also Schwarz 2005).

(82) $[\![\text{AT LEAST}]\!]^{\text{ALT}_\leq}(p) =$
$\lambda w : \forall p' \in \text{ALT}\ [(p' \neq p) \rightarrow p' > p]\ .\ \exists p'' \in \text{ALT}\ [p'' \geq p \wedge p''(w)]$

AT LEAST requires that the alternatives in ALT be ordered by some contextual ranking ($\leq$). In its presupposition, AT LEAST is scalar: it presupposes that its prejacent (p) is lowest-ranked among alternatives. With respect to assertion, AT LEAST makes an existential claim: it says that some alternative ranked at least as high as p is true. In general, AT LEAST takes its prejacent and delivers a disjunction with its prejacent as one disjunct: either p is true or some higher-ranked alternative is. In this way, AT LEAST can yield a *weaker* proposition than the one it takes as input. The existential assertive component of AT LEAST will help resolve the prejacent problem, while the presupposition underlies the easiness inference.

To start, the minimal sub-constituent of (81b) containing AT LEAST is isolated in (83). AT LEAST associates with the focused DP, and its prejacent is ϕ_{NE}.

(83) [AT LEAST [$_{vP_1}$ you go to [the North End]$_F$]] *(fragment)*

example, it is crucial that AT LEAST be a separate operator, since it takes lower scope than overt *only*.

The alternatives that AT LEAST sees express the propositions in the set in (84a). These alternatives must be ranked. A natural contextual ordering is (84b). This ordering correlates with how much effort it would take to go to the North End, New York, or Switzerland, starting from MIT.[13]

(84) a. $\{\phi_{NE}, \phi_{NY}, \phi_{S}\}$
b. $\phi_{NE} < \phi_{NY} < \phi_{S}$

The scalar presupposition, stated in (85), requires that ϕ_{NE} be lowest-ranked among alternatives, as it is in (84b). Given the effort-based ranking, (85) amounts to saying that the North End is the least effortful of the salient places to go.[14]

(85) P: $\lambda w\,.\,\forall p \in \text{ALT}\,[(p \neq \phi_{NE}) \rightarrow p > \phi_{NE}]$

The assertive contribution is, then, (86): that you go to the North End or somewhere more effortful. Because the North End is the least effortful option, that you go there or somewhere more effortful amounts to saying just that you go *somewhere*. AT LEAST weakens the asserted content from the proposition that you go to the North End (at vP_1) to that existential (at the top node in (83)).

(86) A: $\lambda w\,.\,\exists p \in \text{ALT}\,[p \geq \phi_{NE} \wedge p(w)] \Leftrightarrow \phi_{NE} \vee \phi_{NY} \vee \phi_{S}$

With (86) in hand, we turn to the full prejacent of *only*. The constituent sister to *only* is (87), which adds the modal to the prior fragment. With the universal modal scoping over existential AT LEAST, (87) asserts that, in every cheese-world, you go somewhere, potentially different from world to world, as in (88).

(87) $[_{vP_2}$ have [AT LEAST $[_{vP_1}$ you go to [the North End$]_F$]]] *(prejacent)*

(88) A: $\Box(\phi_{NE} \vee \phi_{NY} \vee \phi_{S})$

13 We will make the assumption that global geography is constant across relevant worlds.

14 The ranking does not require that the North End, New York, and Switzerland are all necessarily places where you can actually get good cheese. A reviewer notes that it must be possible to construct a ranking among alternatives which are not all actual ways of achieving the goal in the purpose clause: in *To turn on the light, you only have to flip the switch*, for instance, flipping the switch might be the only way to turn on the light. See also vF&I's Section 4.2.

The proposition in (88) is the prejacent of *only* and is presupposed, due to *only*'s prejacent presupposition. While asserted at vP_2, (88) is converted to a presupposition by the top node in the full LF, repeated as (89).

(89) $[_{TP}$ only $[_{vP_2}$ have [AT LEAST $[_{vP_1}$ you go to [the North End]$_F$]]]]

(90) P: $\Box(\phi_{NE} \vee \phi_{NY} \vee \phi_S)$

The scalar presupposition projects globally, as well. We have assumed that presuppositions project universally from the scope of a universal modal. If p encodes a presupposition p', then ⟦have⟧(p) presupposes that p' is true at all worlds in the modal's domain of quantification. vP_2 will thus presuppose that ϕ_{NE} is lowest-ranked among alternatives *at all cheese-worlds*. Yet, the scalar presupposition, repeated in (91), is not world-dependent. The λw prefix does not bind any world variable.

(91) P: $\lambda w\,.\,\forall p \in \text{ALT}\,[(p \neq \phi_{NE}) \rightarrow p > \phi_{NE}]$

As a result, once the alternatives and their contextual ranking are determined, the presupposition in (91) is either true at all worlds (if ϕ_{NE} is lowest-ranked), or false at all worlds (if ϕ_{NE} is ranked higher). The requirement that the presupposition hold at *certain* worlds amounts to a requirement that it hold at *all* worlds — and the presupposition thus projects over the modal. The scalar presupposition projects over *only* in turn to become a global presupposition of the sentence.[15,16]

In sum, we have thus far derived two presuppositions. The remaining step is to determine the *asserted* contribution of *only*.

15 In baseline data, presuppositions triggered in the prejacent of *only* are generally inherited by the next node up, as in: *#Only John stopped smoking, but John was never a smoker*.

16 We have encoded a scalar presupposition in AT LEAST, but note that our LFs are not necessarily incompatible with the idea that *only* triggers a scalar presupposition (see Beaver & Clark 2008, among many others). The scalar presupposition of AT LEAST is met with the effort-based ranking in (i-a). If (i-a) holds, worlds in $(\phi_{NE} \vee \phi_{NY} \vee \phi_S)$ must, on average, involve less effort than worlds in $(\phi_{NY} \vee \phi_S)$ or ϕ_S, leading to (i-b). Assuming that the modalized alternatives *only* sees could be ranked based on average effort in cheese-worlds, (i-c) follows. AT LEAST could enforce (i-a), *only* could enforce (i-c), or both expressions could enforce respective scalar requirements.

(i) a. $\phi_{NE} < \phi_{NY} < \phi_S$

b. $⟦\text{AT LEAST}⟧^{\text{ALT}_\leq}(\phi_{NE}) < ⟦\text{AT LEAST}⟧^{\text{ALT}_\leq}(\phi_{NY}) < ⟦\text{AT LEAST}⟧^{\text{ALT}_\leq}(\phi_S)$

c. $\Box⟦\text{AT LEAST}⟧^{\text{ALT}_\leq}(\phi_{NE}) < \Box⟦\text{AT LEAST}⟧^{\text{ALT}_\leq}(\phi_{NY}) < \Box⟦\text{AT LEAST}⟧^{\text{ALT}_\leq}(\phi_S)$

5.2 Step 2: negating alternatives

In the present LF, *only* and AT LEAST both associate with the same focus — the object DP — but they operate over different alternatives. Due to its widest scope, *only* sees the alternatives in (92), each containing AT LEAST. Because AT LEAST is focus-sensitive, it is crucial that the alternatives be syntactic structures, and that F-marking be retained in the alternatives to identify the associate of AT LEAST. In (92a)–(92c), the object DP itself is replaced, but F-marking is retained. The three alternatives express the assertions in (93a)–(93c), respectively.[17]

(92) a. $[_{vP_2}$ have [AT LEAST $[_{vP_1}$ you go to [the North End]$_F$]]]
b. $[_{vP_2}$ have [AT LEAST $[_{vP_1}$ you go to [New York]$_F$]]]
c. $[_{vP_2}$ have [AT LEAST $[_{vP_1}$ you go to [Switzerland]$_F$]]]

(93) a. $\Box[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NE}) \Leftrightarrow \Box(\phi_{NE} \vee \phi_{NY} \vee \phi_S)$
b. $\Box[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NY}) \Leftrightarrow \Box(\phi_{NY} \vee \phi_S)$
c. $\Box[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_S) \Leftrightarrow \Box\phi_S$

The alternative in (93a) is equivalent to the prejacent, while (93b) and (93c) are both stronger. *Only* will negate the latter two, deriving the assertion in (94): that you do not go to New York or Switzerland in all cheese-worlds.

