

A square of necessities: X-marking weak and strong necessity modals*

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Abstract Many languages express the modal concept known as weak necessity by adding a conventional marking to their strong necessity modals (von Fintel & Iatridou 2008). This extra marking has been dubbed X-marking by von Fintel & Iatridou (2023) who have also shown that it is the same marking that often appears on so-called subjunctive (counterfactual) conditionals as well as on attitude verbs expressing unattainable desires. We discuss Portuguese weak and strong necessity modals (*dever* and *ter que*) and claim that both can be X-marked, although no weakening in their modal force is attested. We conclude that necessity modals can host a parametrized version of X-marking affecting their modal parameters (modal bases and ordering sources) and generating a square of semantically related necessities.

Keywords: X-marking, weak necessity, strong necessity, Portuguese, modal verbs, counterfactuality

1 Introduction

English modal verbs *ought* and *should* express a modal concept known as WEAK NECESSITY. The core intuition is that the modal force they express

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is weaker than the (strong) necessity expressed by *must* and *have to* but stronger than the (mere) possibility expressed by auxiliaries *may* and *can*:

- (1) In order to get to the airport, ...
 - a. You have to take a cab.
 - b. You should take a cab.
 - c. You can take a cab.

Roughly put, (1a) conveys that taking a cab is the only way to get to the airport, (1b) conveys that taking a cab is the best (not necessarily the only) way to get to the airport, and (1c) only conveys that taking a cab is a possible (not necessarily the only or the best) way to get to the airport. The example illustrates the teleological (goal oriented) reading of the modal verbs, but the intuition is stable across different modal flavors:

- (2) Concerning the COVID-19 protocol for this building ...
 - a. Adults have to wear a mask.
 - b. Children ought to wear a mask.
- (3) Children ought to wear a mask, but they don't have to.

In this deontic setting, wearing a mask is mandatory for adults, and advisable for children.

- (4) Where is John?
 - a. He must be in his office.
 - b. He ought to be in his office.
 - c. He may be in his office.

Here we entered the domain of epistemic modality. Roughly put, the proposition that John is in his office is presented as an inevitable conclusion by (4a), as a likely conclusion by (4b), and as a mere possibility, something not to be discarded, by (4c). In other words, given the available evidence, the *ought* sentence express an epistemic bias by the speaker towards the truth of John being in his office, a bias which is boosted by replacing *ought* by *must* (and which vanishes with the use of *may*):

- (5) John ought to be in his office. In fact, he must be.

While English expresses weak necessity by dedicated modal verbs *ought* and *should*, many other languages express a similar notion by a morphological

marking on their strong necessity modals. This was one of the core findings in von Fintel & Iatridou's (2008) seminal paper on the crosslinguistic encoding of weak necessity, followed up more recently by von Fintel & Iatridou (2023). It can be nicely illustrated by comparing languages as different from each other as Hungarian, Greek, and Spanish:

- (6) Péter-nek el *kell-ene* mosogat-ni-a az edény-ek-et, de senki Péter PART must-nA wash-INF the dishes but noone nem követeli meg tőle.
not require PART he
'Péter ought to do the dishes, but he is not obliged to.'
[Hungarian, von Fintel & Iatridou 2023: ex.58]
- (7) *Tha eprepe* na plinis ta piata ala dhen ise ipexreomenos na to FUT must.PST na wash the dishes but NEG are obliged NA it kanis.
do
'You ought to do the dishes but you are not obliged to do it.'
[Greek, von Fintel & Iatridou 2008: p. 120]
- (8) *Deberia* limpiar los platos, pero no estoy obligado.
must.COND clean the dishes but not am obliged
'I ought to do the dishes but I am not obliged.'
[Spanish, von Fintel & Iatridou 2023: ex.61a]

Thus we have a crosslinguistic contrast between English tripartite modal system comprising separate lexical roots expressing possibility, weak and strong necessity, and bipartite systems of several languages, comprising possibility and strong necessity modal roots, to which weak necessity can be added via morphological marking on strong necessity verbal roots.

Although the nature of this morphological marking may differ from language to language, another major point made by von Fintel and Iatridou is that the same marking often appears on the consequent of so-called counterfactual conditionals, as in (9)-(11), and also on bouletic verbs expressing unattainable desires, as in (12)-(14):

- (9) Ha János tudná a választ, Mari is *tudná* a választ.
if János know-nA the answer Mari too know-nA the answer
'If János knew the answer, Mari would know the answer.'
[Hungarian, von Fintel & Iatridou 2023: ex.9]

- (10) An efevge simera *tha eftane* tin ali evdhomadha.
 if left today FUT arrive.PST.IMP the other week
 ‘If he left today, he would arrive next week’
 [Greek, von Fintel & Iatridou 2008: p. 121]
- (11) Si fuera más alto *sería* un jugador de baloncesto.
 If be.PST.SUBJ more tall be.COND a player of basketball
 ‘If s/he was taller, s/he would be a basketball player.’
 [Spanish, von Fintel & Iatridou 2023: ex. 41]
- (12) *Szeretném* ha Marcsi tudná a választ.
 like-nA if Marcsi know-nA the answer
 ‘I wish Marcsi knew the answer.’
 [Hungarian, von Fintel & Iatridou 2023: ex.40]
- (13) *Tha ithela* na imun psiloteri.
 FUT want.PST na was taller
 ‘I wish I was taller.’
 [Greek, von Fintel & Iatridou 2008: p. 133]
- (14) *Querría* que fuera más alto de lo que es.
 want.COND that be.PST.SUBJ more tall than it that be.PRES
 ‘I wish s/he was taller than s/he is.’
 [Spanish, von Fintel & Iatridou 2023: ex.42]

This extra marking, which can show up on strong necessity modals, conditionals and desire verbs, was dubbed *X-marking* by von Fintel & Iatridou (2023). The authors have set a new research agenda aimed at understanding the semantics of X-marking and envisaging a possible common core underlying the combination of X and the roots to which they attach.

In this paper, we would like to broaden the empirical landscape of X-marking and the expression of weak necessity by bringing up data from Portuguese which we believe can shed light on theoretical issues connected to this agenda.¹ The main points of the paper are:

- Weak necessity modals in Portuguese do not fit either of the two main crosslinguistic patterns identified by von Fintel & Iatridou (2008, 2023). The language contains lexicalized weak necessity modals but

¹ All Portuguese data in this paper are from Brazilian Portuguese. However, to the best of our knowledge, there are no relevant differences between Brazilian and European varieties concerning the central judgments and the empirical domain under discussion.

also distinguishes between morphologically X-marked and non-X-marked versions of them.

- In Portuguese, both weak and strong necessity modals can be X-marked. However, neither does X-marking on strong necessity modals yield an interpretation akin to weak necessity, nor does X-marking on (already) weak necessity modals yield an even weaker necessity.
- English weak necessity modals are ambiguous between X-marked and non-X-marked versions, with readings corresponding to forms that are overtly distinguished in Portuguese.
- Necessity modals can host two types of X-marking: one targeting the modal base and the other the ordering source. In both cases, X-marking acts as a parameter shifter. This gives rise to four semantically related necessity modalities (the square of necessities), and Portuguese instantiates all of them.

The paper is organized as follows: in Section 2, we introduce Portuguese modal verb *dever* and show that it carries the characteristic semantic and pragmatic marks of weak necessity. In Section 3, we start reviewing [Stalnaker's \(1975\)](#) insight that conditional constructions may contain conventional devices which indicate suspension of some default assumption, as well as [von Stechow & Iatridou's \(2023\)](#) rendition of Stalnaker's original work via the notion of X-marking. Then, we bring Portuguese *dever* and English *ought/should* to the scene, claiming that their modal bases can be target by X-marking either overtly (Portuguese) or covertly (English). In Section 4, we discuss X-marking which targets the ordering source of strong necessity modals, weakening their modal force. We suggest a formal implementation which approximates both kinds of X-markings, generating a square of semantically related necessities. Finally, in Section 5 we offer a brief summary and highlight some open issues which deserve further investigation.

2 Portuguese *dever* as a weak necessity modal

We start by introducing Portuguese modal verb *dever*, which we claim is a typical weak necessity modal operator, and its companions *poder* and *ter*

que, which express possibility and strong necessity, respectively.^{2,3} We begin with epistemic readings: having analyzed all the evidence relating to a man’s body found in a dark alley, a criminal investigator could announce his findings about the case in the following ways:⁴

- (15) a. Este homem *tem que* ter sido assassinado.
 This man SN.PRES have been murdered
 ‘This man must have been murdered.’
- b. Este homem *deve* ter sido assassinado.
 This man WN.PRES have been murdered
 ‘This man ought to have been murdered.’
- c. Este homem *pode* ter sido assassinado.
 This man POS.PRES have been murdered
 ‘This man may have been murdered.’

Intuitions are very clear. (15a) leaves no room for an alternative conclusion: given all the evidence, it cannot be the case that the man died from an accident, a heart attack, etc. (15b) implies that the most likely cause of death was murder, but does not dismiss the possibility of alternatives, and (15c) presents murdering as a mere possibility. Thus, while (16) below sounds contradictory, both (17) and (18) sound perfectly fine and consistent:⁵

- (16) *ter que p & poder ¬p*

#Este homem *tem que* ter sido assassinado, mas ele *pode* não
 This man SN.PRES have been murdered but he POS.PRES not
 ter sido.
 have been

² For a discussion of entailment relations involving Portuguese modal verbs, see Pessotto 2014.

³ The forms *poder*, *dever*, and *ter que* are infinitival forms, also used as citation forms for Portuguese verbs. For a comprehensive reference grammar of Portuguese, see Raposo et al. 2013.

⁴ The following abbreviations will be used in our glosses: WN: weak necessity modal; SN: strong necessity modal; POS: possibility modal; PRES: present; PST: past; IMP: imperfective; SUBJ: subjunctive.

⁵ We use # to mark examples which are grammatically correct, but semantically and/or pragmatically anomalous.

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(17) *dever p & poder ¬p*

Este homem *deve* ter sido assassinado, mas ele *pode* não
This man WN.PRES have been murdered but he POS.PRES not
ter sido.
have been

(18) *poder p & poder ¬p*

Este homem *pode* ter sido assassinado, mas ele *pode* não
This man POS.PRES have been murdered but he POS.PRES not
ter sido.
have been

Along with these intuitions come judgments about meaning relations among (15a)-(15c), suggesting that (15a) asymmetrically entails (15b) and (15c), and that (15b) asymmetrically entails (15c):

(19) *ter que p & ¬dever p*

#Este homem *tem que* ter sido assassinado, mas ele não *deve*
This man SN.PRES have been murdered but he not WN.PRES
ter sido.
have been

(20) *dever p & ¬ter que p*

Este homem *deve* ter sido assassinado, mas ele não *tem que*
This man WN.PRES have been murdered but he not SN.PRES
ter sido.
have been

(21) *dever p & ¬poder p*

#Este homem *deve* ter sido assassinado, mas ele não *pode*
This man WN.PRES have been murdered but he not POS.PRES
ter sido.
have been

(22) *poder p & ¬dever p*

Este homem *pode* ter sido assassinado, mas não *deve* ter
This man POS.PRES have been murdered but not WN.PRES have
sido.
been

Moreover, while possibility *poder* can be consistently conjoined with a minimal pair containing a negated prejacet, neither *dever* nor *ter que* can, the resulting sentences being blatantly contradictory:

(23) *poder p & poder ¬p*

Este homem *pode* ter sido assassinado, mas ele *pode* não
 This man POS.PRES have been murdered but he POS.PRES not
 ter sido.
 have been

(24) *dever p & dever ¬p*

#Este homem *deve* ter sido assassinado, mas ele *deve* não
 This man WN.PRES have been murdered but he WN.PRES not
 ter sido.
 have been

(25) *ter que p & ter que ¬p*

#Este homem *tem que* ter sido assassinado, mas ele *tem que* não
 This man SN.PRES have been murdered but he SN.PRES not
 ter sido.
 have been

Similar considerations can be made about non-epistemic readings, for which paradigms parallel to the ones we have seen in (15)-(25) can be easily constructed, and clear judgments obtained. We briefly illustrate the basic facts here with a teleological, goal-oriented reading: Suppose Mary is downtown and needs to go to the airport to catch a flight that departs in a few hours. Since she is not familiar with the local transportation system, she asks some local person, who offers her some help. (26a)-(26) illustrate three possible relevant answers that she may get:

- (26) a. Você *tem que* pegar um taxi.
 You SN.PRES take a cab
 ‘You have to take a cab.’
 b. Você *deve* pegar um taxi.
 You WN.PRES take a cab
 ‘You ought to take a cab.’
 c. Você *pode* pegar um taxi.
 You POS.PRES take a cab
 ‘You can take a cab.’