(94) A: $\neg\Box(\phi_{NY} \vee \phi_S) \wedge \neg\Box\phi_S \Leftrightarrow \neg\Box(\phi_{NY} \vee \phi_S)$

17 The scalar presupposition of AT LEAST must be inactive in the alternatives. Otherwise, (92b) would carry the presupposition that ϕ_{NY} is lowest-ranked among alternatives, as in (i-a), and (92c) that ϕ_S is lowest-ranked, as in (i-b). If (i-a) or (i-b) projected, they would contradict the scalar presupposition in (91) above, projected from the prejacent of *only*. (i-a) and (i-b) are not locally accommodated within the alternatives either. Since (91) entails that (i-a) and (i-b) are false, if they were locally accommodated, the alternatives would have to be false, irrespective of the existential contribution of AT LEAST. In turn, the assertion of *only* would be trivialized. Crnič (2011) suggested that the scalar presupposition of AT LEAST is inactive in the alternatives for a higher *even*, as well. We suspect that this behavior may relate to a broader pattern of what Sauerland (2013) calls "weakened projection" where a range of presuppositions seem to be ignored in focus alternatives. Weakened projection has been observed, for instance, with phi-features (e.g. Kratzer 1998, von Heusinger 2007, Heim 2008, Bassi 2021), as well as with factive and change of state verbs (Walker 2012).

(i) a. $\lambda w\,.\,\forall p \in \text{ALT}\,[(p \neq \phi_{NY}) \rightarrow p > \phi_{NY}]$
b. $\lambda w\,.\,\forall p \in \text{ALT}\,[(p \neq \phi_S) \rightarrow p > \phi_S]$

5.3 Step 3: putting the pieces together

Overall, we have derived three meaning components: the scalar presupposition in (95a), the prejacent presupposition of *only* in (95b), and the assertion in (95c).

(95) a. P: $\lambda w . \forall p \in \text{ALT} [(p \neq \phi_{NE}) \rightarrow p > \phi_{NE}]$
b. P: $\Box(\phi_{NE} \vee \phi_{NY} \vee \phi_{S})$
c. A: $\neg\Box(\phi_{NY} \vee \phi_{S})$

Key to resolving the prejacent problem are (95b) and (95c) — which are precisely the same meaning components vF&I derived. For direct comparison, our LF is repeated in (96), together with vF&I's in (97).

(96) $[_{\text{TP}}$ only $[_{vP_2}$ have [AT LEAST $[_{vP_1}$ you go to [the North End]$_F$]]]]

(97) $[_{\text{TP}}$ NEG $[_{vP_2}$ have [[EXC [the North End]$_F$] $\lambda 1$ $[_{vP_1}$ you go to t_1]]]]

In both analyses, there is an operator below the modal, and one above the modal. Moreover, in both, the lower operator makes an existential contribution. In vF&I's analysis, EXC directly introduces a presupposition that you go somewhere, and that projects universally through the modal, delivering (95b). In our analysis, AT LEAST asserts that you go somewhere, and the modal operates on that assertion. Strong *only* converts the result to a presupposition by presupposing its prejacent, again deriving (95b). Regarding the assertion, in vF&I's analysis, EXC asserts that you go somewhere other than the North End, and the modal and negation operate on that to assemble the inference in (95c). In our analysis, (95c) results from *only* above the modal negating alternatives, each containing AT LEAST. While you must at least go to the North End, negating alternatives says that you don't have to go anywhere more effortful, as (95c) does.[18]

Since we derive the same meaning components as vF&I, we likewise predict the target possibility inference. Neither (95b) nor (95c) entails that you go to the North End in all cheese-worlds. (95b) says that you go somewhere in all cheese-worlds, and (95c) that you don't go to places other than the North End in all cheese-worlds. It follows that you go to the North End in *some* cheese-worlds.

18 Krasikova & Zhechev (2005a,b) derive a similar assertion by re-analyzing *only*. Their *only* identifies more effortful alternatives to the complement of the modal by taking the modal and the proposition expressed by its complement as separate arguments. By positing AT LEAST within the complement, we maintain *only* as a propositional operator.

The scalar presupposition in (95a) captures the easiness effect. Given the effort-based ranking of alternatives, the scalar presupposition requires that the prejacent of AT LEAST, here ϕ_{NE}, convey the least effortful option. In contrast to vF&I's example, the scalar presupposition would fail, for instance, in (98), parsed as (99a). ϕ_S is highest-ranked by effort, not lowest, so (99b) is false at all worlds.[19]

(98) #To get good cheese, you only have to go Switzerland.

(99) a. $[_{TP}$ only $[_{vP_2}$ have [AT LEAST $[_{vP_1}$ you go to [Switzerland]$_F$]]]]
b. Scalar Ps: $\lambda w\,.\,\forall p \in \text{ALT}\,[(p \neq \phi_S) \rightarrow p > \phi_S]$

5.4 Next steps

So far, we have solved the prejacent problem while maintaining that *only* itself is a simplex operator triggering a strong prejacent presupposition. The source of weakening in vF&I's example is a separate covert AT LEAST which can occur in the scope of *only*. Since *only* is simplex, there is no expectation that it should exhibit characteristic properties of sentential negation, as transparent NEG + EXC constructions do, and challenges for decomposition raised in Section 3 can be resolved. Our task now is to understand why weakening is not *always* observed and, to that end, we return to the data from Section 4.

6 Restricting AT LEAST

Consider, again, the cases where *only* associates with a conjunction, or co-occurs with negation. The data below all license an inference that the propo-

19 The LF in (99a) faces other problems, too. Notably, with ϕ_S highest-ranked among alternatives, $\Box[\![\text{AT LEAST}]\!]^{\text{ALT}_{\leq}}(\phi_S)$ entails $\Box[\![\text{AT LEAST}]\!]^{\text{ALT}_{\leq}}(\phi_{NY})$ and $\Box[\![\text{AT LEAST}]\!]^{\text{ALT}_{\leq}}(\phi_{NE})$, so there are no alternatives for *only* to negate. This LF may, then, be blocked because *only* is vacuous, even if no scalar presupposition were triggered. If the relevant places are the North End, Switzerland, and Israel, however, vacuity concerns are avoided. Suppose there is no good cheese in the North End, but there is in Switzerland and Israel. Aside from the scalar presupposition of AT LEAST, (99a) could express a sufficiency reading (that you can go to Switzerland, and don't have to go to Israel). Yet, (i) still seems odd. The intuition in (i) can be isolated to the scalar presupposition failing.

(i) a. Among the NE, Switzerland, and Israel, where can I go to get good cheese?
b. #You only have to go to Switzerland.

sition expressed by the transparent material in the complement of *only* is true.

(100) a. John only visited the North End and New York. $(\rightsquigarrow \phi_{NE} \wedge \phi_{NY})$
b. John didn't only visit the North End. $(\rightsquigarrow \phi_{NE})$
c. John only didn't visit the North End. $(\rightsquigarrow \neg\phi_{NE})$

In general, our approach, unlike vF&I's, never *under-generates* transparent prejacent inferences. Due to the prejacent presupposition of strong *only*, the target readings can derive from the parses in (101), also familiar from Section 4. Insertion of AT LEAST is optional, not obligatory, and AT LEAST is absent in these parses. Without AT LEAST, transparent prejacent inferences emerge.[20]

(101) a. $[_{TP}$ only $[_{vP}$ John visit [the North End and New York]$_F]]$
b. $[_{TP}$ not [only $[_{vP}$ John visit [the North End]$_F]]]$
c. $[_{TP}$ only [not $[_{vP}$ John visit [the North End]$_F]]]$

At the same time, it does still remain a concern that we might *over-generate* too weak readings. If AT LEAST may be inserted in vF&I's modal case, why does AT LEAST not lead to weakening in the conjunction and negation data, too?

We will entertain possible LFs for the data in (100) with AT LEAST in the scope of *only*. We begin with (100a), and then address the negation data in (100c) and (100b) in that order. Over-generation in (100a) will be restricted by a condition on the scales that AT LEAST invokes. In the negation cases, we will suggest that pragmatic constraints prevent AT LEAST from being inserted altogether at certain scope positions. In (100c), AT LEAST will be blocked because it would render *only* vacuous, and we will consider the idea that AT LEAST is blocked in (100b) because its own contribution would be vacuous within a local domain.

6.1 Conjunction

The example in (102), recall, licenses the conjunctive inference that John visited the North End and New York. We sketch in (103) an LF with AT LEAST.

20 A parse without AT LEAST should exist also in vF&I's modal case, but seems pragmatically difficult. In other data, the reading is intuited. In a class, *You only have to write a paper* can convey the transparent prejacent that you do *have* to write a paper (and don't have to do other work).

(102) John only visited the North End and New York. ($\rightsquigarrow \phi_{NE} \wedge \phi_{NY}$)

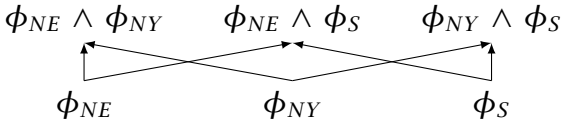
(103) [$_{TP}$ only [AT LEAST [$_{vP}$ John visit [the North End and New York]$_F$]]]

If the set of alternatives is determined by replacing the focused constituent with elements of equal or lesser structural complexity, as we have assumed throughout the paper, AT LEAST in (103) operates over alternatives expressing the propositions in (104). The alternative set is familiar from Section 4.

(104) $\{\phi_{NE}, \phi_{NY}, \phi_S, \phi_{NE} \wedge \phi_{NY}, \phi_{NE} \wedge \phi_S, \phi_{NY} \wedge \phi_S\}$

Since the set includes conjunctive and non-conjunctive alternatives, certain members stand in an entailment relation with others. One natural way to order elements of the set is thus based on entailment, with stronger alternatives ranked higher, and weaker alternatives lower. The result is a partial ordering, sketched in (105).[21]

(105)



We will conjecture that the scales over which AT LEAST operates must be constructed in accord with the principle in (106), requiring that ranking positively correlate with entailment.[22]

(106) *Entailment Preservation*
A well-formed scale is such that ∀p, p' [p ⊂ p' → p > p'].

There is, in fact, some independent evidence that the construction of scales must be constrained such that 'stronger' alternatives must be ranked above 'weaker' alternatives. The evidence we are aware of involves strength in pragmatic, rather than logical terms. Consider again the example:

(107) a. #To get good cheese, you only have to go to Switzerland.
b. [$_{TP}$ only [$_{vP_2}$ have [AT LEAST [$_{vP_1}$ you go to [Switzerland]$_F$]]]]

As we discussed earlier, if (singular) alternatives are ranked in direct proportion to effort, the scalar presupposition of AT LEAST fails in (107b), since ϕ_S

21 An arrow from p to q indicates that q is higher on the scale than p.
22 For a related constraint formulated specifically for likelihood scales, see Crnič 2011.

is ranked highest, not lowest. Yet, if the inverse scale in (108) were available too, (107a) should have a felicitous reading, contrary to fact.

(108) $\phi_S < \phi_{NY} < \phi_{NE}$

In (108), the ranking is inversely proportional to effort, so an alternative p is ranked *lower* the *more* effort you exert on average at p-worlds. With that ranking, ϕ_S is lowest ranked and the scalar presupposition in (107b) would be satisfied: (107b) would felicitously presuppose that Switzerland is the *hardest* place to go. Since AT LEAST can convey that its prejacent is easiest, but not most difficult, the inverse ranking must be unavailable. The 'stronger' (more effortful) alternatives must outrank the 'weaker' (less effortful) alternatives. Entailment Preservation extends the constraint to logical strength. With the set of alternatives in (104), Entailment Preservation forces scales like (105), and the scalar presupposition of AT LEAST then fails in (103), since its prejacent, $(\phi_{NE} \wedge \phi_{NY})$, is not lowest ranked: it is ranked above the atomic alternatives ϕ_{NE} and ϕ_{NY}. The LF is blocked on that basis.[23]

To satisfy the presupposition, the alternative set would have to change. We entertain the possibility that there could be a pruning mechanism, where ϕ_{NE} and ϕ_{NY} can be excluded from the alternative set. For convenience, we consider the reduced alternative set in (109), pruning all atomic alternatives:

(109) $\{(\phi_{NE} \wedge \phi_{NY}), (\phi_{NE} \wedge \phi_S), (\phi_{NY} \wedge \phi_S)\}$

None of the remaining alternatives stand in a logical entailment relation, so any scale would trivially respect Entailment Preservation. With respect to pragmatic strength, the scale in (110) may be constructed in direct proportion to effort. On average, more effort is exerted in $(\phi_{NY} \wedge \phi_S)$ worlds than in $(\phi_{NE} \wedge \phi_S)$ worlds, and more effort is exerted in those than in $(\phi_{NE} \wedge \phi_{NY})$ worlds. Here, the prejacent of AT LEAST — $(\phi_{NE} \wedge \phi_{NY})$ — is lowest-ranked, and the presupposition of AT LEAST is satisfied.

23 In principle, scales could be constructed which would satisfy the scalar presupposition and yield weak readings, such as the scale in (i) (suppose alternatives shown at the same level in the ranking are equally ranked to one another here). As the reader may verify, the LF in (103) would then express (ii), equivalent to the reading in Section 4.1: (ii) conveys that John visited the North End *or* New York, not necessarily both. By Entailment Preservation, such scales are ill-formed.

(i) $\underline{\phi_{NE} \wedge \phi_{NY}} < \phi_{NE}, \phi_{NY} < \phi_S, \phi_{NE} \wedge \phi_S, \phi_{NY} \wedge \phi_S$

(ii) a. P: $\phi_{NE} \vee \phi_{NY} \vee \phi_S$ b. A: $\neg\phi_S$

(110) $(\phi_{NE} \wedge \phi_{NY}) < (\phi_{NE} \wedge \phi_{S}) < (\phi_{NY} \wedge \phi_{S})$

Given the alternatives in (109) and the scale in (110), the prejacent of *only* in (103) is the disjunction in (111a) — conveying that John visited *two* places — and that is presupposed, as in (112a). The alternatives in (111b) and (111c) are negated, deriving the assertion in (112b): that John did not visit both the North End and Switzerland, nor did he visit both New York and Switzerland. The only way to satisfy the presupposition and assertion is if the two places John visited are the North End and New York. Hence, taking presupposition and assertion together, the correct inferences obtain: that John visited the North End and New York, and did not visit Switzerland.[24] With pruning, the LF with AT LEAST is unproblematic, yielding the same inferences as an LF without AT LEAST.

(111) a. $[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NE} \wedge \phi_{NY}) \Leftrightarrow (\phi_{NE} \wedge \phi_{NY}) \vee (\phi_{NE} \wedge \phi_{S}) \vee (\phi_{NY} \wedge \phi_{S})$

b. $[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NE} \wedge \phi_{S}) \Leftrightarrow (\phi_{NE} \wedge \phi_{S}) \vee (\phi_{NY} \wedge \phi_{S})$

c. $[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NY} \wedge \phi_{S}) \Leftrightarrow (\phi_{NY} \wedge \phi_{S})$

(112) a. P: $(\phi_{NE} \wedge \phi_{NY}) \vee (\phi_{NE} \wedge \phi_{S}) \vee (\phi_{NY} \wedge \phi_{S})$

b. A: $\neg((\phi_{NE} \wedge \phi_{S}) \vee (\phi_{NY} \wedge \phi_{S})) \wedge \neg(\phi_{NY} \wedge \phi_{S}) \Leftrightarrow \neg(\phi_{NE} \wedge \phi_{S}) \wedge \neg(\phi_{NY} \wedge \phi_{S})$

In fact, even with conjunction, evidence for AT LEAST does emerge in the right environments. Consider the case in (113), which returns to vF&I's configuration with a modal. Suppose your task is to find two pieces of cheese and, for some reason, you must get them at different places. It then seems possible to use (113) to convey a sufficiency reading, that the North End and New York are the easiest two places to go, and you can — not must — go to them for the cheeses.

(113) To get the cheeses, you only have to go to the North End and New York.

Now, compare (114) and (115), given the scale after pruning above. Here, the LF with AT LEAST is distinguishable from the one without AT LEAST. The LF in (115) licenses an inference that you have to go to the North End and New

24 In the discussion here, we do not consider homogeneity effects when the alternatives are negated. Assuming that a plural analysis of the conjunction is at least available, it would be worthwhile in the future to consider how homogeneity interacts with AT LEAST on such a parse.