Intuitions are very clear: (26a) conveys that taking a cab is the only way to get to her destination. (26b) sounds like a piece of advice: although there are other means to get to the airport, taking a cab is the best option. And (26) only says that taking a cab is one possible way (among others) to get there. The paradigms and judgments above reveal that *dever* fits the test proposed in Rubinstein 2021 as a working definition for weak necessity modals:

- (27) *Weak Necessity (WN)* [Rubinstein 2021: p. 3156]
A modal word o is a WN modal iff for any proposition q and holding the type of modality constant:
- i. $o(q)$ is entailed by $n(q)$ but not vice versa, for some necessity modal n , and
 - ii. the conjunction $o(q)$ and $o(\neg q)$ is a contradiction as the conclusion of a deliberation.

The emerging picture is the familiar scale of ascending modal force as we move from *poder* to *dever* to *ter que*. Fixing the conversational backgrounds and whatever contextual parameters which determine the modal flavor, the modal sentences seem to be related by (asymmetric) entailment:

- (28) *Ascending scale of modal force:*
 $poder\ p <\ dever\ p < ter\ que\ p$

- (29) *Entailments:*
 $ter\ que\ p \models dever\ p \models poder\ p$
 $poder\ p \not\models dever\ p \not\models ter\ que\ p$

Finally, judgments typically found in the literature about scalar implicatures are easily reproduced with our triplet of modals, with *poder p* conversationally implying $\neg\textit{dever p}$ and *dever p* conversationally implying $\neg\textit{ter que p}$. Conversational implicatures, as widely assumed, can be reinforced, canceled, and suspended. We illustrate the relevant facts with deontic modality this time:

- (30) *Concerning the COVID-19 protocol of this establishment, ...*
- a. Clientes *devem* usar máscara, mas eles não *têm que*.
Clients WN.PRES use mask but they not SN.PRES
'Clients ought to wear a mask, but they don't have to.'

- b. Clientes *devem* usar máscara. Na verdade, eles *têm que*.
 Clients WN.PRES use mask In fact they SN.PRES
 ‘Clients ought to wear a mask. In fact, they have to.’
- c. Clientes *devem* usar máscara. Talvez eles *tenham que*.
 clients WN.PRES use mask Maybe they SN.PRES.SUBJ
 ‘Clients ought to wear a mask. Maybe they have to.’

Before concluding this section, it is worth mentioning that the Portuguese modal system differs in crucial respects from the modal systems of closely related romance languages, including Spanish, which has three modal verbs etymologically related to *poder*, *dever* and *ter que*. The core observation is that Spanish *deber* and *tener que*, the morphological counterparts of Portuguese *dever* and *ter que*, both behave as strong necessity modals, as can be seen in data such as (31), taken from von Fintel & Iatridou (2023):

- (31) *Spanish* [von Fintel & Iatridou 2023: ex.60]
- a. #Tengo que limpiar los platos, pero no estoy obligado.
 have COMP clean the dishes but not am obliged
 - b. #Debo limpiar los platos, pero no estoy obligado.
 must clean the dishes but not am obliged

Transposing the examples into Portuguese produces a contrast, suggesting once again that *dever* is weaker than *ter que*:

- (32) *Portuguese*
- a. #Tenho que limpar os pratos, mas não estou obrigado.
 I_have to clean the dishes but not am obliged
 ‘I must clean the dishes, but I am not obliged.’
 - b. Devo limpar os pratos, mas não estou obrigado.
 I_ought clean the dishes but not am obliged
 ‘I ought to clean the dishes, but I am not obliged.’

3 X-marked weak necessity modals: Portuguese and English

In the previous section we argued that Portuguese *dever* is a weak necessity modal, which puts it on a pair with English *ought* and *should* in terms of modal strength. Both *dever* and *ought/should* are flexible in terms of modal flavors, being weaker than strong necessity modals *ter que* and *must/have to*, and stronger than possibility modals *poder* and *may/can*.

In this section we argue that both *dever* and *ought/should* can be X-marked in the sense of von Stechow & Iatridou (2023), a notion that has its roots in Stalnaker's (1975, 2014) work on the distinction between so-called indicative and subjunctive (counterfactual) conditionals, and his idea of a conventional device which indicates the suspension of some presupposition while performing a modal assertion. We claim that X-marking in English weak necessity modals *ought/should* has no overt exponent, which creates an ambiguity, whereas X-marking in Portuguese *dever* is realized as past imperfect morphology, the same morphology which appears in the consequent of X-marked conditionals in the language. We will start with a brief presentation of Stalnaker/von Stechow and Iatridou's main points on X-marking in conditionals (Section 3.1) which will set the stage for our proposal about x-marking on epistemic (Section 3.2) and non-epistemic (Section 3.3) weak necessity modals.

3.1 Preliminary: X-marked conditionals

According to Stalnaker (1968) (see also Stalnaker & Thomason 1970), a conditional *if A, (then) C* always express a proposition which is true in a possible world w if, and only if, the consequent C is true in the world in which the antecedent A is true which is most similar to w . Thus, both (33) and (34), when uttered in the actual world w_0 express that the world in which John is in the building which is most similar to w_0 is a world in which he is in his office:

(33) If John is in the building, he is in his office.

(34) If John were in the building, he would be in his office.

Stalnaker (1975, 2014) proposes that the difference between (33) and (34) comes from a pragmatic defeasible presumption according to which the selected worlds at which the truth of the consequent is to be evaluated should belong to the context set of a conversation at the moment the conditional is uttered. The context set is the set of worlds compatible with everything the participants are presupposing (the common ground) at a given moment. Indicative conditionals such as (33) are unmarked and understood as complying to this pragmatic presumption. Its assertion would be suitable for instance in a context in which John's whereabouts is an open issue, and the speaker attempts to eliminate from the context set the possibility that John

is the building but not in his office. Subjunctive conditionals such as (34) are marked structures. This extra marking (past tense) indicates that the pragmatic default is being suspended and that the selected worlds may reach outside the context set. Asserting (34) would be suitable for instance in a context in which the participants are taking for granted that John is not in the building.

If we follow Stalnaker and model the common ground of a conversation as a set of propositions, we may say that subjunctive conditionals carry a conventional marking signaling that some proposition(s) belonging to this common ground is (are) being temporarily suspended for the evaluation of the consequent.

As for the propositions which are being suspended in the case of subjunctive conditionals, it may be the negation of the antecedent of the conditional, in which case we would have a bona fide counterfactual hypothesis, as in (35) below, but it can also be another proposition, as in cases in which the truth of the antecedent is under discussion and the speaker may be arguing either in support of it, as in (36), or against it, as in (37):

(35) Unfortunately John isn't here. If he were here, we would be happy.

(36) If the butler had done it, we would have found just the clues which we in fact found.

[Stalnaker 1975, example adapted from Anderson 1951]

(37) The murderer used an ice pick. But if the butler had done it, he wouldn't have used an ice pick. So, the murderer must have been someone else.

[Stalnaker 1975, example credited to John Watling]

Whereas in (35) the antecedent of the conditional is indeed presupposed to be false, in (36) and (37) it does not seem to be. As pointed out by Stalnaker, presupposing that a proposition is false while arguing for it would be self-defeating, and presupposing that it is false while arguing against it would be begging the question. Thus what (35), (36), and (37) have in common and which seem to motivate the use of a marked conditional is the suspension of some propositions belonging to the common ground, i.e, some propositions that the participants are taking for granted at the moment the conditional is uttered. It is to this derived context, modeled after a premise set which excludes these propositions that the antecedent is added, and the hypothetical reasoning expressed by the conditional proceeds by checking the truth

of the consequent. In (35), the proposition would be that John isn't here. In (36), it would be the proposition that we have found the clues we did. And in (37), it would be the proposition that the murderer used an ice-pick.⁶

As for the nature of the extra morphosyntactic marking, there is crosslinguistic variation, as documented by von Fintel & Iatridou (2023) and briefly reviewed in the introduction to this paper. English, as can be seen in (35)-(37) above, uses the past tense, whereas Portuguese, as can be seen below, uses past subjunctive in the antecedent and past imperfect in the consequent:⁷

(38) Se Pedro estivesse aqui, ele *estava* feliz.
if Pedro be.PST.SUBJ here he be.PST.IMP happy
'If Pedro were here, he would be happy.'

(39) Se eu ganhasse na loteria, eu *comprava* um carro novo.
if I win.PST.SUBJ in_the lottery I buy.PST.IMP a car new
'If I won the lottery, I would buy a new car.'

This extra marking was dubbed X-MARKING by von Fintel & Iatridou (2023), who avoided associating it with specific grammatical categories. Following von Fintel & Iatridou (2023) and von Fintel (1998), we opt here for a formal implementation of Stalnaker's idea using a Kratzerian framework for conditionals, according to which *if*-clauses interact with a modal base *f* and an ordering source *g* to restrict a (possibly covert) modal quantifier. Technically, *f* and *g* are functions from worlds (the world of evaluation) to sets of

⁶ This line of analysis may be extended to cases involving so-called future less vivid conditionals, which are future oriented conditionals with an antecedent describing an eventuality whose occurrence is taken to be unlikely:

(i) If he took the medicine, he would get better. [Iatridou 2000]

The extra marking here might be indicating the suspension of a bias against the truth of the antecedent. It would signal that the modal quantification is reaching outside some privileged zone of the common ground or the epistemic alternatives of the speaker (the most likely worlds). Implementing such an analysis would require a sort of context probabilism (Yalcin 2012) which would assign probabilities to subsets of the worlds selected by the context set, but we do not intend to pursue it here.

⁷ The past subjunctive forms in the antecedent are traditionally called 'pretérito do subjuntivo' and the past indicative forms in the consequent are called 'pretérito imperfeito'. Verb forms in the so-called 'conditional tense' (also called 'futuro do pretérito' (future of the past) in some traditional grammars) would also be possible in the consequent, with no obvious shift in meaning, except that for many speakers the use of conditional tense sound more formal.

propositions. When applied to a world w , f and g deliver the sets of propositions f_w and g_w , respectively. Together with the proposition expressed by the antecedent A , the job of the modal base is to pre-select a set of worlds ($\bigcap(f_w \cup \{A\})$) which will then be ranked by the ordering source. In the examples we are discussing, the modal quantifier would be universal, the modal base would be the common ground cg , and the ordering source would be totally realistic, ranking worlds based on how similar they are to the actual world (SIM_w). For non-X-marked conditionals, we have the following schema:⁸

(40) *Non-X-marked ('indicative') conditionals*

$$\llbracket \text{if } A, C \rrbracket = \lambda w. \forall w' \in \text{BEST}_{SIM_w} \bigcap (cg \cup \{A\}) : w' \in \llbracket C \rrbracket$$

- (41) a. SIM_w is a set of propositions which uniquely characterizes w :
for any world w , $\bigcap SIM_w = \{w\}$
b. For any set (domain) of worlds D , set of propositions O , and worlds u, v :
- (i) $\text{BEST}_O D = \{w' \mid \neg \exists w'' \in D : w'' \leq_O w' \wedge w' \not\leq_O w''\}$
 - (ii) $u \leq_O v$ iff $[\forall p \in O : \text{if } v \in p, \text{ then } u \in p]$

As for X-marked conditionals, the idea of presupposition suspension can be modeled as a shift in the modal base (the common ground cg) obeying the following constraint: the shifted modal base, which we will represent by cg^* , should be such that the context set it delivers ($\bigcap cg^*$) be a proper superset of the original one ($\bigcap cg$):⁹

(42) *X-marked ('subjunctive') conditionals*

$$\llbracket \text{if}_X A, C \rrbracket = \lambda w. \forall w' \in \text{BEST}_{SIM_w} \bigcap (cg^* \cup \{A\}) : w' \in \llbracket C \rrbracket$$

⁸ As pointed out to me by Anthony Gillies, this Kratzerian rendition departs in an important respect from Stalnaker's (1968) original formulation, since the induced ordering permits ties and incomparabilities in what worlds in a set are best.

⁹ We avoid committing ourselves to any formal relation imposed directly on f/cg , since that would require additional assumptions about f/cg which would be under-motivated in the present context. For instance, one might propose that the shifted modal base be a proper subset of the common ground (Mackay 2019). However, this might not be enough to guarantee a proper extension of the context set. Take, for instance, the set $\{p, p \cap q\}$ and its proper subset $\{p \cap q\}$, both of which entail p and q . Rather, in order to guarantee that any proper subset of a modal base f_w will result in a larger intersection set $\bigcap f_w$, one should assume that the modal base is non-redundant in the following sense (see Kratzer 2012: p. 132): a set of propositions is redundant if it contains propositions p and q such that $p \neq q$ and $p \subseteq q$. In other words, a non-redundant modal base does not contain both a proposition and its non-trivial logical consequences. Whether or not this is a natural assumption is a question which we do not intend to address here.

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(43) for any modal base f and world w : $\bigcap f_w \subset \bigcap f_w^*$.