York, while (114) does not. The presupposition and assertion in (114) together entail just that it is possible to go to the North End and New York only and get the cheeses, deriving sufficiency, as observed. The LF with AT LEAST must be available.[25]

(114) [$_{TP}$ only [$_{vP_2}$ have [AT LEAST [$_{vP_1}$ you go to [the NE and NY]$_F$]]]]

a. P: $\Box[(\phi_{NE} \wedge \phi_{NY}) \vee (\phi_{NE} \wedge \phi_S) \vee (\phi_{NY} \wedge \phi_S)]$

b. A: $\neg\Box[(\phi_{NE} \wedge \phi_S) \vee (\phi_{NY} \wedge \phi_S)] \wedge \neg\Box(\phi_{NY} \wedge \phi_S) \Leftrightarrow \neg\Box[(\phi_{NE} \wedge \phi_S) \vee (\phi_{NY} \wedge \phi_S)]$

(115) [$_{TP}$ only [$_{vP_2}$ have [$_{vP_1}$ you go to [the NE and NY]$_F$]]]

a. P: $\Box(\phi_{NE} \wedge \phi_{NY})$

b. A: $\neg\Box(\phi_{NE} \wedge \phi_S) \wedge \neg\Box(\phi_{NY} \wedge \phi_S)$

We conclude that AT LEAST does not face any substantive over-generation concern with conjunction. With a full set of alternatives, any entailment preserving scale necessarily leads to failure of the scalar presupposition, and the LF with AT LEAST is simply blocked. With pruning, the scalar presupposition is satisfiable with a licit effort-based scale, and attested readings are derived.

It bears note that pruning perhaps could be incorporated into vF&I's system to account for the conjunction example in (102), as well. As defined earlier in the paper, EXC does not carry a scalar presupposition. As a result, while AT LEAST necessitates pruning of the atomic alternatives, EXC would not, and without pruning, its existential presupposition would not derive that John visited two places. Yet, EXC may in fact carry a scalar presupposition (see vF&I's Section 4.2), and if EXC were obligatorily scalar, then it would require pruning, similar to AT LEAST.[26] Regardless, pruning is not a general

25 Our thinking on pruning in the conjunction data has been impacted by Bernhard Schwarz and a reviewer, whom we thank. Example (113) was suggested to us by Bernhard Schwarz.

26 Suppose the existential in EXC were restricted to quantify only over entities containing two atomic parts, analogous to how we pruned the set of propositional alternatives for AT LEAST. The LF would be (i), with EXC quantifying over (ii). EXC would introduce the presupposition that John visited some two places. If EXC is defined based on Overlap in its assertion, (i) would assert that John didn't visit anywhere non-overlapping with the sum of the North End and New York. Yet, each element of (ii) does overlap with that sum, so the assertion would be trivial. If EXC were defined based on identity, as vF&I originally stated it, triviality can be avoided. (i) would assert that John didn't visit NE⊕S or NY⊕S. If no homogeneity effects arise, the predicted meaning is correct.

(i) [$_{TP}$ NEG [[EXC the North End and New York] λ1 [$_{vP}$ John visit t_1]]]

(ii) {NE⊕NY, NE⊕S, NY⊕S}

solution to the problem of vF&I deriving too weak readings. We turn now to the negation cases. There, we will propose that AT LEAST is blocked wholesale at certain scope sites, and offer principled reasons for why it is blocked based on constraints on *only* and AT LEAST.

6.2 Negation (*only* > *not*)

It will be most expedient to start with the negation example in (116), where *only* scopes over *not*. Recall the crucial inferences exhibited in (116): that John did not visit the North End, but did visit the other places.

(116) John only didn't visit the North End. ($\rightsquigarrow \neg\phi_{NE}$)

With *only* taking wide scope, there are two positions where AT LEAST could occur beneath *only*: in the immediate scope of *only* above negation (Section 6.2.1), or with lowest scope beneath negation (Section 6.2.2). We consider each LF in turn. The former will either be blocked by the scalar presupposition of AT LEAST or yield an unproblematic reading. The latter will be blocked because *only* is vacuous.

6.2.1 LF 1: *only* > AT LEAST > *not*

AT LEAST is immediately below *only* in (117). The alternatives that AT LEAST sees include negation, and the scalar presupposition requires $\neg\phi_{NE}$ to be ranked below $\neg\phi_{NY}$ and $\neg\phi_{S}$. One metric for a contextual ranking might be inverse proportion to effort. Worlds in $\neg\phi_{NE}$ (including worlds in ϕ_{NY} and ϕ_{S}) would on average involve more effort than worlds in $\neg\phi_{NY}$ (including worlds in ϕ_{NE} and ϕ_{S}) and, in turn, $\neg\phi_{S}$ (including worlds in ϕ_{NE} and ϕ_{NY}), yielding (118).

(117) $[_{\text{TP}}$ only [AT LEAST [not $[_{v\text{P}}$ John visit [the North End]$_{\text{F}}$]]]]

(118) $\neg\phi_{NE} < \neg\phi_{NY} < \neg\phi_{S}$

In the preceding section, we have seen that there are reasons to doubt that inverse proportional metrics make for licit contextual scales. If not, (117) will be blocked on the basis of the scalar presupposition of AT LEAST and the unavailability of scales like (118). Still, even if rankings like (118) were possible, (117) would be unproblematic. If (118) were available, the predicted meaning would be (119), which yields the right overall inferences: it is presupposed

that there's somewhere John did not visit, and asserted that he visited everywhere other than the North End, from which it follows that he did not visit the North End.

(119) a. P: $[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\neg\phi_{NE}) \Leftrightarrow \neg\phi_{NE} \vee \neg\phi_{NY} \vee \neg\phi_{S}$
b. A: $\neg[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\neg\phi_{NY}) \wedge \neg[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\neg\phi_{S}) \Leftrightarrow$
$\neg(\neg\phi_{NY} \vee \neg\phi_{S}) \wedge \neg\neg\phi_{S} \Leftrightarrow \phi_{NY} \wedge \phi_{S}$

6.2.2 LF 2: *only* > *not* > AT LEAST

We now entertain an LF with AT LEAST below both *only* and negation. Because AT LEAST scopes below negation, it operates over positive alternatives, and its scalar presupposition is easily satisfied with the familiar effort-based ranking ($\phi_{NE} < \phi_{NY} < \phi_{S}$). Yet, a pathology arises when we flesh out the meaning.

(120) $[_{\text{TP}}$ only [not [AT LEAST $[_{vP}$ John visit [the North End]$_{\text{F}}$]]]]

By making an existential contribution, AT LEAST locally weakens the complement of negation. Since negation is an entailment reversing operator, however, the overall prejacent of *only* is strengthened by AT LEAST. The prejacent conveys that John didn't visit *any* place, as in (121). Without AT LEAST, the prejacent would be $\neg\phi_{NE}$, which (121) asymmetrically entails.

(121) P: $\neg[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NE}) \Leftrightarrow$
$\neg(\phi_{NE} \vee \phi_{NY} \vee \phi_{S}) \Leftrightarrow \neg\phi_{NE} \wedge \neg\phi_{NY} \wedge \neg\phi_{S}$

Herein lies the problem: the alternatives *only* sees express the propositions in (122), and with the prejacent strengthened, all alternatives are weaker than the prejacent. Since *only* negates non-weaker alternatives, there are no excludable alternatives, and the assertive component of *only* is trivialized to a tautology. The assertion, shown in (123), says that every non-weaker alternative to the prejacent is false — and that is trivially so when there are none.

(122) a. $\neg[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NE})$
$\Leftrightarrow \neg(\phi_{NE} \vee \phi_{NY} \vee \phi_{S}) \Leftrightarrow \neg\phi_{NE} \wedge \neg\phi_{NY} \wedge \neg\phi_{S}$
b. $\neg[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NY})$ $\Leftrightarrow \neg(\phi_{NY} \vee \phi_{S}) \Leftrightarrow \neg\phi_{NY} \wedge \neg\phi_{S}$
c. $\neg[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{S})$ $\Leftrightarrow \neg\phi_{S}$

(123) a. A: $\lambda w\,.\,\forall p \in \{(122a), (122b), (122c)\}\,[p(w) \rightarrow (121) \subseteq p] \Leftrightarrow$
b. $\lambda w\,.\,\forall p \in \{(122a), (122b), (122c)\}\,[(121) \not\subseteq p \rightarrow \neg p(w)]$

Alxatib (2013) provides independent evidence that for *only* to be licit, there must be some alternative for it to exclude (and, moreover, one whose negation is not already entailed by the prejacent). Because *only* vacuously quantifies over non-weaker alternatives, the LF in (120) is predictably undetectable.