With this much in mind, we will then return immediately to our discussion of Portuguese *dever* and English *ought* and *should*.

3.2 Epistemic necessity modals

In this section we will focus on epistemic weak necessity which can be used to express some (tentative) conclusion or expectation based on available evidence. English epistemic *ought/should* have been shown to be particularly complex, generating recalcitrant data and standing out in their intricacies from their epistemic companions *may/might/must/have to* (cf. Copley (2006), Yalcin (2016), and Hawthorne (2021), *inter alia*). As we will see shortly, a comparison with epistemic uses of Portuguese *dever* together with the notion of X-marking as just reviewed will indicate some new prospects for the analysis of their modal profile.

We begin with von Stechow & Iatridou's (2008: p. 126) quick pass on epistemic uses of English *ought*. They provide the following context and example:

- (44) Let's say you are on your way to Morris's office, which is down the hall from mine, and ask me whether I think that Morris is in his office. Neither of us knows whether he is, in fact, there.
- (45) It's 3pm. Given what I know about Morris's habits, *he ought to be in his office*. Why don't you go check?

Appropriate paraphrases for cases like this include:

- (46) a. He is likely to be in his office.
b. He is more likely to be there than not to be there.
c. It is probable that he is in his office.

But things get more complicated and interesting when we widen our dataset:

- (47) Morris ought to be here by now, but he isn't.
- (48) The beer should be cold by now, but it isn't. [Copley 2006]
- (49) They left an hour ago, and there isn't any traffic. So they should be here by now. But they're not. [Swanson 2008]

The examples show that the conjunction of *ought/should p* and *not p* isn't always inconsistent. This is not the case with other epistemic modals:

- (50) a. #The beer must be cold by now, but it isn't.
 b. #The beer may be cold by now, but it isn't. [Copley 2006]
- (51) a. #They left an hour ago, and there isn't any traffic. So they might be here by now. But they're not.
 b. #They left an hour ago, and there isn't any traffic. So they probably are here by now. But they're not. [Yalcin 2016]

As has been frequently remarked in the literature, this is unexpected if *ought/should* merely express weak epistemic necessity of the same flavor that *may* and *must* do.

Let us keep this in mind and shift back to Portuguese and its weak necessity modal *dever*. The paradigm we will try to replicate is (52), which sets *ought* apart from both *may* and *must* and displays the possibility of consistently conjoining *ought p* and *not p*:

- (52) a. ✓ *ought p* & $\neg p$
 He ought to be here by now, but he isn't.
 b. ✗ *may p* & $\neg p$
 #He may be here by now, but he isn't.
 c. ✗ *must p* & $\neg p$
 #He must be here by now, but he isn't.

When we turn to Portuguese, we notice that the contrast between possibility and strong necessity on one side and weak necessity on the other vanishes if we control for tense marking. First, all present tense versions of the three sentences above sound incoherent and pragmatically inadequate under an epistemic reading:

- (53) a. #Ele *deve* estar aqui agora, mas não está.
 he WN.PRES be here now but not is
 b. #Ele *pode* estar aqui agora, mas não está.
 he POS.PRES be here now but not is
 c. #Ele *tem que* estar aqui agora, mas não está.
 he SN.PRES be here now but not is

A square of necessities

However, switching to past tense modals makes all three sentences sound coherent and perfectly fine:¹⁰

- (54) a. Ele *devia* estar aqui agora, mas não está.
he WN.PST.IMP be here now but not is
b. Ele *podia* estar aqui agora, mas não está.
he POS.PST.IMP be here now but not is
c. Ele *tinha que* estar aqui agora, mas não está.
he SN.PST.IMP be here now but not is

Comparing (52) and (53)-(54), we see that epistemic *may* and *must/have to* side with present tense *poder* and *ter que*, while epistemic *ought* sides with past imperfect *dever*.¹¹

However, this isn't always the case. Returning to von Stechow and Iatridou's example from the beginning of this section, we have (55) uttered in a context in which you ask me about Morris' whereabouts and neither of us knows for sure where he is:

- (55) He ought to be in his office.

In this case, present tense *dever* (as well as present tense *poder* and *ter que*) is fine whereas past tense *dever* sounds awkward:

- (56) [I suspect Morris is in his office, but I don't know for sure]
a. Ele deve estar na sala dele.
he WN.PRES be in_the office his
b. #Ele devia estar na sala dele.
he WN.PST.IMP be in_the office his

What is behind the present/past tense split in Portuguese and the corresponding flexibility of epistemic *ought* in English? To begin addressing this question it might be useful to consider the presumably related fact that two tokens of epistemic *ought* can occur close to each other in a discourse but with apparently different meanings, as in the following passage from Thomson (2008), quoted in Yalcin 2016: p. 233:

¹⁰ For discussion of past tense marking on the possibility modal *poder*, see Pessotto 2011.

¹¹ As we noted before when discussing conditionals, here too the past imperfect forms could be replaced by conditional tense with no obvious shift in meaning.

Consider Rasputin. He was hard to kill. First his assassins poisoned him, then they shot him, then they finally drowned him. Let us imagine that we were there. Let us suppose that the assassins fed him pastries dosed with a powerful, fast-acting poison, and then left him alone for a while, telling him they would be back in half an hour. Half an hour later, one of the assassins said to the others, confidently, “He ought to be dead by now.” The others agreed, and they went to look. Rasputin opened his eyes and glared at them. “He ought to be dead by now!” they said, astonished. It might be thought that when they first said the words, they meant that it was then probable that he was dead. Not so when they second said the words. By the time they second said the words, they knew perfectly well that he wasn’t dead.

[...] what it calls for is simply that we distinguish: if I say “The car keys ought to be on the hall table,” then I assert different propositions, according as my state of knowledge is different. If (i) I don’t know that the car keys are, or that they aren’t, on the hall table, then if I say “They ought to be on the hall table,” what I mean is that it is probable that they are there. If (ii) I know that they aren’t there, then if I say “They ought to be on the hall table,” what I mean is that it *was* probable that they would be there.

[Thomson 2008: pp. 202–203]

Here too we observe a split when we switch from English to Portuguese: the first occurrence of *ought* is translated into present tense *deve* whereas the second one translates into past tense *devia*:

- (57) *not knowing whether Rasputin is dead*
 - a. He ought to be dead.
 - b. Ele *deve*/#*devia* estar morto.
- (58) *having found out that Rasputin is alive*
 - a. He ought to be dead.
 - b. Ele *devia*/#*deve* estar morto.

Thus, Portuguese provides an overt case for the idea voiced by Thomson for two different propositions being expressed by the *ought* sentences in

the Rasputin scenario. Moreover, as also proposed by Thomson for English, the difference between the two *oughts* seem to be related to the temporal perspective associated with the modality they express: (57) is based on the interlocutors present epistemic state, while (58) reports on a past epistemic state that no longer matches the speaker's present state of knowledge.

Contrasts of this sort can also be easily replicated with Portuguese possibility and strong necessity modals. Suppose, for instance, that a nurse has given a patient a sedative which takes effect between one and two hours. Having left the patient alone in his room, one hour later she says (59) to herself:

- (59) Ele já *pode* estar dormindo. Eu vou checar.
he already POS.PRES be sleeping I will check

Then she goes to his room to check and notices that he is still awake. Now she says (60) to herself:

- (60) Ele já *podia* estar dormindo. Vou voltar em quinze minutos.
he already POS.PST.IMP be sleeping I_will return in fifteen minutes

As for strong necessity modals, a minimally different story could be told about a very powerful sedative which is guaranteed to take effect in no longer than one hour. The nurse's words to herself would now be as in (61) and (62), before and after she checks the patient, respectively:

- (61) Ele já *tem que* estar dormindo. Eu vou checar.
he already SN.PRES be sleeping I will check
- (62) Ele já *tinha que* estar dormindo. Há algo errado.
he already SN.PST.IMP be sleeping there_is something wrong

Assuming that this flexibility in temporal perspective attributed to English *ought* does not extend to *may* and *must* would then explain why [*ought p and not p*] sounds consistent whereas [*may/must p and not p*] does not. That this contrast is related to temporal perspective and not to weak necessity *per se* is evidenced by the Portuguese data, in which possibility, weak necessity and strong necessity modals can all be marked for tense and express either a past or a present modal perspective. Present tense perspective yields inconsistent conjunctions, but past tense perspective does not.

At this point it is natural to assume that from a grammatical point of view there are two *oughts* in English, sharing the same modal core, a root morpheme expressing weak necessity modality (WN), and differing in their temporal perspective, either present or past. Portuguese would then be minimally contrasting, with each morphological complex being spelled out differently.¹²

- (63) First pass on English *ought* vs. Portuguese *dever*
- a. *English*
 ought₁: WN+present tense
 ought₂: WN+past tense
 - b. *Portuguese*
 deve: WN+present tense
 devia: WN+past tense

However, as is so often the case with the interaction between tense and modality, things are more complicated than they appear to be and as we will see now tense marking might not be doing its usual job of temporal location in these examples. The point we would like to make is that reference to a past epistemic state expressing a previous expectation or bias towards the truth of its prejacent *p* is neither necessary nor sufficient for the felicity and truth of an utterance of a past tense weak necessity sentence *devia p*.

That a past expectation is not necessary can be shown with the following scenario: a nurse is starting her shift and is about to enter a room to check on one of her patients. Without knowing the patient's conditions, she enters the room. After opening the door, she notices that he is awake and greets him. She then reads his medical records which says that he has taken a powerful sedative one hour before and which normally takes effect in about 50-60 minutes. She then says (64) to him:

- (64) Você *devia* estar dormindo.
 you WN.PST.IMP be sleeping
 'You ought to be sleeping.'

¹² As pointed out by an anonymous reviewer, English weak necessity modals *ought* and *should* are historically descended from past tense forms (*ought* was the past tense of the ancestor of *owe* and similarly with *should* and *shall*), a fact that might add to the plausibility of the analysis. However, as the reviewer stressed, the import of etymological points in contemporary semantics is a debatable matter.

Notice that at no point prior to her utterance she expected the patient to be sleeping. At first, she was not opinionated and then she knew he was not sleeping. In other words, at no point prior to the moment of her utterance she was entitled to expect that he was or would be sleeping. Nevertheless her utterance with past tense *devia* is felicitous and intuitively true.

It might be argued that the major piece of evidence on which the modal claim in (64) was made is the past occurrence of an event (the patient took the sedative an hour before) and that (64) is a case of metaphysical or circumstantial modality expressing that a past event determined a normal future course of events according to which the patient would be sleeping in an hour or so. But this is questionable, since a slight variation on the example can be constructed in which the relevant evidence is tied to the current state of the patient. For instance, the nurse might be looking at a sophisticated monitor next to the patient's bed which is displaying some sort of brain wave typical of sleeping periods. Knowing that he is awake, she utters (64). In this modified scenario there is no salient past event nor any prior expectation that the patient would be sleeping, and yet (64) is still appropriate and true.

That a past expectation is not sufficient is made clear by contrasts such as the following:

- (65) A: Where is Peter?
B: Probably in his office.
A: But today is a holiday!
B: Oh, I didn't know it was ...
B: #(*É por isso que*) *ele devia estar lá.*
(That's why) he WN.PST.IMP be there
[cf. ✓*That's why I expected him to be there.*]
- (66) A: Where is Peter?
B: Probably in his office.
A: I have just checked and he isn't there.
B: *Estranho!* ✓ *Ele devia estar lá.*
strange he WN.PST.IMP be there

In both situations, there was a point prior to B's second utterance at which speaker B expected Pedro to be in his office and might even have uttered a modal sentence with present tense *deve*. However, only in (66) is past tense *devia* acceptable. Notice moreover that in both situations at the moment B makes his final remarks he no longer expects the prejacent to be true. If contextual salience of a past epistemic state which favored the truth of the

prejacent and which no longer holds were sufficient for the licensing of the past tense necessity modal, there would have been no contrast between (65) and (66).

What then is the licensing factor for the use of the past tense in (64)? And what is behind the contrast between (65) and (66)?

We will start with three negative answers. The first, already anticipated by the discussion above, is that past imperfect *devia* is not a necessity modal under the immediate scope of a temporal operator that changes the modal perspective to the past, as in (67):

(67) [PRET [MODAL_{epis} *p*]]

This is indeed possible with Portuguese epistemic modals when they are preceded by expressions that make explicit the past perspective:

(68) Dado tudo que a polícia sabia até aquele momento, o autor
 given all that the police knew until that moment, the author
 do atentado *devia* ser o Pedro.
 of_the attack WN.PST.IMP be the Pedro.
 ‘Given all the police knew at that point, it was likely (at that moment)
 that the author of the attack was Pedro.’

However, as we have seen already, in the context we discussed, (64) is not a case of past perspective.

The second negative answer is that (64) is not a past tense under the scope of a necessity modal:

(69) [MODAL_{epis} [PRET *p*]]

Such cases of temporal raising (Stowell 2004) in which the imperfect tense morphology appears attached to the necessity modal, but expresses the past orientation of the prejacent, are also possible in Portuguese:

(70) Dado tudo que a polícia sabe agora, o paciente devia
 given all that the police knows now the patient WN.PST.IMP
 estar dormindo (naquele momento).
 be sleeping on_that moment
 ‘Given all the police knows, it is likely that the patient was sleeping
 then.’

The interpretation is that, given what the police knows, it is likely (in the present) that the patient was sleeping (in the past). However, it is clear that we were not dealing with a past-oriented prejacent in (64). (64) did not mean that it is likely that the patient was sleeping.

At last, a third negative answer: In all the cases we had presented before (see (59)-(64)), the modals in the past imperfect had been used in contexts in which the prejacent was known to be false. Moreover, as we saw in Section 3.1, this very same morphological marking appears on the inflected verb of the consequent of Portuguese conditionals expressing counterfactual modality, even when the situations under discussion are not located in the past:

- (71) Se Pedro estivesse aqui agora, ele *estava* feliz.
if Pedro BE.PST.SUBJ here now he BE.PST.IMP happy
'If Pedro were here now, he would be happy.'

Therefore, it is conceivable that the cases we had been analyzing are part of a more general paradigm of non-temporal uses of past imperfect morphology in which our modal verb *dever* appears in the consequent of a counterfactual conditional structure with an implied antecedent. In such cases, *dever* would be embedded under a covert modal, and the consequent would express weak epistemic necessity in some 'counterfactual' worlds:

- (72) If ..., then [MODAL_{CF} [*dever* p]]

However, there is solid evidence that this is not the case. Still confining ourselves to epistemic necessity, consider (73) and (74), both uttered in a scenario where I don't know for sure if Pedro is his office, although I expect him to be there. Then I go there to check but what I see is an empty room:

- (73) O Pedro *devia* estar aqui.
the Pedro WN.PST.IMP be here
'Pedro ought to be here.'

- (74) ??Se eu não estivesse vendo esta sala vazia, o Pedro *devia*
If I not were seeing this room empty the Pedro WN.PST.IMP
estar aqui.
be here
'??If I were not seeing this empty room, Pedro ought to be here.'

(73), as we have seen already, is a perfectly fine WN statement. (74) is an unsuccessful attempt to paraphrase (73) with a conditional structure based

on a counterfactual antecedent expressing the negation of some salient piece of evidence or knowledge.¹³ Notice that this could be easily achieved if we used a propositional attitude verb in the main clause:

- (75) Se eu não estivesse vendo a sala vazia, eu ia achar que o Pedro estava aqui.
 If I not were seeing the room empty I would think that the Pedro was here
 'If I were not seeing this empty room, I would be thinking that Pedro was here.'

With a modal verb, however, this cannot be done. (74) sounds pretty awkward, contrasting sharply with (73). Epistemic strong necessity modal *ter que* also provides contrasting pairs analogous to (73)-(74):

- (76) O Pedro *tinha que* estar aqui.
 the Pedro SN.PST.IMP be here
- (77) ??Se eu não estivesse vendo essa sala vazia, o Pedro *tinha que* estar aqui.
 If I not were seeing this room empty the Pedro SN.PST.IMP be here

The upshot is that none of the instances of necessity modals marked for past imperfect which we discussed in this section stand for a conditional structure with an implicit antecedent and a dominating modal expressing counterfactuality.

Back to our original question, what then is the licensing factor for the use of the past tense in examples like (64) (and what is behind the contrast be-

¹³ This remark is based on von Fintel & Iatridou's (2023) analogous observations for WN in English and Greek. However, as they noticed, it is not easy to express weak modality in a counterfactual scenario in these languages. English *ought*, accompanied or not by *would* produces ungrammatical results:

- (i) *If you wanted to please your roommate, you would have to/ought/would ought to do the dishes.

[von Fintel & Iatridou 2023: fn.45]

As for Greek (and some other languages discussed by them), WN modals are already morphologically marked and do not allow for a second layer of marking. As we have shown here, Portuguese is more generous with a dedicated WN modal root which can be marked for past tense or not.

tween (65) and (66)? Our (positive) answer to these questions is that the past tense is indicating that some salient evidence bearing on the speaker's actual current epistemic state is being intentionally ignored and that the prejacent is being inferred based on this smaller premise set (plus some normality presumption which we will discuss later). This smaller premise set might match some past epistemic state of the speaker towards the prejacent, but, as we have seen, it does not have to. In the scenarios discussed above in connection to (64), the evidence that is being suspended is that the patient is awake (the negation of the prejacent). The speaker (the nurse, in those cases) is looking at the patient, talking to him, and it is absolutely clear that he is not sleeping. Were it not for this direct evidence against the prejacent p , the speaker would be entitled to expect or to have an epistemic bias towards the truth of the prejacent p .

Based on these facts, we propose that the past tense markings on Portuguese necessity modals *dever* and *ter que* are instances of X-marking signaling suspension of the belief/knowledge in the negation of the prejacent. In formal terms, X-marking acts as a domain shifter, replacing the modal base f by a revision of f for the prejacent p , which we will represent below as f^{*p} . The net effect of this revision is the widening of the set induced by f ($\cap f_w$) with the inclusion of some worlds in which the prejacent is true:¹⁴

(78) For any modal base f and ordering source g :

- a. $\llbracket \text{devia} \rrbracket^{f,g} = \lambda p. \lambda w. \llbracket \text{deve} \rrbracket^{f^{*p},g}(p)(w)$
- b. $\llbracket \text{tinha que} \rrbracket^{f,g} = \lambda p. \lambda w. \llbracket \text{tem que} \rrbracket^{f^{*p},g}(p)(w)$
- c. f^{*p} is a $*$ -revision of f for p .

(79) For any proposition p and modal bases f, f' :

f' is a $*$ -revision of f for p if, and only if, for any world w ,
 $\cap f'_w = \cap f_w \cup \{w' \mid \exists w'' \in \cap f_w : w' \in \text{BEST}_{\text{SIM}_{w''}}(p)\}$ ¹⁵

In the examples we have been discussing in this section, the modal base is epistemic, encoding the speaker's knowledge/belief (or all the evidence available to him) at utterance time in the world of evaluation. As for the ordering

¹⁴ I am indebted to an anonymous reviewer for their thoughts on how to constrain this revision of f and the suggestion of tying it directly to the negation of the prejacent.

¹⁵ $\text{BEST}_{\text{SIM}_{w''}}(p)$ is the set of p -worlds that are most similar to w'' . The revision of f for p will then add to $\cap f_w$ all p -worlds that are most similar to some world in $\cap f_w$. The idea comes from Grano & Phillips-Brown's (2022) article on desire-ascriptions, and was adapted in von Fintel & Iatridou's (2023) discussion of X-marked desires.

source it encodes an ideal of normality or plausibility.¹⁶ When compared to the present tense modals *deve* and *tem que*, the past tense modals *devia* and *tinha que* operate on the same ordering sources, but on shifted modal bases, signaling a modal reasoning based on suppression of the speaker's knowledge/belief in the negation of the prejacent.¹⁷

Finally, we can get back to the contrast between (65) and (66), which we reproduce below as (80) and (81):

- (80) A: Where is Peter?
 B: Probably in his office.
 A: But today is a holiday!
 B: Oh, I didn't know it was ...
 B: #(*É por isso que*) *ele devia estar lá.*
 (That's why) he WN.PST.IMP be there
 [*cf. ✓ That's why I expected him to be there.*]
- (81) A: Where is Peter?
 B: Probably in his office.
 A: I have just checked and he isn't there.
 B: *Estranho! ✓ Ele devia estar lá.*
 strange he WN.PST.IMP be there

In (81), suspending the belief/knowledge that Peter is not in his office reinstates a bias towards the truth of the prejacent and the use of the past tense *devia* is correctly predicted to be true. Contrastingly, in (80), even if we suspend the belief/knowledge that the Peter is not in his office, the previous information that the day was a holiday blocks the inference that he is, and the use of a past tense modal is correctly predicted to be false.¹⁸

¹⁶ The exact nature of this ordering source has been the subject of some controversies (see, for instance, Yalcin 2016). See also von Stechow & Gillies 2010, 2021 for arguments against the presence of an ordering source in the semantics of some epistemic strong necessity modals.

¹⁷ A proposal along similar lines has been made in Laca (2012) for conditional morphology on French and Spanish modals. However things get more complicated in those languages, since, as we saw for Spanish in the introduction, conditional mood can also be used to turn strong necessity modals into weak necessity ones (von Stechow & Iatridou 2008). We will return to this issue in Section 4 after we discuss von Stechow & Iatridou's (2008, 2023) formal analysis of the weak/strong necessity contrast, and introduce our square of necessities.

¹⁸ Notice that the contrast between (80)-(81) argues in favor of a revision of *f* for the prejacent of the modal and not for any contextually salient proposition. In particular, suspension of the evidence that the day was a holiday in (80) would reinstate a bias towards the truth of the prejacent, incorrectly predicting the truth of B's modal assertion.

We will return in Section 4 to a formal account of the difference in modal force between weak and strong necessity modals, but, as can already be noticed in (78), the past imperfect modals are not semantically weaker or stronger than their respective present tense versions. Although the domains ($\bigcap f_w^{*p}$) which will be passed on to the ordering source are indeed larger than their unmarked counterparts ($\bigcap f_w$), selection of the best worlds from these different sets delivered by the modal bases need not yield identical or even overlapping sets. As a consequence, there is no entailment relation between our pairs of present (*deve/tem que*) and past (*devia/tinha que*) modals:

- (82) a. *deve p* $\not\equiv$ *devia p*
 devia p $\not\equiv$ *deve p*
 b. *tem que p* $\not\equiv$ *tinha que p*
 tinha que p $\not\equiv$ *tem que p*

More generally, and summing up what we have proposed in this section, we see Portuguese as a language in which both weak and strong necessity modals can be X-marked, with the marking signaling a shift to a wider domain, but with no change in modal force:

- (83) *Portuguese X-marked necessity modals:*
 a. *devia*: WN_X (weak necessity+X-marking)
 b. *tinha que*: SN_X (strong necessity+X-marking)
- (84) For any modal base f and ordering source g :
 a. $\llbracket WN_X \rrbracket^{f,g} = \lambda p. \lambda w. \llbracket WN \rrbracket^{f^{*p},g}(p)(w)$
 b. $\llbracket SN_X \rrbracket^{f,g} = \lambda p. \lambda w. \llbracket SN \rrbracket^{f^{*p},g}(p)(w)$
 c. f^{*p} is a $*$ -revision of f for p .

3.3 Non-epistemic necessity

We now turn to non-epistemic readings of necessity modals, checking how our X-marking proposal extends to other modal flavors.

We will start with a brief excursus on Arregui's (2010) analysis of English *should* which focuses on deontic modality and the connection she makes with Stalnaker's idea of a conventional marking of presupposition suspension. Then we discuss Portuguese necessity modals taking advantage of their freedom in combining with both present and past tense morphology. As in

our discussion of epistemic necessity, this will give further support for our reassessment of the English data (as well as of Arregui’s analysis).

3.3.1 Arregui (2010) on *should*

In her important work on English (mostly deontic) *should*, Arregui (2010) contrasts simple *should* and *should have* statements explicitly linking the difference to the Stalnakerian idea of presupposition suspension and modal quantification over domains which may reach outside the context set. This of course puts her work in direct contact with our proposal for X-marked necessity modals in Portuguese and English. Here is one of her basic contrasts (Arregui 2010: p. 247):

- (85) a. Sara should return the library book on time.
 b. Sara should have returned the library book on time.

Arregui locates the relevant interpretive differences between simple *should* and *should have* in the aspectual make-up of the prejacent of the modal verb, more specifically in a contrast between perfective and perfect aspectual heads (Arregui 2010: p. 266):

- (86) a. Sara should return the library book on time. *Simple perfective*
 b. [should [$\emptyset_{\text{perfective}}$ [Sara return the library book on time]]]
 (87) a. Sara should have returned the library book on time. *Perfect*
 b. [should [have_{perfect} [Sara return the library book on time]]]

Following Kratzer (1998), she assumes that aspectual heads relate properties of events to properties of times. The covert perfective head in (86) encodes temporal inclusion, expressing that the running time of an event $\tau(e)$ is included in some reference time t . The innovative point of her aspectual analysis is an additional semantic-pragmatic ingredient, encoded as a presupposition triggered by $\emptyset_{\text{perfective}}$ (we underline this presupposition in (88), taken from Arregui (2010: p. 247)):

$$(88) \quad \llbracket \emptyset_{\text{perfective}} \rrbracket^c(P) = \lambda t. \lambda s : \frac{\exists w [w \in c \ \& \ s \leq w]}{\exists e [P(e)(s) = 1 \ \& \ \tau(e) \subset t]}^{19}$$

¹⁹ Arregui’s proposal is couched within a situations-based framework in which situations are parts of worlds, and events are a type of situation. Her terminology: P is a property of events, and $P(e)(s) = 1$ iff e is a P -event that occurs in s .