6.3 Negation (*not* > *only*)

We conclude the section with the case in (124), where *only* takes scope under negation, intuitively licensing the inferences that John visited the North End and somewhere else. In (124), there is one position where AT LEAST could occur beneath *only*: in its immediate scope, as in the LF in (125). This derivation raises the most difficult over-generation challenge for our account.

(124) John didn't only visit the North End. ($\rightsquigarrow \phi_{NE}$)

(125) [$_{TP}$ not [only [AT LEAST [$_{vP}$ John visit [the North End]$_F$]]]]

The prejacent of *only* says that John visited somewhere, which is presupposed by *only* and projects over negation, as in (126a). *Only* yields the assertion that John did not visit any place more effortful than the North End, and negation derives that he did visit somewhere more effortful, as in (126b).

(126) a. P: $[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NE}) \Leftrightarrow \phi_{NE} \vee \phi_{NY} \vee \phi_S$

b. A: $\neg(\neg[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NY}) \wedge \neg[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_S)) \Leftrightarrow$
$\neg(\neg\phi_{NY} \wedge \neg\phi_S) \Leftrightarrow \phi_{NY} \vee \phi_S$

The presupposition is too weak. It does not entail that John visited the North End, and even in combination with the assertion, the ϕ_{NE} inference is not captured. The reading is identical to the one vF&I's account derived (see Section 4.2). While vF&I predicted this reading to be the only one for (124), we improve by deriving it alongside the observed reading, which comes from an available parse without AT LEAST, as repeated in (127) below.

(127) [$_{TP}$ not [only [$_{vP}$ John visit [the North End]$_F$]]]

a. P: ϕ_{NE}

b. A: $\neg(\neg\phi_{NY} \wedge \neg\phi_S) \Leftrightarrow \phi_{NY} \vee \phi_S$

Still, to avoid over-generation, the LF in (125) must be blocked. Our strategy is to capitalize on the ambiguity in our system between parses with and without AT LEAST. We suggest that these compete, and present a reason why we can expect the parse without AT LEAST to win over the parse with AT LEAST.

The LFs in (125) (with AT LEAST) and (127) (without AT LEAST) differ in presupposition, but the two LFs express an equivalent assertion. Accordingly, one possibility might be that a more complex LF with AT LEAST is licensed only when that LF differs in global assertion from a counterpart without AT LEAST. This hypothesis is captured in (128). To isolate assertion, equivalence is evaluated based on von Fintel's (1999) notion of Strawson entailment.[27]

(128) *Anti-Vacuity (First Version)*
Let LF_1 be a matrix TP of the form [$_{TP}$ … [only [… AT LEAST [X] …]]].
Let LF_2 be a matrix TP of the form [$_{TP}$ … [only [… [X] …]]]
(where LF_2 replaces AT LEAST [X] in LF_1 with [X]).
LF_1 is disallowed if $[\![LF_1]\!] \Leftrightarrow_{Strawson} [\![LF_2]\!]$.

Illustrating in a trivalent setting, one proposition Strawson entails another just in case at every world where the former is true, the latter is *not false*, as in (129). Presupposition failure results in a stigmatized truth-value, #. For worlds where the presupposition of either α or β fails, the underlined conditional in (129) is necessarily true. If α has a stigmatized value, the antecedent is false. If β has a stigmatized value, the consequent is true. Either way, the conditional is true. To assess Strawson entailment, then, we can consider only worlds where both the presuppositions of α and β are met. For those, the conditional holds just in case the truth of α guarantees the truth of β. With the presuppositions of α and β met, truth or falsity is determined by assertion — and (128) isolates assertion in kind.

(129) *Strawson Entailment*
If α and β are of type <s,t>,
$\alpha \Rightarrow_{Strawson} \beta$ iff for all w, $\underline{\alpha(w) = 1 \rightarrow \beta(w) = 1 \text{ or } \#}$.

The presuppositions of (125) and (127) are both met in worlds where John visited the North End. At those worlds, whether (125) and (127) are true rests on their equivalent assertions: at worlds where John additionally visited New York or Switzerland, both are true; at worlds where John just visited the North End, both are false. The two LFs express Strawson equivalent meanings, and the LF without AT LEAST will block the LF with AT LEAST, avoiding over-generation in (124).

27 Note that at least one focus operator is present in each competitor LF. We assume that *only* and AT LEAST associate with the same focus, and that the constraint is evaluated within a particular context, so the same replacements for the focused element are salient in both competitors.

Moreover, Anti-Vacuity still correctly allows AT LEAST in vF&I's modal data, given the set of alternatives that we have been assuming. The LFs with and without AT LEAST are repeated for reference below.

(130) $[_{TP}$ only $[_{vP_2}$ have [AT LEAST $[_{vP_1}$ you go to [the North End]$_F$]]]]

a. P: $\Box(\phi_{NE} \vee \phi_{NY} \vee \phi_S)$
b. A: $\neg\Box(\phi_{NY} \vee \phi_S)$

(131) $[_{TP}$ only $[_{vP_2}$ have $[_{vP_1}$ you go to [the North End]$_F$]]]

a. P: $\Box\phi_{NE}$
b. A: $\neg\Box\phi_{NY} \wedge \neg\Box\phi_S$

While the weakening effect of AT LEAST is felt in the presupposition, an effect of AT LEAST is retained globally in the assertion as well. The assertion in (130b) is stronger than that in (131b). Suppose that you go to the North End in all cheese-worlds. Then, the presuppositions in (130a) and (131a) are both satisfied. If you also go to a second place in all cheese-worlds — New York in some of the worlds, and Switzerland in the others — (131b) is true, while (130b) is false. The stronger assertion in (130b) says that there are some cheese-worlds where you go to neither New York nor Switzerland. As a result, the two LFs do not express Strawson equivalent meanings, and Anti-Vacuity would permit the LF with AT LEAST.

Anti-Vacuity has promise. But, a problem arises. Modal cases where there are just *two*, rather than three, salient alternatives argue that presupposition cannot be ignored. Consider vF&I's example as an answer to the question in (132a). The question makes salient only the alternatives ϕ_{NE} and ϕ_{NY}. As before, (132b) conveys just the possibility claim that you *can* get good cheese at the North End, indicating that a parse with AT LEAST is still available.

(132) a. Can I get good cheese at the North End, or do I need to go to NY?
b. To get good cheese, you only have to go to the North End.

The LF with AT LEAST is repeated in (133), and the two alternatives *only* sees express (134a) and (134b). (134a) is *only*'s prejacent, and says that you have to go to the North End or New York. The only excludable alternative is (134b). As ϕ_{NY} is highest-ranked, $\Box[\![\text{AT LEAST}]\!]^{\text{ALT}\leq}(\phi_{NY})$ is equivalent to just $\Box\phi_{NY}$, and the assertion says that you don't have to go to New York. The meaning is (135).

(133) $[_{TP}$ only $[_{vP_2}$ have [AT LEAST $[_{vP_1}$ you go to [the North End]$_F$]]]]

(134) a. $\Box[\![\text{AT LEAST}]\!]^{\text{ALT}_{\leq}}(\phi_{NE}) \Leftrightarrow \Box(\phi_{NE} \vee \phi_{NY})$
b. $\Box[\![\text{AT LEAST}]\!]^{\text{ALT}_{\leq}}(\phi_{NY}) \Leftrightarrow \Box\phi_{NY}$

(135) a. P: $\Box(\phi_{NE} \vee \phi_{NY})$
b. A: $\neg\Box\phi_{NY}$

As usual, the LF with AT LEAST carries a weaker presupposition than the one without. In (136), the presupposed prejacent of *only* says that you have to go to the North End itself, as in (136a). Yet, the assertions are now identical. *Only* in (136) directly negates $\Box\phi_{NY}$, as in (136b). If Anti-Vacuity only cares about assertion, it should block AT LEAST, leaving the sufficiency reading unexplained.[28]

(136) $[_{\text{TP}}$ only $[_{vP_2}$ have $[_{vP_1}$ you go to [the North End]$_{\text{F}}]]]$
a. P: $\Box\phi_{NE}$
b. A: $\neg\Box\phi_{NY}$

It appears that AT LEAST is licensed in the modal data even if it only impacts the presupposition. If that conclusion is correct, an Anti-Vacuity constraint based on Strawson entailment must be abandoned.