The presupposition indicates that the output function is defined only for (situations in) worlds in the context set c . Moreover, Arregui assumes that presuppositions in the nuclear scope of a modal verb can be accommodated within the restriction of the quantifier over possible worlds expressed by the verb. As a result, all the worlds being quantified over in a statement such as (86) will be presumed to be in the context set. Such a statement would be appropriate, for instance, in contexts in which the book she borrowed from the library is due on the next day and whether she will return it on time or not is an open issue. According to (86), returning the book on time is the best possibility within the worlds in c .

The perfect head in (87) induces no such presupposition. It only encodes temporal precedence, expressing that the running time of an event $\tau(e)$ precedes some reference time t (Arregui 2010: p. 268):

$$(89) \quad \llbracket \text{have}_{\text{perfect}} \rrbracket(P) = \lambda t. \lambda s. \exists e [P(e)(s) = 1 \ \& \ \tau(e) < t \ \& \ t \subset \tau(s)]$$

As a result, the worlds being quantified over in cases such as (87) may include worlds outside the context set. (87) may be appropriate, for instance, if the due date has passed, Sara can no longer return the book on time and a fine will apply.

Arregui's focus on the contrast between *should* and *should have* makes sense since English *should* does not inflect for tense (nor do other necessity modals *ought* and *must*) and, as examples such as (85) reveal, aspect matters. Her proposal allows for a compositional treatment of *should (have)* statements based on a single lexical entry for *should*.

As it is clear from the above presentation, Arregui's analysis builds on Stalnaker's insight of (not) reaching outside the context set while performing a modal statement. At the same time, she reverses the insight, so to speak. Whereas for Stalnaker what is conventionally marked in the contrast between indicative and subjunctive conditionals is the 'out-of-the-context-set' signal, with the 'within-the-context-set' presumption left unmarked and understood as a default, in Arregui's analysis it is the latter that is marked, encoded in her analysis as part of the meaning of a perfective head.

This would also contrast with von Stechow and Iatridou's X-marking, which subsumes Stalnaker's original insight, as well as with my extension of their ideas to Portuguese necessity modals and English epistemic *ought/should*. Clearly, it would be desirable to have a unifying analysis if we could cover both English non-epistemic *should* and Portuguese necessity modals *dever* and *ter que* under the same formal analysis.

We would like to claim that an alternative to Arregui’s analysis is possible which would bring us closer to Stalnaker’s and von Fintel and Iatridou’s proposals. It would also align perfectly with our previous discussion of epistemic *ought/should*. The proposal is that non-epistemic *should* can be X-marked, but that this optional marking does not have any phonetic content, which gives rise to an ambiguity. Their prejacent, whether containing perfect aspect or not, will carry no conventional signal (presuppositional or otherwise) regarding the Stalnakerian common ground/context set. As a direct consequence, both simple *should* and *should have* statements can express modal quantification whose domain is either within the context set or extending outside it. This, we believe, is a welcome result. As for simple *should*, we have seen examples in which a future oriented prejacent is an open issue:

(90) Sara should return the library book on time.

But similar examples can be given in which the prejacent is taken to be false or at least unlikely:

(91) A: Sara will not return the library book on time.
 B: That’s too bad! She should do it.

(92) Sara should return the library book on time, but she won’t.

Our analysis is that (90) is an instance of non-X-marked *should*, whereas both (91) and (92) are instances of X-marked *should*.²⁰ Since the prejacent is future oriented, the interpretations are expected to be akin to future less vivid X-marked conditionals, as discussed by von Fintel and Iatridou and reviewed before:

(93) I don’t think he will come to the party tonight. That’s too bad because if he came, he would have a good time.

[von Fintel & Iatridou 2023: ex.18a]

As for *should have*, we have seen examples of counterfactual prejacent:

(94) Sara should have returned the library book on time (but she didn’t).

²⁰ Spoiler to our next section: Portuguese most natural translations have present tense *deve* in (90) and past imperfect *devia* on (91)/(92).

But similar examples in which the prejacent is taken to be an unbiased open issue are easy to find, as in the following announcement about prerequisites for a course:

(95) Students should have taken at least one of the following modules: ...

Our analysis is that (95) is an instance of unmarked *should*, whereas (94) instantiates an X-marked modal.²¹

It is just unfortunate that the impoverished morphological system of English necessity modals ends up masking these contrasts based on X-marking. However, if our proposal is on the right track, they are still there, as they were in the case of epistemic necessity discussed in the previous section when we compared English and Portuguese. Moreover, and still in line with our comparative approach, Portuguese will once again provide us with a transparent case in which we can easily construct minimal pairs sharing the same temporal-aspectual profile and overtly differing only with respect to the tense marking on their necessity modals. In the next section, we explore Portuguese richer modal morphology in connection to our proposal of X-marking on non-epistemic necessity modals.

3.3.2 Portuguese necessities

We start with examples of teleological modality. To set the background, we give a minimal pair with present tense weak and strong necessity modals, both uttered in the following scenario: After her doctor's recommendation, a patient arrives at the hospital and says she wants to have a blood test. (96)-(97) are two possible responses that she may hear from the employee she is talking to:

(96) Para fazer esse exame de sangue agora, você *deve* ter
In_order_to make this exam of blood now you WN.PRES have
começado seu jejum 12 horas atrás.
started your fast twelve hours ago
'In order to make this blood test now, you ought to have started your
fast twelve hours ago.'

²¹ Another spoiler: Portuguese translations have present tense *deve* in (95) and past imperfect *devia* in (94).

- (97) Para fazer esse exame de sangue agora, você *tem que* ter
 In_order_to make this exam of blood now you SN.PRES have
 começado seu jejum doze horas atrás.
 started your fast twelve hours ago
 'In order to make this blood test now, you have to have started your
 fast twelve hours ago.'

One way of stating the difference in meaning between (96) and (97) is the following: (96) expresses that fasting is not, strictly speaking, necessary for the blood test, but it is necessary if the patient wants more reliable results, whereas (97) expresses that there is no alternative and fasting is the only way to get the test done. This is expected given what we saw in Section 2 about the contrast between weak and strong necessity modals *dever* and *ter que*.

We now replace the present tense modals in (96)-(97) by their past imperfect counterparts, keeping the rest of the sentences as well as the utterance context introduced above intact:

- (98) Para fazer esse exame de sangue agora, você *devia*
 In_order_to make this exam of blood now you WN.PST.IMP
 ter começado seu jejum doze horas atrás.
 have started your fast twelve hours ago
- (99) Para fazer esse exame de sangue agora, você *tinha que* ter
 In_order_to make this exam of blood now you SN.PST.IMP have
 começado seu jejum doze horas atrás.
 started your fast twelve hours ago

First of all, a very clear intuition about the interpretation of (98)-(99) is that the prejacentes are understood either as counterfactual or very unlikely, with remarks such as *but you didn't* or *but you didn't, right?* being natural follow-ups. For instance, (98)-(99) would sound natural in contexts in which the speaker is entitled to infer that the patient has not started fasting twelve hours before, either because it is, say, 3PM, or because the patient said something like *I felt dizzy earlier this morning after having breakfast*. Contrastingly, in (96)-(97) with present tense modals, the prejacentes are understood as open issues, and a natural follow-up would be a neutral question such as *did you?*

Except for this contrast related to the speaker's stance towards the openness/falsity of the prejacentes, everything else remains the same when we pass from (96)-(97) to (98)-(99). We still have present perspective, weak and

strong teleological necessities with past oriented prejacent. In a sense, this is expected, given that there were no changes in the roots of the modal verbs, in the present oriented preposed adverbial clauses, and in the presence of perfect aspect (have + past participle) in the complements of the modals. What remains to be given is an account of the role of past imperfect morphology on the modal verbs, one which should not mess with the common temporal-modal profile we have just attested and yet deliver the contrast related to the speaker's stance towards the openness/falsity of the prejacent in (96)-(97)/(98)-(99).

We propose that the past imperfect morphology on the necessity modals is an instance of X-marking, along the same lines discussed in the previous section in connection with epistemic readings and also in 'counterfactual' conditionals: a conventional marking signaling that some presupposed or circumstantial/factual assumption is being suspended while performing a modal assertion. As desired, there will be no changes in modal force, flavor, temporal perspective or prejacent orientation.

Consideration of examples with future-oriented prejacent gives further support to the proposal and highlights the similarities with what we saw before with epistemic modals and 'counterfactual' conditionals. Suppose Mary is coughing a lot. (100) is a possible recommendation from her doctor:

- (100) *Você deve* *tomar um xarope.*
you WN.PRES take a syrup
'You ought to take a syrup.'

The modalized sentence expresses that given the actual circumstances, the best thing Mary can do to get better is to take the syrup. Let us now replace the present tense modal with its past imperfect counterpart. An alternative recommendation from the doctor would be (101):

- (101) *Você devia* *tomar um xarope.*
you WN.PST.IMP take a syrup
'You ought to take a syrup.'

The use of past imperfect *devia* would sound particularly natural in contexts in which you are manifestly reluctant to take the syrup or even in which you had told the doctor before that you would not take any medicine. And it would sound odd if the patient has not expressed resistance to take a syrup nor is the doctor anticipating any such reluctance. This sense of modal re-

moteness (it is unlikely that you will take the syrup) is absent in (100) with the present tense *deve*. Identical remarks apply to cases with strong necessity modal *ter que*:

- (102) Você tem que tomar um xarope.
 you SN.PRES take a syrup
 ‘You have to take a syrup.’
- (103) Você tinha que tomar um xarope.
 you SN.PST.IMP take a syrup

Both (102) and (103) say that taking the medicine is necessary for you to get better. Only (103) conveys that your taking the syrup is a remote possibility.

We emphasize here that the uses of past imperfect forms in (101) and (103) are not anchored in temporality. We may even make the scenario more specific, making it clear that the doctor’s recommendation is based on current evidence (coughing, etc.) and that at no time in the past was it recommendable for the patient to take a syrup or any other medicine, since the doctor does not endorse self-medication. The doctor might even have started his talk to the patient with something like *you did well not having taken anything before consulting me*.

These contrasts in (100)-(103) are parallel to what we saw before with respect to so-called future less vivid conditionals. For instance, from a semantic-pragmatic perspective, (102) is to (103) as (104) is to (105):

- (104) Se você tomar um xarope, você melhorará rapidamente.
 if you take.FUT.SUBJ a syrup you heal.FUT quickly
 ‘If you take a syrup, you will get better quickly.’
- (105) Se você tomasse um xarope, você melhorava rapidamente.
 if you take.PST.SUBJ a syrup you heal.PST.IMP quickly
 ‘If you took a syrup, you would get better quickly.’

Moreover, as we remarked in the case of epistemic necessity, it is important to tease apart uses of X-marked modals expressing modality anchored in real world situations and non-X-marked modals embedded under a conditional structure, and expressing modality projecting from a counterfactual scenario:

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- (106) A: Pedro needs to be at the airport in less than an hour.
B: Então ele *devia* pegar um Uber.
then he WN.PST.IMP take an Uber
'Then, he ought to take an Uber.'
- (107) A: Pedro doesn't need to be at the airport soon.
B: OK. Se ele precisasse, ele *devia* pegar um Uber.
ok if he needed he WN.PST.IMP take an Uber

(106)-(107) makes the point for teleological weak necessity. What (106) expresses in that given the actual circumstances, the best alternative (though not the only one) is to take an Uber to the airport. The use of *devia* is justified by the fact that Fred seems to be or might be inclined towards the use of a different means of transportation. Contrastingly, what (107) expresses is a counterfactual reasoning: taking an Uber would be the best alternative if Pedro needed to get to the airport in less than an hour. Similar remarks apply to X-marked instances of strong necessity modal *ter que*:

- (108) A: Pedro needs to be at the airport in less than an hour.
B: Então ele *tinha que* pegar um Uber.
then he SN.PST.IMP take an Uber
- (109) A: Pedro doesn't need to be at the airport soon.
B: OK. Se ele precisasse, ele *tinha que* pegar um Uber.
ok if he needed he SN.PST.IMP take an Uber

According to (108), given the actual circumstances, the only way to get to the airport is to take an Uber. Contrastingly, (109) expresses a counterfactual reasoning: taking an Uber would be the only alternative if Pedro needed to get to the airport in less than an hour.