We reach a challenging situation. In vF&I's modal case, AT LEAST is allowed even if it just affects presupposition. Yet, in the negation case, AT LEAST would just affect presupposition, and must be blocked. For reference, the LFs with and without AT LEAST for the negation example in (124) are repeated in (137) and (138), returning to three alternatives. Due to the weaker presupposition, the presupposition and assertion of the AT LEAST LF together convey that John visited somewhere other than the North End, while the LF without AT LEAST conveys, on top of that, that John visited the North End. As we saw, only (138) is available.

(137) $[_{\text{TP}}$ not [only [AT LEAST $[_{vP}$ John visit [the North End]$_{\text{F}}]]]]$
a. P: $\phi_{NE} \vee \phi_{NY} \vee \phi_{S}$
b. A: $\phi_{NY} \vee \phi_{S}$

28 One could think that other alternatives can be accommodated, but this strategy does not seem to be generally viable. Consider, for instance, (i), in a situation where both interlocutors know that there are only two grades as high as a B+: a B+ and an A. B's statement conveys sufficiency without forcing accommodation of extra grade options.

(i) a. A: To make it to the Dean's list, you can't get less than a B+.
b. B: That's right, but you only need to get a B+.

(138) [$_{TP}$ not [only [$_{vP}$ John visit [the North End]$_F$]]]

a. P: ϕ_{NE}
b. A: $\phi_{NY} \vee \phi_S$

So, why *is* AT LEAST disallowed in the negation case, but allowed in the modal case? We note a distinction in how local AT LEAST is to *only*. In the negation case, *only* is beneath negation, and AT LEAST is in its immediate scope. In vF&I's example, *only* is above the modal and, in the crucial LF, the modal intervenes between *only* and AT LEAST. The LFs are repeated side by side. Once we take the relationship between *only* and AT LEAST into consideration, there is a sense in which AT LEAST makes a vacuous contribution in the negation case, but not in the modal case — even when both presupposition and assertion are factored in.

(139) [$_{TP}$ not [only [AT LEAST [$_{vP}$ John visit [the North End]$_F$]]]]

(140) [$_{TP}$ only [$_{vP_2}$ have [AT LEAST [$_{vP_1}$ you go to [the North End]$_F$]]]]

Consider just the smallest projection in the negation LF in (139) containing *only* and AT LEAST, as isolated in (141). This constituent has the presupposition in (142a) (that John visited somewhere) and contributes the assertion in (142b) (that he did not visit any place more effortful than the North End). The presupposition and assertion are conjoined in (143). Taken together, they entail that John visited the North End, and not New York or Switzerland.

(141) [only [AT LEAST [$_{vP}$ John visit [the North End]$_F$]]]

(142) a. P: $[\![\text{AT LEAST}]\!]^{\text{ALT}_\leq}(\phi_{NE}) \Leftrightarrow \phi_{NE} \vee \phi_{NY} \vee \phi_S$
b. A: $\neg[\![\text{AT LEAST}]\!]^{\text{ALT}_\leq}(\phi_{NY}) \wedge \neg[\![\text{AT LEAST}]\!]^{\text{ALT}_\leq}(\phi_S)$
$\Leftrightarrow \neg\phi_{NY} \wedge \neg\phi_S$

(143) $(\text{P} \wedge \text{A}) \Leftrightarrow (\phi_{NE} \vee \phi_{NY} \vee \phi_S) \wedge (\neg\phi_{NY} \wedge \neg\phi_S)$
$\Leftrightarrow \phi_{NE} \wedge \neg\phi_{NY} \wedge \neg\phi_S$

We observe that the entailments in (143) are identical to those expected from the counterpart fragment with *only*, but not AT LEAST, given in (144). In (144), it is directly presupposed that John visited the North End, and the negated alternatives express that John visited New York and Switzerland, respectively. The conjunction of presupposition and assertion in (145) is equivalent to (143).

(144) [only [$_{vP}$ John visit [the North End]$_F$]]

a. P: ϕ_{NE}
b. A: $\neg\phi_{NY} \wedge \neg\phi_S$

(145) (P $\wedge$ A) $\Leftrightarrow$ $\phi_{NE} \wedge \neg\phi_{NY} \wedge \neg\phi_S$ $\Leftrightarrow$ (143)

A new possibility arises: that Anti-Vacuity takes into account presupposition and assertion together, but is evaluated not globally, but locally, at the point that *only* merges in the structure. We re-formulate Anti-Vacuity in (146). The revised constraint applies to the smallest constituent containing *only* and AT LEAST. Moreover, it blocks that constituent if the proposition which it expresses is *strictly* equivalent to that expressed by a simpler counterpart without AT LEAST.

(146) *Anti-Vacuity (Second Version)*
Let C_1 be a constituent of the form [only [...AT LEAST [X] ...]].
Let C_2 be a constituent of the form [only [... [X] ...]]
(where C_2 replaces AT LEAST [X] in C_1 with [X]).
C_1 is disallowed if $[\![C_1]\!] \Leftrightarrow_{\text{strict}} [\![C_2]\!]$.

Strict entailment is defined in (147), again illustrating in a trivalent setting. One proposition strictly entails another just in case at every world where the former is true, the latter is true. For either proposition to be true, its presupposition and assertion must both be true. In effect, then, what matters for evaluation of strict entailment is the conjunction of presupposition and assertion.

(147) *Strict Entailment*
If α and β are of type <s,t>,
$\alpha \Rightarrow_{\text{strict}} \beta$ iff for all w, $\alpha(w) = 1 \rightarrow \beta(w) = 1$.

The constituent with AT LEAST in (141) conveys a meaning strictly equivalent to that of the counterpart without AT LEAST in (144). As a result, (141) runs afoul of Anti-Vacuity, and the full negation LF with AT LEAST, containing (141), is blocked in turn. The over-generation problem in the negation data is thus resolved.[29]

29 In assessing Anti-Vacuity, we suppress the scalar presupposition of AT LEAST. Since the scalar presupposition is not world-dependent, recall that it is either contradictory or tautologous. When it is contradictory, this presupposition will block the LF with AT LEAST independent of Anti-Vacuity. When tautologous, Anti-Vacuity will be blind to it, since conjoining a tautology with the contingent prejacent presupposition of *only* and the contingent asser-

The blocking effect is quite general. Compare the schematic constituents in (148a) and (148b). For simplicity, suppose that the alternatives to the XP express propositions that are all logically independent of each other. (148a) carries the presupposition that some alternative is true, and asserts that no alternative higher-ranked than ⟦XP⟧ is true. If ⟦XP⟧ is lowest-ranked, as required by AT LEAST, that amounts to saying that ⟦XP⟧ is true, and no other alternative is. *Only* in (148b) directly triggers a presupposition that ⟦XP⟧ is true, and negates the other alternatives. The conjunction of presupposition and assertion should thus be equivalent in the two fragments, and so any LF containing (148a) should be blocked in turn.

(148) a. [only [AT LEAST [$_{XP}$ …[…]$_F$ …]]]
b. [only [$_{XP}$ …[…]$_F$ …]]

By this reasoning, given the independent alternatives to ϕ_{NE} — $\{\phi_{NE}, \phi_{NY}, \phi_S\}$ — the revised Anti-Vacuity constraint directly predicts that AT LEAST must be separated from *only* by some other operator in order to occur. In vF&I's example, repeated in (149), separation does obtain and Anti-Vacuity is respected.

(149) [$_{TP}$ only [$_{vP_2}$ have [AT LEAST [$_{vP_1}$ you go to [the North End]$_F$]]]]
a. P: $\Box(\phi_{NE} \vee \phi_{NY} \vee \phi_S)$
b. A: $\neg\Box(\phi_{NY} \vee \phi_S)$

(150) [$_{TP}$ only [$_{vP_2}$ have [$_{vP_1}$ you go to [the North End]$_F$]]]
a. P: $\Box\phi_{NE}$
b. A: $\neg\Box\phi_{NY} \wedge \neg\Box\phi_S$

Because *only* takes widest scope in (149), the matrix node is the relevant constituent for Anti-Vacuity. As we have seen, the conjunction of presupposition and assertion in (149) entails just that you *can* go to the North End for good cheese. In the competitor in (150), their conjunction entails that you *must* go there. The two structures do not express strictly equivalent meanings, and AT LEAST is therefore licensed. Since both presupposition and assertion are taken into account together, the result is general, whether there are three alternatives, as shown, or two.

tion yields an equivalent result to just the conjunction of the prejacent presupposition and assertion alone. As noted in fn. 14, it is also possible that *only* encodes a scalar presupposition, or that both *only* and AT LEAST do. If so, a scalar presupposition would be triggered in both competitors, in any case.