To broaden our empirical domain, we finish this section with some examples with a more deontic flavor, arguing they are also consistent with the past imperfect as X-marking proposal. Consider (110) said by a lawyer to a client who did not appear at his hearing with the judge:

- (110) Você *devia* ter comparecido à audiência.
you WN.PST.IMP have appeared at_the hearing

The preajacent is counterfactual and the lawyer's words convey that the client's behavior was not in his best interest. Deontic strong necessity is similar. Consider (111) uttered by a judge to a defendant who did not appear at his hearing, justifying a penalty he is about to announce:

- (111) Você tinha que ter comparecido à audiência.
 you SN.PST.IMP have appeared at_the hearing

In these examples, we have past oriented prejacent and the most natural scenarios which instantiate the truth of the modal statements involve counterfactual prejacent which are known to be false. Nevertheless, even in cases like these it is possible to envisage scenarios in which the sentences are adequate and true and yet the prejacent is not taken to be false. Suppose, for instance, that you realize it is 12PM and your friend John is at home. Then, you say (112):

- (112) Mas ele *devia/tinha que* ter comparecido à audiência com
 but he WN/SN.PST.IMP have appeared at_the hearing with
 o juiz às 11h30.
 the judge at 11:30
 ‘But he should have appeared at the hearing with the judge at 11:30.’

And then you continue your reasoning, either as in (113) or as in (114):

- (113) I think it’s unlikely that he went and got back already, but let’s ask him.
 (114) Being a responsible guy, it is quite likely that he went there, the hearing didn’t last long, and the traffic back home was good.

The modal claim in (112) appear in the middle of a reasoning whose conclusion supports the possibility or even the likelihood of the prejacent, as highlighted in (113) and (114), respectively. As in the cases of conditionals discussed by Stalnaker, it would be awkward to presuppose that a proposition is false while arguing in favor of its possibility or likelihood. We propose that suspension of presuppositions in these cases might be seen as a way of unbiassing the context, detaching the modal claim from some salient circumstantial evidence (John is at home only half an hour after a scheduled hearing far away downtown) which makes it unlikely (though not impossible) that the prejacent is true. It would highlight a potential conflict between an obligation and what is being observed by the speaker and which may lead someone to believe that the obligation has not been fulfilled. Replacing the X-marked, past imperfect modals by their unmarked, present tense versions would preserve the respective deontic necessities but would not express any

sort of modal remoteness, and the prejacent would be understood as unbiased open issues.

With all this in mind, we can import the same Kratzerian framework we used for epistemic necessities into the formal analysis of Portuguese X-marked non-epistemic necessities in (83)-(84). The only difference is that instead of an epistemic modal base and a normal/stereotypical ordering source, we are now dealing with a circumstantial modal base, which encodes some relevant facts holding in the actual world, and ordering sources encoding ideals of various types (teleological, deontic, etc.). X-marking on the modal verb signals suppression of the negation of the prejacent from the salient circumstances, just like it did for pieces of knowledge or evidence in the case of epistemic modals.

3.3.3 Two notes

We finish this section on X-marked non-epistemic necessity modals with two brief remarks on the broader topic concerning non-epistemic X-marked necessities.

A note on sneezes and X-marking

Here is an intriguing case of what looks like a prototypical example of circumstantial modality in the literature (see, for instance, [Kratzer 1991](#): p.640):

- (115) a. I must sneeze.
b. I have to sneeze.
- (116) a. I ought to sneeze.
b. I should sneeze.

[Yalcin \(2016\)](#) has raised doubts about the availability of a circumstantial reading for English *ought*. His comments on these examples are worth quoting:

It is just strange to say these [(116)] in the relevant kind of context. Why? Suppose you sense a sneeze coming on, but you are not convinced it is inevitable. Why don't [(116a)] and [(116b)] seem like natural words of warning, slightly weaker than the warning conveyed by [(115a)] and [(115b)]? (As deontic ought is thought to be weaker than deontic must.) We have the intuition that the flavor of modality is qualitatively different—not just

weaker—when we move from the strong necessity modals here to the weak ones. This is surprising. Even if [...] [(116a)] and [(116b)] are marked without some additional setup in this kind of scenario, we naively might have thought that there should be a pure circumstantial reading of ought and should available, such that [(116a)] and [(116b)] can be appropriate when you feel a sneeze approaching, in the way [(115a)] and [(115b)] are. But such a reading seems not to be available. This requires explanation.

[Yalcin 2016: p. 242]

Indeed, the sentences in (116) do not sound as merely weaker versions of (115). They would sound natural, for instance, if I have inhaled some sort of sneezing powder which normally makes people sneeze, but which surprisingly did not take any effect in my case. A comparative analysis with Portuguese weak (and strong) necessity modals is enlightening:

- (117) Eu tenho que espirrar.
I SN.PRES sneeze
- (118) Eu devo espirrar (a qualquer momento).
I WN.PRES sneeze (at any moment)
- (119) Eu tinha que espirrar /estar espirrando.
I SN.PST.IMP sneeze /be sneezing
- (120) Eu devia espirrar /estar espirrando.
I WN.PST.IMP sneeze /be sneezing

(117) with present tense strong necessity modal *ter que* conveys what its English *must (have-to)* counterparts in (115) do: in view of the circumstances (the state of my nose), it is inevitable that I sneeze. (118) with present tense weak necessity modal *dever* is the missing piece in Yalcin’s English puzzle, a sentence that does sound like a weaker version of (117), conveying the reading Yalcin was looking for in a *ought/should* sentence but couldn’t find. Indeed, when we translate into Portuguese the *ought/should* sentences in (116), what we get is a past tense necessity modal, as in (120). Completing the paradigm, we have (119) with past tense strong necessity *ter que* which, as expected, is just like (120) except for its stronger force.

We now have a much better prospect for circumstantial weak necessity which was masked by English impoverished tensed modal system: all exam-

ples seen above — English (115)-(116) and Portuguese (117)-(120) — are cases of circumstantial modality: in classic Kratzerian terms, they express inferences based on a realistic modal base and a normality-based ordering source. In the case of English *must/have to* in (115) and Portuguese present tensed modals in (117) and (118), nothing needs to be added. As for the English *ought/should* examples in (116) and the Portuguese past tensed modals in (119) and (120), they can be analyzed as X-marked circumstantial necessity, signaling that some factual premise is being suspended and the domain of quantification might include worlds which are epistemically inaccessible. Natural contexts for these utterances would include cases in which the prejacent is taken as false (I should be sneezing, but I am not; I should sneeze at any moment, but I think I won't).

The upshot is that English *ought* and *should* are idiosyncratic in that they cannot express non-X-marked circumstantial necessity the way *have to* and *must* do. One shouldn't go deeper than this since the full paradigm of circumstantial necessity, weak and strong, X-marked or not, are displayed by Portuguese necessity modals *dever* and *ter que*.

A note on X-marking on Portuguese desire verbs

Still in the realm of non-epistemic necessities, we highlight here that our discussion in this section aligns perfectly with von Stechow & Iatridou's (2023) cross-linguistic morpho-semantic analysis of another construction which hosts X-marking: attitude reports expressing what they call 'unattainable desires', a type of attitude which has been analyzed as expressing a type of bouletic modal necessity. English has a dedicated verb (*wish*) for this particular type of attitude:

(121) John wishes Mary were happy.

This sentence expresses John's desire that Mary be happy, but it also conveys that he believes she is not. Portuguese, as several languages discussed by von Stechow and Iatridou, uses past imperfect morphology on its *want*-type verb on the main clause and past subjunctive morphology on its finite complement clause:

(122) Pedro *quer* que Maria *esteja* feliz.
Pedro WANT.PRES that Maria BE.PRES.SUBJ happy
'Pedro wants Mary to be happy.'

- (123) Pedro *queria* que Maria *estivesse* feliz.
 Pedro WANT.PST.IMP that Maria BE.PST.SUBJ happy
 ‘Pedro wishes Mary were happy.’

Both sentences express Pedro’s desire that Maria be happy. However, only (123) implies that Pedro believes she is not. (122), at least as a default, express that Maria being happy is consistent with Pedro’s beliefs about her current state. It should be emphasized that the desire expressed by (123) is not counterfactual nor located in the past. (122) and (123) are about desires that Pedro has in the actual world at the utterance time. Facts like these have led von Fintel and Iatridou to analyze this type of desire reports as another instance of X-marking, the same conventional extra marking indicating departure from a default assumption. Further support for this assimilation is the fact that we also find the same morphosyntactic profile of (123) in subjunctive/counterfactual conditionals, with past imperfect tense on the main clause and past subjunctive on the subordinate clause:

- (124) Se Pedro *estivesse* aqui, ele *estava* feliz.
 if Pedro BE.PST.SUBJ here he BE.PST.IMP happy
 ‘If Pedro were here, he would be happy.’

This sameness of form was shown by von Fintel and Iatridou to hold in several unrelated languages, even when the pieces of morphology are not borrowed from the tense-aspect-mood domain.

As for a formal implementation, and glossing over some details and controversies discussed by von Fintel and Iatridou, their point of departure is a Kratzerian modal analysis for desire verbs such as English *want*, according to which it introduces restricted universal quantification over possible worlds. They assume that, as a default, the domain of quantification is formed by a doxastic modal base (the set $B_{a,w}$ of propositions believed by the attitude holder a in the world of evaluation w) which defines the doxastic alternatives of an agent a in world w ($\bigcap B_{a,w}$) and a bouletic ordering source $D_{a,w}$, which ranks those pre-selected worlds based on the agent’s desires/preferences in the world of evaluation:^{22,23}

$$(125) \quad \llbracket a \text{ wants } p \rrbracket = \lambda w. \forall w' \in \text{BEST}_{D_{a,w}} \bigcap B_{a,w} : w' \in p$$

22 For simplicity, we will ignore the possibility of agent a having inconsistent beliefs in a world w .

23 For an alternative analysis, see Heim 1992.

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X-marking the desire verb would then indicate the suspension of *a*'s belief in the negation of the proposition *p* expressed by the embedded clause, yielding a superset of his doxastic alternatives:

$$(126) \quad \llbracket a \text{ wants}_X p \rrbracket = \lambda w. \forall w' \in \text{BEST}_{D_{a,w}} \cap B_{a,w}^{*p} : w' \in p$$

$B_{a,w}^{*p}$ is a revision of $B_{a,w}$ for p .²⁴

The emerging idea is that both English (121) and Portuguese (123) instantiate (126) and could be used in a scenario in which Pedro believes (knows) that Mary is not happy.²⁵

We are then left with a good prospect for a unifying proposal for X-marking encompassing conditionals and desire verbs, as discussed and analyzed by von Stechow and Iatridou. And if our proposal about past imperfect marking on Portuguese necessity modals are on the right track, they too fit nicely into the picture, given what we discussed in detail in the last two sections.

4 X-marking and the weak/strong contrast

Having made a proposal for Portuguese past imperfect necessity modals, and having shown how they fit into von Stechow and Iatridou's X-marking (re)analysis of so-called subjunctive conditionals and unattainable desire reports, we now turn our attention to the third case discussed by von Stechow and Iatridou as a possible host for X-marking, namely, strong necessity modals. In the languages they discuss (Portuguese not included), X-marking has the semantic effect of softening the modal force, turning strong necessity into weak necessity. We repeat an example from Spanish, a language in which so-called conditional tense is used as the exponent of X-marking as we saw before in conditionals and desire verbs:

²⁴ This is analogous to the revision of modal bases introduced in (79) and discussed above in connection to modal verbs and their prejacent.

²⁵ Since desires are often future-oriented, examples of what might be called future less vivid desires can be easily constructed in analogy to future less vivid conditionals:

- (i) Eu queria que Maria chegasse a tempo para o almoço, mas eu
I want.PST.IMP that Maria arrive.PST.SUBJ in time for the lunch but I
acho isso improvável.
consider this unlikely

- (127) #Debo limpiar los platos, pero no estoy obligado.
 must clean the dishes but not am obliged
 [von Fintel & Iatridou (2023: ex.60a)]
- (128) Deberia limpiar los platos, pero no estoy obligado.
 must.COND clean the dishes but not am obliged
 'I ought to do the dishes but I am not obliged.'
 [von Fintel & Iatridou (2023: ex.61a)]

The discussion of X-marking on strong necessity modals by von Fintel and Iatridou is particularly relevant to our purposes for two reasons. First, there is an apparent clash between our proposal for X-marking on Portuguese modals and von Fintel and Iatridou's proposal associating X-marked necessities with weakening of modal force, and which was based on empirical evidence coming from several languages, as we saw in the introduction of this paper. X-marking on strong necessity modals in Portuguese does not yield a weak necessity modal, nor X-marking on an (already) weak necessity modal yields an even weaker modal. Second, according to von Fintel & Iatridou (2023), this is the place where "the theory of X-marking has serious trouble to provide a unified analysis". We will review the issues behind these difficulties shortly, but divorcing X-marking on Portuguese necessity modals from any weakening in modal force seems to have put our proposal in a even more difficult position towards unification.

In order to make clear what we have in mind, we first outline von Fintel and Iatridou's proposal for the relation between weak and strong necessity modals. Then we introduce a slight change of perspective in terms of formalization, and return to the way Portuguese necessity modals fit the picture.

According to von Fintel and Iatridou, both strong and weak necessity modals express universal, restricted quantification over possible worlds. As in the standard Kratzerian framework for modal verbs, this quantification is parametrized by contextually supplied conversational backgrounds: modal bases and ordering sources. The difference between strong and weak necessities is that the former employs one ordering source whereas the latter employs two. Since the job of an ordering source is to rank the worlds pre-selected by a modal base and extract the top-ranked, best elements, weak necessity is viewed as selecting the best of the best, with its secondary ordering applying on top of the primary ordering which is part of strong necessities. In formal terms:

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- (129) For any modal base f and ordering sources g^1, g^2 :
- a. $\llbracket \text{SN} \rrbracket^{f, g^1} = \lambda p. \lambda w. \forall w' \in \text{BEST}_{g^1_w}(\cap f_w) : w' \in p$
 - b. $\llbracket \text{WN} \rrbracket^{f, g^1, g^2} = \lambda p. \lambda w. \forall w' \in \text{BEST}_{g^2_w}(\text{BEST}_{g^1_w}(\cap f_w)) : w' \in p$

As can be seen in (129), for any given choice of modal base and ordering sources, the worlds over which a weak necessity modal universally quantifies is a subset of the worlds over which a strong necessity modal quantifies, making weak necessity modals semantically weaker than strong necessity modals.

Intuitions across different modal flavors were provided by von Fintel & Iatridou (2008), hinting at what might be behind the primary/secondary ordering source split:²⁶

In the goal-oriented case, the first ordering source is simply the goal proposition designated by an (*in order*) *to*-adjunct or an *if you want to*-anankastic conditional. The second, subsidiary ordering source contains considerations such as how fast, how comfortable, how cheap, ... the means for achieving the goal are. [...]

[E]pistemic *ought* differs from epistemic *must/have to* in being sensitive not just to the hard and fast evidence available in a situation but also to a set of propositions that describe what is *normally* the case [...]

And in the deontic case, *ought* might be sensitive to less coercive sets of rules and principles in addition to the laws and regulations that strong necessity modals would be interpreted with respect to.

[von Fintel & Iatridou 2008: p. 119]

With this much in mind, it is natural to see the X-marking on a strong necessity modal as signaling the addition of a secondary ordering source, which in turn leads to a modal statement weaker than the original strong necessity.

²⁶ Rubinstein (2012) refines these intuitions, trying to ground the primary/secondary ordering source split on more solid pragmatic notions. In particular, she ties the split to a distinction between negotiable and non-negotiable priorities among conversational participants. See also Rubinstein 2021 for an insightful overview of this and related issues concerning weak necessity as well as for pointers to the relevant literature.

Let us now recast von Fintel & Iatridou’s (2023) implementation in terms of a parameter shift, along the lines we did in our discussion of Portuguese necessities. Our point of departure is von Fintel & Iatridou’s (2023) insight that X-marking can target different modal parameters, modal bases and ordering sources. There is, however, a striking asymmetry between the X-marking that targets modal bases and the X-marking that targets ordering sources. Whereas the first signals presupposition suspension and domain widening, the latter signals addition of premises and narrowing of the set of ideal (best) worlds. At first sight, they seem to point in opposite directions. Let us then try a slightly different perspective.²⁷

As pointed out to me by Anthony Gillies, ordering sources come with an implicit presupposition that only the propositions in them count toward betterness. From this higher level, meta-semantic perspective, the X-marking that targets ordering sources could be viewed as signaling that this presupposition is being suspended and that additional ordering information might come into play. We would like to capitalize on this intuition and offer a lower level, formal rendition of it.

We start by highlighting that although modal bases and ordering sources are of the same semantic type (functions from possible worlds to sets of propositions), they play very different roles in building the meaning of modal operators. It is the business of a modal base f to deliver a set of worlds which provide an initial domain for a modal quantifier ($\bigcap f_w$). As we have already seen in detail, X-marking that targets modal bases (we will refer to it as X_f) imposes a widening of this set ($\bigcap f_w \cup \dots$).

On the other hand, it is the business of an ordering source to specify what counts as *being better than* and rank worlds of a given domain D accordingly. Let the set $\text{BTT}_{g_w}(D)$ in (130) be a formal exponent of this central aspect of an ordering source g :

$$(130) \quad \text{BTT}_{g_w}(D) = \{\langle u, v \rangle \mid u, v \in D \ \& \ u <_{g_w} v\}^{28}$$

The idea we would like to suggest is that this set be taken as the ordering source analogue of $\bigcap f_w$, the set induced by a modal base f . The proposal is that the X-marking that targets ordering sources g (we will refer to it as X_g) imposes a widening of this set ($\text{BTT}_{g_w} \cup \dots$). More concretely, we pro-

²⁷ I am indebted to editor Anthony Gillies and an anonymous reviewer for their help in addressing this issue.

²⁸ D ranges over sets (domains) of worlds and $u <_{g_w} v$ means that world u is better than world v according to g_w .

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pose that X_g signals a revision of g for some proposition p , a revision which discriminates among the worlds previously treated as top-ranked by g . As formalized in (131), this revision, which we will refer to as g^{**p} , favors p -worlds over non- p -worlds:

- (131) For any proposition p and ordering sources g, g' :
- g' is a $**$ -revision of g for p if, and only if, for any domain D and world w ,
- $$\text{BTT}_{g'_w}(D) = \text{BTT}_{g_w}(D) \cup \{\langle u, v \rangle \mid u \in p \ \& \ v \notin p \ \& \ u, v \in \text{BEST}_{g_w}(D)\}$$

With this much in place, X_g -marking on a strong necessity modal can be seen as a modal shifter affecting the ordering source g , but leaving the modal base f intact:

- (132) For any modal base f and ordering source g :
- a. $\llbracket \text{SN} \rrbracket^{f,g} = \lambda q. \lambda w. \forall w' \in \text{BEST}_{g_w}(\cap f_w) : w' \in q$
 - b. $\llbracket \text{SN}_{X_g} \rrbracket^{f,g} = \lambda q. \lambda w. \llbracket \text{SN} \rrbracket^{f,g^{**p}}(q)(w)$, for some proposition p
 - c. g^{**p} is a $**$ -revision of g for p .

Finally, weak necessity modals (WNs) can be taken as equivalent to X_g -marked strong necessity modals (SNs):

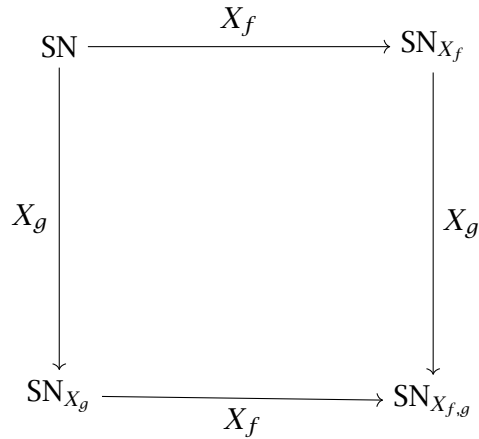
- (133) $WN \equiv \text{SN}_{X_g}$

We will not claim to have arrived at a truly unified theory of X-marking. Our modest goal here was to approximate the two types of X-marking from a formal perspective and improve the prospects for an eventual unification. We now have X_f and X_g as parameter shifters targeting modal bases f and ordering sources g , respectively, both yielding the widening of sets which can be seen as semantic signatures of f and g . Moreover, the revisions imposed on f and g are both centered on propositions p : X_f leads to a p -diverse domain, and X_g to a p -sensitive ranking. Notwithstanding these facts, an obvious asymmetry remains: the revision imposed on a modal base f by X_f specifically targets the prejacent of the modal verb, whereas the revision imposed on an ordering source g by X_g targets a different proposition. We will leave this asymmetry as a topic for future investigation.

We are then left with the following picture of X-marking on necessity modals: X-marking can be viewed as a cover concept encompassing the semantic mappings X_f and X_g . When applied to necessity modals, they give

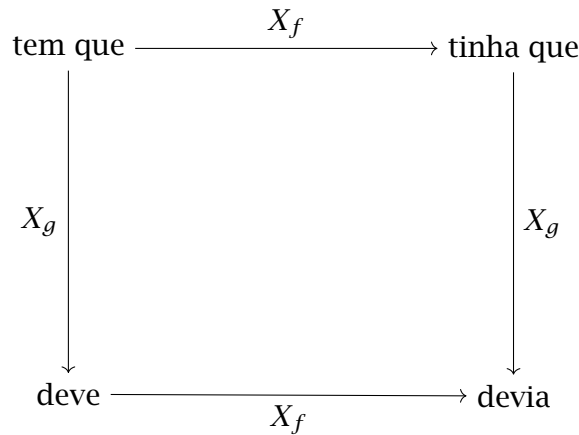
rise to the following SQUARE OF NECESSITIES in which the vertices represent the modals and the edges the mappings that relate them semantically:

(134) *Necessity operators and X-marking*



If our proposals for Portuguese necessity modals are on the right track, they instantiate both X-markings and occupy the four vertices of (134):

(135) *Portuguese necessity modals and X-marking*

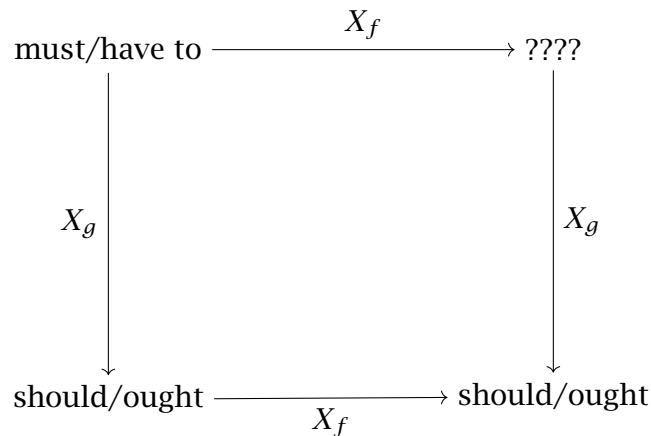


As von Stechow and Iatridou have documented, X-marking can manifest cross-linguistically either lexically or morphologically. In the case of Portuguese necessity modals, X_g is a lexical operation applying to a verb root (*ter que*) and resulting in another verb root (*dever*), there being no overt morphophonological relation between them. X_f is a morphological (affixal) operation adding past imperfect morphology to (weak or strong) necessity verb roots. And if

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our proposals about English *ought* and *should* are also on the right track, *ought* and *should* are ambiguous, expressing weak necessity with or without X-marking. In our formal setting, this means that *ought* and *should* stand for both SN_{X_g} and $SN_{X_f, g}$ (the latter being a composite of X_f and X_g applied to SN). Together with *must/have to* they occupy three vertices of the square:²⁹

(136) *English necessity modals and X-marking*



Finally, languages like Greek and Spanish, also discussed by von Stechow and Iatridou, obtain SN_{X_g} via morphological marking on their SN modals. However, as discussed by Laca (2012), at least for Spanish and French the same morphological marking (so called conditional tense) can also play the role of X_f , indicating domain widening. As Laca points out, “[a]n indication of the domain-widening effect of conditional morphology is the fact that modals in the conditional are much more easily compatible with negative belief assertions than indicative modals” (Laca 2012):

(137) Ce livre devrait être sur l’étagère de droite.
 this book MUST.COND be on the shelf of righthand
 (va vérifier/ et non pas là où il est)
 ‘This book ought to be on the right-hand shelf (go check/ and not where it is).’

²⁹ At this point we assume that English does not have a lexical item occupying the top right vertex, but we will leave further discussion of this point to another occasion.

- (138) Ese libro debería estar en el estante de la derecha.
 this book MUST.COND be in the shelf of the righthand
 (fíjate/y no donde está)
 ‘This book should be on the right-hand shelf (go check/ and not
 where it is).’

As her translations seem to indicate, a weakening in modal force (*must* → *should*) accompanies the widening of the modal domain. If this is the case, the meaning of tensed modals *devrait/deberia* can be seen as the output of X_g (as in von Fintel and Iatridou’s data) and also of $X_{f,g}$, occupying the lower left and lower right vertices of the square, just like English *ought/should*. If, on the other hand, Laca’s tensed modal examples still convey strong necessity, then they would occupy the lower left and the upper right vertices of the square. At this point, we will leave open the details about whether or how these languages fill the remaining vertices of our square of necessities.