Overall, we have suggested that AT LEAST is restricted in the negation data by a constraint prohibiting AT LEAST from making a vacuous contribution. We entertained a global Anti-Vacuity constraint based on Strawson entailment, and a local constraint based on strict entailment. The local constraint seems to best distinguish the negation case from vF&I's modal case, where AT LEAST is productively licensed.

6.4 Summary and open questions

Our proposal has a general solution to the under-generation of transparent prejacent inferences discussed in Section 4, since AT LEAST is not obligatory. At the same time, the proposal does still face over-generation concerns, since AT LEAST is optional. We proposed that insertion of AT LEAST is restricted due to a constraint on scale construction (in the conjunction case), together with plausible principles against vacuity of *only* and AT LEAST (in the negation cases).

The local Anti-Vacuity constraint in the preceding subsection leaves questions open. Most notably, why would Anti-Vacuity care about the smallest constituent containing *only* and AT LEAST? We merely have a speculation to offer. We note that AT LEAST seems to be severely restricted or unavailable when not in the scope of *only* (or, as Crnič (2011) discusses, *even* in some languages). Consider (151). If AT LEAST were licensed in unembedded environments, (151) would allow the parse in (152a), which asserts that John went somewhere, as in (152b).

(151) John went to the North End.

(152) a. $[_{\text{TP}}$ AT LEAST $[_{\nu\text{P}}$ John go to [the North End]$_{\text{F}}]]$
b. A: $\phi_{NE} \vee \phi_{NY} \vee \phi_{S}$

Yet, (152a) is not available. Unlike a parse without AT LEAST, which asserts ϕ_{NE}, the assertion in (152b) does not entail ϕ_{NE}. To clarify the intuition, a purpose clause is added in (153). The baseline in (153a) licenses no inference about where John's parents live. In contrast, in (153b) it is inferred that they live in the North End, suggesting that the sentence must entail that John went to the North End.

(153) a. To visit his parents, John went somewhere.
b. To visit his parents, John went to the North End.

Moreover, just any embedding is not sufficient to license AT LEAST. It appears, for instance, that AT LEAST cannot freely occur under a modal when *only* is not also present above. Consider (154), and a parse with AT LEAST in (155).

(154) To get good cheese, you have to go to the North End.

(155) a. [$_{TP}$ have [AT LEAST [$_{vP}$ you go to [the North End]$_F$]]]
b. A: $\Box(\phi_{NE} \vee \phi_{NY} \vee \phi_S)$

The assertion in (155b) says just that to get good cheese, you have to go somewhere. Yet, as shown in (156), (154) does not allow the interlocutor to accept that they will follow the speaker's advice and, at the same time, indicate that they will go somewhere other than the North End. (156b) is deviant, in contrast to (156a). $\Box\phi_{NE}$ is the intuited assertion for (154), as expected without AT LEAST.

(156) a. A: To get good cheese, you have to go somewhere.
B: Ok, I'll go to NY.
b. A: To get good cheese, you have to go the North End.
B: # Ok, I'll go to NY.

If there is some dependency between *only* and AT LEAST involved in licensing AT LEAST, the locality domain for Anti-Vacuity would dovetail: Anti-Vacuity would be concerned with the constituent containing AT LEAST and its licensing operator. In this section, we have confined our attention to elucidating constraints restricting the distribution of AT LEAST within the scope of *only*.

7 Conclusion

We started with vF&I's observation that a necessity modal in the scope of *only* can result in a sufficiency reading conveying a global possibility inference. We argued against vF&I's proposal that the source of weakening is *only* itself. In our analysis, *only* is a simplex adverbial, which triggers a strong prejacent presupposition. Instead, we drew upon a covert weakening operator, AT LEAST, which may occur within the prejacent of *only*. AT LEAST is optional and restricted in data with conjunction and negation. In those cases, transparent prejacent inferences are detected.

Our aim has been to understand sufficiency readings with *only*. There are, however, other routes to sufficiency. For one, copular constructions with

all.[30] The counterpart to vF&I's example is (157), which, despite not containing *only*, still conveys that the North End is an easy place to go, and that you can go there for good cheese — not that you must go there. Perhaps AT LEAST is licensed in the environment of *all*, just as it is licensed beneath *only*, and scopes under *have* in (157). If *all* contributes exhaustivity in place of *only* (Homer 2019 develops a proposal along these lines, treating *all* as a quantity superlative), a composition similar to the one we have put forward for vF&I's case might have promise.

(157) To get good cheese, all you have to do is go to the North End.

A reviewer adds the example in (158), with overt *at most* (see vF&I's fn. 3 for a similar case). With *at most* taking scope under *have*, (158) might be expected to convey that in all cheese-worlds you go to the North End and nowhere more effortful. Yet, the intuition seems to resemble a sufficiency reading. (158) conveys that the North End is an easy place to get good cheese, and raises the possibility that the North End might be the easiest of all. At the same time, it is not excluded that you could also go somewhere more effortful.

(158) To get good cheese, you have to at most go to the North End.

Because we have re-sourced weakening from *only* into a separate covert operator, it is broadly consistent with our approach for sufficiency readings to have a wider distribution. Generalizing an analysis based on AT LEAST to a broader range of sufficiency data is, we hope, a fruitful direction to pursue next.

References

Alonso-Ovalle, Luis & Aron Hirsch. 2018. Keep *only* strong. *Semantics and Linguistic Theory (SALT)* 28. 251–270. https://doi.org/10.3765/salt.v28io.4439.

Alxatib, Sam. 2013. *Only and association with negative antonyms*. Cambridge, MA: Massachusetts Institute of Technology dissertation. https://doi.org/1721.1/84414.

30 We thank Vincent Homer and Bernhard Schwarz for making us aware of these data.

Bassi, Itai. 2021. *Fake features and valuation from context*. Cambridge, MA: Massachusetts Institute of Technology dissertation. https://doi.org/1721.1/140990.

Beaver, David & Brady Clark. 2008. *Sense and sensitivity: How focus determines meaning*. Malden, MA: Wiley-Blackwell. https://doi.org/10.1002/9781444304176.

Bosque, Ignacio. 1980. *Sobre la negación*. Madrid: Cátedra.

Cepeda, Paola. 2015. Word order and polarity in Spanish postnominal *alguno*. Talk presented at the 2015 Hispanic Linguistics Symposium (HLS), University of Illinois at Urbana-Champaign, Sept. 25. https://tinyurl.com/ybs44hpz.

Cepeda, Paola. 2016. Word order and polarity in Spanish postnominal *alguno*. Poster presented at the Linguistic Society of America 2016 Annual Meeting, Washington DC, Jan 8. https://tinyurl.com/4xhmx6ck.

Chierchia, Gennaro. 2006. Broaden your views: Implicatures of domain widening and the 'logicality' of language. *Linguistic Inquiry* 37(4). 535–590. https://doi.org/10.1162/ling.2006.37.4.535.

Chierchia, Gennaro. 2013. *Logic in grammar: Polarity, free choice and intervention*. Oxford: Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199697977.001.0001.

Chierchia, Gennaro, Danny Fox & Benjamin Spector. 2012. Scalar implicature as a grammatical phenomenon. In Claudia Maienborn, Klaus von Heusinger & Paul Portner (eds.), *Semantics: An international handbook of natural language meaning*, vol. 3, 2297–2331. Berlin: Mouton de Gruyter. https://doi.org/10.1515/9783110253382.2297.

Coppock, Elizabeth & David Beaver. 2014. Principles of the exclusive muddle. *Journal of Semantics* 31. 371–432. https://doi.org/10.1093/jos/fft007.

Crnič, Luka. 2011. *Getting even*. Cambridge, MA: Massachusetts Institute of Technology dissertation. https://doi.org/1721.1/68912.

Crnič, Luka. 2021. Exceptives and exhaustification. Draft of forthcoming proceedings paper from *West Coast Conference on Formal Linguistics (WCCFL)* 39. http://lukacrnic.com/pdfs/wccfl.pdf.

Espinal, Maria Teresa, Susagna Tuba, Joan Borràs-Comes & Pilar Prieto. 2016. Double negation in Catalan and Spanish: Interaction between syntax and prosody. In Pierre Larrivée & Chungmin Lee (eds.), *Negation and polarity: Experimental perspectives*, 145–176. Cham: Springer. https://doi.org/10.1007/978-3-319-17464-8_7.

von Fintel, Kai. 1999. NPI licensing, Strawson-entailment, and context-dependency. *Journal of Semantics* 16(2). 97–148. https://doi.org/10.1093/jos/16.2.97.

von Fintel, Kai & Sabine Iatridou. 2007. Anatomy of a modal construction. *Linguistic Inquiry* 38(3). 445–483. https://doi.org/10.1162/ling.2007.38.3.445.

Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and implicature in compositional semantics*, 71–120. Basingstoke: Palgrave MacMillan. https://doi.org/10.1057/9780230210752_4.

Fox, Danny & Roni Katzir. 2011. On the characterization of alternatives. *Natural Language Semantics* 19(1). 87–101. https://doi.org/10.1007/s11050-010-9065-3.

Franke, Michael. 2006. Teleological necessity and *only*. In Janneke Huitink & Sophia Katrenko (eds.), *Proceedings of the 11th European School Summer School in Logic, Language and Information (ESSLLI) student session*, 14–25. http://www.home.uni-osnabrueck.de/michfranke/Papers/TelNecOnly.pdf.

Gajewski, Jon. 2011. Licensing strong NPIs. *Natural Language Semantics* 19(2). 109–148. https://doi.org/10.1007/s11050-010-9067-1.

Geurts, Bart & Robert van der Sandt. 2004. Interpreting focus. *Theoretical Linguistics* 30(1). 1–44. https://doi.org/doi:10.1515/thli.2004.005.

Giannakidou, Anastasia. 2006. *Only*, emotive factives, and the dual nature of polarity dependency. *Language* 82. 575–603. https://doi.org/10.1353/lan.2006.0136.

Guerzoni, Elena. 2006. Intervention effects on NPIs and feature movement: Towards a unified account of intervention. *Natural Language Semantics* 14(4). 359–398. https://doi.org/10.1007/s11050-007-9008-9.

Heim, Irene. 2008. Features on bound pronouns. In Daniel Harbour, David Adger & Susana Béjar (eds.), *Phi theory: Phi-features across modules and interfaces*, 460–469. Oxford: Oxford University Press.

von Heusinger, Klaus. 2007. Alternative semantics for definite NPs. In Kerstin Schwabe & Susanne Winkler (eds.), *Information structure and the architecture of grammar: A typological perspective*, 485–508. Amsterdam: John Benjamins Publishing Company. https://doi.org/10.1075/la.100.26heu.

Hirsch, Aron. 2016. An unexceptional semantics for expressions of exception. *University of Pennsylvania Working Papers in Linguistics* 22(1). https://doi.org/20.500.14332/45082.

Homer, Vincent. 2011. *Polarity and modality*. University of California, Los Angeles dissertation. https://linguistics.ucla.edu/general/dissertations/HomerDissertationUCLA2011.pdf.

Homer, Vincent. 2019. That's *all*. *West Coast Conference on Formal Linguistics (WCCFL)* 36. 1–21. http://www.lingref.com/cpp/wccfl/36/paper3441.pdf.

Homer, Vincent. 2021. Negative polarity. In Daniel Gutzmann, Lisa Matthewson, Cécile Meier, Hotze Rullmann & Thomas Ede Zimmermann (eds.), *The Wiley Blackwell companion to semantics*. Malden, MA: Wiley Blackwell. https://doi.org/10.1002/9781118788516.sem057.

Horn, Laurence. 1969. A presuppositional analysis of *only* and *even*. *Chicago Linguistics Society (CLS)* 5. 98–107.

Horn, Laurence. 1996. Exclusive company: *Only* and the dynamics of vertical inference. *Journal of Semantics* 13(1). 1–40. https://doi.org/10.1093/jos/13.1.1.

Ippolito, Michela. 2007. On the meaning of *only*. *Journal of Semantics* 25(1). 45–91. https://doi.org/10.1093/jos/ffm010.

Katzir, Roni. 2007. Structurally-defined alternatives. *Linguistics and Philosophy* 30(6). 669–690. https://doi.org/10.1007/s10988-008-9029-y.

Krasikova, Sveta & Ventsislav Zhechev. 2005a. Scalar use of *only* in conditionals. *Amsterdam Colloquium* 15. 137–143. http://events.illc.uva.nl/AC/AC2005/uploaded_files/AC05Proceedings.pdf.

Krasikova, Sveta & Ventsislav Zhechev. 2005b. You only need a scalar *only*. *Sinn und Bedeutung (SuB)* 10. 199–211. https://doi.org/10.21248/zaspil.44.2006.310.

Kratzer, Angelika. 1998. More structural analogies between pronouns and tenses. *Semantics and Linguistics Theory Conference (SALT)* 8. 92–110. https://doi.org/10.3765/salt.v8i0.2808.

Laka, Itziar. 1990. *On the nature of functional categories and projections*. Cambridge, MA: Massachusetts Institute of Technology. https://doi.org/1721.1/13667.

Linebarger, Marcia C. 1987. Negative polarity and grammatical representation. *Linguistics and Philosophy* 10. 325–387. https://doi.org/10.1007/BF00584131.

Link, Godehard. 1983. The logical analysis of plural and mass terms: A lattice-theoretical approach. In Christoph Schwarze Rainer Bäuerle & Arnim von Stechow (eds.), *Meaning, use and interpretation of language*, 302–323. Berlin: de Gruyter. https://doi.org/10.1515/9783110852820.302.

López, Cristina Sánchez. 2000. La negación. In Ignacio Bosque & Violeta Demonte (eds.), *Gramática descriptiva de la lengua española*, vol. 2, 2561–2634. Madrid: Espasa-Calpe.

Rivero, María Luisa. 1970. A surface structure constraint on negation in Spanish. *Language* 46. 640–66. https://doi.org/10.2307/412311.

van Rooj, Robert & Katrin Schulz. 2007. *Only*: Meaning and implicatures. In Maria Aloni Paul Dekker & Alastair Butler (eds.), *Questions in dynamic semantics*, 193–223. Bingley, UK: Emerald. https://doi.org/10.1163/9780080470993_010.

Sauerland, Uli. 2013. Presuppositions and the alternative tier. *Semantics and Linguistic Theory (SALT)* 23. 156–173. https://doi.org/10.3765/salt.v23i0.2673.

Schwarz, Bernhard. 2005. Scalar additive particles in negative contexts. *Natural Language Semantics* 13(2). 125–168. https://doi.org/10.1007/s11050-004-2441-0.

Thomas, Guillaume. 2011. Another additive particle. *Semantics and Linguistic Theory (SALT)* 21. 634–651. https://doi.org/10.3765/salt.v21i0.2589.

Vostrikova, Ekaterina. 2021. Conditional analysis of clausal exceptives. *Natural Language Semantics* 29. 159–227. https://doi.org/10.1007/s11050-021-09177-z.

Walker, Andreas. 2012. Focus, uniqueness and soft presupposition triggers. In Maria Aloni, Vadim Kimmelman, Floris Roelofsen, Galit W. Sassoon, Katrin Schulz & Matthijs Westera (eds.), *Logic, language and meaning: 18th Amsterdam Colloquium, Amsterdam, the Netherlands, December 19-21, 2011, Revised selected papers* (Lecture Notes in Computer Science), 460–469. Berlin & Heidelberg: Springer. https://doi.org/10.1007/978-3-642-31482-7_47.

Zeijlstra, Hedde. 2004. *Sentential negation and negative concord*. University of Amsterdam dissertation. https://www.lotpublications.nl/Documents/101_fulltext.pdf.

Zeijlstra, Hedde. 2008. Negative concord is syntactic agreement. University of Amsterdam, ms. https://ling.auf.net/lingbuzz/000645.

Zeijlstra, Hedde. 2016. Negation and negative dependencies. *Annual Review of Linguistics* 2(1). 233–254. https://doi.org/10.1007/s11050-004-2441-0.

Zwarts, Frans. 1996. Three types of polarity. In Fritz Hamm & Erhard Hinrichs (eds.), *Plurality and quantification*, 177–238. Dordrecht: Kluwer. https://doi.org/10.1007/978-94-017-2706-8_5.

Keep *only* strong

Luis Alonso-Ovalle
McGill University
Department of Linguistics
1085 Avenue du Docteur Penfield
Montréal, Québec, Canada H3A1A7
luis.alonso-ovalle@mcgill.ca

Aron Hirsch
Leibniz-Center General Linguistics (ZAS)
18 Schützenstraße
Berlin, Germany 10117
hirsch@leibniz-zas.de