5 Conclusion

In this paper we have discussed instances of Portuguese necessity modals *dever* and *ter que* carrying a morphological marking (part imperfect) which we claimed express something akin to what Kai von Fintel and Sabine Iatridou dubbed X-marking. On the one hand, we have assimilated the semantic effect of this marking to the one showing up on so-called counterfactual conditionals and unattainable desire ascriptions, analyzed in von Fintel & Iatridou 2008, 2023 as suspension of some assumption on which a modal statement is based. On the other hand, we have contrasted the X-marking on Portuguese necessity modals with the X-marking on strong necessity modals showing up in many languages which downgrades the modal force from strong to weak necessity, also analyzed by von Fintel and Iatridou. No such weakening is observed in the case of Portuguese X-marked necessity modals.

We have also compared Portuguese *dever* to English *ought/should* with particular attention to some recalcitrant data concerning its epistemic and non-epistemic uses, and proposed that *ought/should* is ambiguous between X-marked and non X-marked weak necessity. This ambiguity is absent in Portuguese, a language in which X-marking on its weak necessity modal produces a different form (past imperfect). We concluded that a language may have up to four related necessity modals which occupy the vertices of what we called ‘the square of necessities’. Portuguese is such a language.

Among the broader issues raised by our discussions in this paper is the possibility of finding instances of this square of necessities in other modal constructions. For instance, we saw how [von Fintel & Iatridou \(2023\)](#) analyze *wish* reports as X-marked desires in which X signals widening of a modal domain (a doxastic set, in this case). The question here is whether there are cases of languages which X-mark the bouletic ordering source which is part of the lexical semantics of desire predicates and which would produce a weak desire predicate akin to weak necessity modals.

A related point can be made for possibility modals. [Stalnaker \(2014\)](#) briefly entertained possible instantiations of X-marking (not his terminology though) on English *may* and *might*. He noticed that (139) can be said to a child in a context in which it is known or presupposed that she did not start a fire:

- (139) You shouldn't have been playing with matches; *you might have started a fire.*

[[Stalnaker 2014](#): p. 186]

He also pointed out that tense differences play a role, and that English *may* would not be appropriate in this context. (140) conveys that it is an open issue whether a fire started:

- (140) You shouldn't have been playing with matches; *you may have started a fire.*

[[Stalnaker 2014](#): p. 187]

The examples translate straightforwardly into Portuguese, with *may* and *might* being replaced by present tense *pode* and past imperfect *podia*, respectively, both being inflected forms of possibility modal *poder*.³⁰ [Stalnaker](#) highlighted the similarity with indicative/subjunctive conditionals, and the same can be noted for Portuguese. Given our implementation from the last section, *may* (and *pode*) in (140) and *might* (and *podia*) in (139) would then be analyzed as unmarked and X_f -marked modals, respectively, with X signaling widening of the domain yielded by the modal base. Two vertices of a conjectural square of possibilities, in analogy to our square of necessities, would then be occupied. The other two vertices, however, remain vacant, unless we find evidence for X_g -marked possibility modals, in analogy to X_g

³⁰ For discussion of past tense marking on the possibility modal *poder*, see [Pessotto 2011](#).

marked weak necessity ones, which would express some sort of strong possibility.^{31,32}

Finally, as we also said in the introduction, this paper aimed at shedding light on empirical and theoretical issues connected to the expression of weak and strong necessities by modal verbs. Ideally it will serve to trigger semantic (re)-analyses of morphologically marked necessity modals in other languages, perhaps as X-marking along the lines we have discussed above. We have already seen, for instance, how [Laca \(2012\)](#) analyzed some French and Spanish tensed modals, and how her analyses might relate to our square of necessities. Just as an additional illustrative data point, take Italian, which has been shown to apply past imperfect morphology to its necessity modal *dovere*. It was dubbed ‘imperfetto potenziale’ and characterized as “mainly related to modal verbs” and expressing “a sort of supposition” in [Bazzanella 1990](#): p. 443:

- (141) Vincenzo *doveva* essere qui; non capisco cosa gli sia
 Vincenzo *dovere.IMP* be here; not understand what has to.him
 successor.
 happened
 ‘Vincenzo should have been here; I can’t understand what has happened to him.’

[[Bertinetto 1986](#): p. 374 *apud* [Bazzanella 1990](#)]

This looks very much like the core data we have presented here. Hopefully this piece of data as well as data and/or analyses coming from other languages and modals will also benefit from our discussion of Portuguese necessity modals.

References

Anderson, Alan Ross. 1951. A note on subjunctive and counterfactual conditionals. *Analysis* 12(2). 35–38. <https://doi.org/10.1093/analys/12.2.35>.

³¹ The result would be *strong*, rather than *weak*, possibility, because existentially quantifying over a smaller domain (the best of the best worlds) would result in a stronger statement when compared to unmarked possibility which expresses existential quantification over the (merely) best worlds.

³² Things are more complicated with English possibility modals, since the *may have/might have* contrast mentioned by Stalnaker does not seem to extend to ‘simple’ *may/might* statements (i.e, without perfect *have*). See [Condoravdi \(2002\)](#) for extensive discussion of the interaction between tense/aspect and English possibility modals.

- Arregui, Ana. 2010. Detaching if-clauses from *should*. *Natural Language Semantics* 18(3). 241–293. <https://doi.org/10.1007/s11050-010-9055-5>.
- Bazzanella, Carla. 1990. *Modal uses of the Italian indicativo imperfetto in a pragmatic perspective*. *Journal of Pragmatics* 14(3). 439–457. [https://doi.org/10.1016/0378-2166\(90\)90100-R](https://doi.org/10.1016/0378-2166(90)90100-R).
- Bertinetto, Pier Marco. 1986. *Tempo, aspetto e azione nel verbo italiano*. Florence: Accademia della Crusca.
- Condoravdi, Cleo. 2002. Temporal interpretation of modals: Modals for the present and for the past. In David Beaver, Stefan Kaufmann, Brady Clark & Luis Casillas (eds.), *The construction of meaning*, 59–88. Stanford, CA: CSLI Publications.
- Copley, Bridget. 2006. *What should should mean*. Manuscript, CNRS; Presented at the Language Under Uncertainty workshop, Kyoto University: January 2005. <https://shs.hal.science/halshs-00093569>.
- von Fintel, Kai. 1998. The presupposition of subjunctive conditionals. In Uli Sauerland & Orin Percus (eds.), *The interpretive tract (MIT working papers in linguistics 25)*, 29–44. MA: ITWPL. <https://web.mit.edu/fintel/fintel-1998-subjunctive.pdf>.
- von Fintel, Kai & Anthony Gillies. 2010. *Must...stay...strong!* *Natural Language Semantics* 18(4). 351–383. <https://doi.org/10.1007/s11050-010-9058-2>.
- von Fintel, Kai & Anthony Gillies. 2021. Still going strong. *Natural Language Semantics* 29(1). 91–113. <https://doi.org/10.1007/s11050-020-09171-x>.
- von Fintel, Kai & Sabine Iatridou. 2008. How to say *ought* in foreign: The composition of weak necessity modals. In Jacqueline Guéron & Jacqueline Lecarme (eds.), *Time and modality*, 115–141. New York: Springer. https://doi.org/10.1007/978-1-4020-8354-9_6.
- von Fintel, Kai & Sabine Iatridou. 2023. Prolegomena to a theory of X-marking. *Linguistics and Philosophy* 46(6). <https://doi.org/10.1007/s10988-023-09390-5>.
- Grano, Thomas & Milo Phillips-Brown. 2022. (Counter)factual want ascriptions and conditional belief. *Journal of Philosophy* 119(12). 641–672. <https://doi.org/10.5840/jphil20221191241>.
- Hawthorne, John. 2021. The epistemic use of *ought*. In Lee Walters & John Hawthorne (eds.), *Conditionals, paradox, and probability: Themes from the philosophy of Dorothy Edgington*, 164–173. Oxford: Oxford University Press. <https://doi.org/10.1093/os0/9780198712732.003.0010>.

- Heim, Irene. 1992. Presupposition projection and the semantics of attitude verbs. *Journal of Semantics* 9(3). 183–221. <https://doi.org/10.1093/jos/9.3.183>.
- Iatridou, Sabine. 2000. The grammatical ingredients of counterfactuality. *Linguistic Inquiry* 31(2). 231–270. <https://doi.org/10.1162/002438900554352>.
- Kratzer, Angelika. 1991. Modality. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantics: An international handbook of contemporary research*, 639–650. Berlin: de Gruyter. <https://doi.org/10.1515/9783110126969.7.639>.
- Kratzer, Angelika. 1998. More structural analogies between pronouns and tenses. *Semantics and Linguistic Theory (SALT)* 8. 92–110. <https://doi.org/10.3765/salt.v8i0.2808>.
- Kratzer, Angelika. 2012. *Modals and conditionals*. Oxford: Oxford University Press. <https://doi.org/10.1093/acprof:oso/9780199234684.001.0001>.
- Laca, Brenda. 2012. On modal tenses and tensed modals. In Chiyo Nishida & Cinzia Russi (eds.), *Building a bridge between linguistic communities of the old and the new world: Current research in tense, aspect, mood, and modality*, vol. 25 (Cahiers Chronos), 163–198. Amsterdam & New York: Brill. https://doi.org/10.1163/9789401208345_010.
- Mackay, John. 2019. Modal interpretation of tense in subjunctive conditionals. *Semantics and Pragmatics* 12(2). 1–29. <https://doi.org/10.3765/sp.12.2>.
- Pessotto, Ana Lúcia. 2011. ‘pode’ e ‘podia’: Uma proposta semântico-pragmática. *Revista da Abralín* 10(2). 11–41. <https://doi.org/10.5380/rabl.v10i2.32058>.
- Pessotto, Ana Lúcia. 2014. Epistemic and gradable modality in Brazilian Portuguese: A comparative analysis of ‘poder’, ‘dever’ and ‘ter que’. *Revista Virtual de Estudos da Linguagem (ReVEL)*, [Special issue on] *Modality in the formal semantics of natural languages* 12(8). 49–75. <http://www.revel.inf.br/en/edicoes/?id=35>.
- Raposo, Eduardo, Maria Fernanda Bacelar do Nascimento, Maria Antonia Coelho da Mota, Luisa Segura & Amalia Mendes. 2013. *Gramática do português*. Lisboa: Calouste Gulbekian.
- Rubinstein, Aynat. 2012. *Roots of modality*. Amherst, MA: University of Massachusetts Amherst dissertation. <https://doi.org/10.7275/3527519>.
- Rubinstein, Aynat. 2021. Weak necessity. In Daniel Gutzmann, Lisa Matthewson, Cécile Meier, Hotze Rullmann & Thomas Ede Zimmermann (eds.),

- The Wiley Blackwell companion to semantics*, 3153–3196. Oxford: Wiley-Blackwell. <https://doi.org/10.1002/9781118788516.sem105>.
- Stalnaker, Robert. 1968. A theory of conditionals. In Nicholas Rescher (ed.), *Studies in logical theory*, 98–112. Oxford: Blackwell. Reprinted as Stalnaker 1981.
- Stalnaker, Robert. 1975. Indicative conditionals. *Philosophia* 5(3). 269–286. <https://doi.org/10.1007/BF02379021>.
- Stalnaker, Robert. 1981. A theory of conditionals. In William L. Harper, Robert Stalnaker & Glenn Pearce (eds.), *Ifs: Conditionals, belief, decision, chance and time* (The Western Ontario Series in Philosophy of Science 15), 41–55. Holland: D. Reidel. https://link.springer.com/chapter/10.1007/978-94-009-9117-0_2.
- Stalnaker, Robert. 2014. *Context*. Oxford: Oxford University Press.
- Stalnaker, Robert & Richmond Thomason. 1970. A semantic analysis of conditional logic. *Theoria* 36(1). 23–42. <https://doi.org/10.1111/j.1755-2567.1970.tb00408.x>.
- Stowell, Tim. 2004. Tense and modals. In Jacqueline Guéron & Jacqueline Lecarme (eds.), *The syntax of time*, 621–635. Cambridge, MA: MIT Press. <https://doi.org/10.7551/mitpress/6598.003.0025>.
- Swanson, Eric. 2008. Modality in language. *Philosophy Compass* 3(6). 1193–1207. <https://doi.org/10.1111/j.1747-9991.2008.00177.x>.
- Thomson, Judith Jarvis. 2008. *Normativity*. Chicago, IL: Open Court Publishing Company.
- Yalcin, Seth. 2012. Context probabilism. In M. Aloni, V. Kimmelman, F. Roelofsen, G.W. Sassoon, K. Schulz & M. Westera (eds.), *Logic, language and meaning: 18th Amsterdam Colloquium, Amsterdam, the Netherlands, December 19-21, 2011, revised selected papers* (Lecture Notes in Computer Science 7218), 12–21. H: Springer. https://doi.org/10.1007/978-3-642-31482-7_2.
- Yalcin, Seth. 2016. Modalities of normality. In Nate Charlow & Matthew Chrisman (eds.), *Deontic modality*, 230–255. Oxford: Oxford University Press. <https://doi.org/10.1093/acprof:oso/9780198717928.003.0009>.

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