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Free choice and presuppositional exhaustification*

Guillermo Del Pinal
UMass Amherst

Itai Bassi
Leibniz ZAS

Uli Sauerland
Leibniz ZAS

Abstract Sentences such as *Olivia can take Logic or Algebra* ('$\Diamond\vee$-sentences') are typically interpreted as entailing that Olivia can take Logic and can take Algebra. Given a standard semantics for modals and disjunction, those 'Free choice' (FC) readings are not predicted from the surface form of $\Diamond\vee$-sentences. Yet the standard semantics is appropriate for the 'double prohibition' reading typically assigned to $\neg\Diamond\vee$-sentences like *Olivia can't take Logic or Algebra*. Several extant approaches to FC can account for those two cases, but face challenges when $\Diamond\vee$, $\neg\Diamond\vee$ and related sentences appear embedded in certain environments. In this paper, we present a novel account of FC that builds on a 'grammatical' theory of scalar implicatures—proposed by Bassi et al. (2021) and Del Pinal (2021)—according to which covert exhaustification is a presupposition trigger such that the prejacent forms the assertive content while any excludable or includable alternatives are incorporated at the non-at issue, presuppositional level. Applied to $\Diamond\vee$, $\neg\Diamond\vee$, and similar sentences, 'presuppositional exhaustification' predicts that their default interpretations have an assertive component (roughly, the classical interpretation of the prejacent) and a homogeneity presupposition which projects in standard ways. Those predictions, we then show, support a uniform account of the puzzling behavior of $\Diamond\vee$, $\neg\Diamond\vee$, and related sentences when embedded under (negative) factives (Marty & Romoli 2020), disjunctions (Romoli & Santorio 2019), and in the scope of universal, existential (Bar-Lev & Fox 2020) and non-monotonic quantifiers (Gotzner et al. 2020).

Keywords: free choice, scalar implicatures, exhaustification, presuppositions, accommodation, pragmatics.

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1 Introduction

Sentences with disjunction in the scope of a possibility or existential modal, such as (1), give rise to a conjunctive 'free choice' (FC) inference—i.e., that Mary can study in Tokyo and can study in Boston, as in (1a). Yet FC doesn't follow from the surface form of '$\Diamond\vee$'-sentences, given classical lexical entries for modals and disjunction.

(1) Maria can study in Tokyo or Boston. $\Diamond(T \vee B) \Leftrightarrow (\Diamond T \vee \Diamond B)$

  a. $\rightsquigarrow$ *Maria can study in Tokyo*
    $\rightsquigarrow$ *Maria can study in Boston* $\Diamond T \wedge \Diamond B$

Understanding this intriguing tension between our linguistic intuitions and classical modal logic promises to shed light on the interface between our semantic competence, natural logic, and general pragmatic reasoning.

One traditional approach to FC adopts a Gricean strategy: keep a classical model of our core semantic competence, and try to derive FC as a pragmatic inference akin to scalar implicatures (SIs) (Kratzer & Shimoyama 2002). Another approach is to adopt non-classical lexical entries for specific logical terms (Zimmerman 2000). Recently, FC has also been used to support 'Grammatical' theories according to which the SIs of a sentence ϕ are due to exhaustification with a covert operator, **exh**, whose output is akin to asserting $[\![\phi]\!]$ and the negation of each excludable and relevant alternative of ϕ. Grammatical theories have two attractive features.

First, the FC reading of (1) can be derived via recursive exhaustification, a type of operation that is expected on an **exh**-based theory, but harder to capture with a pure pragmatic account of SIs (Champollion et al. 2019, Alsop et al. 2021). Given suitable alternatives at each **exh** site, Fox (2007) established the result in (2a).

(2) a. $[\![\mathbf{exh}[\mathbf{exh}[\Diamond[T \vee B]]]]\!] = (\Diamond T \leftrightarrow \Diamond B) \wedge \Diamond(T \vee B) = \Diamond T \wedge \Diamond B$
  b. $[\![\mathbf{exh}^{IE+II}[\Diamond[T \vee B]]]\!] = \Diamond T \wedge \Diamond B$

Some semanticists now hold that **exh** doesn't just exclude but also includes certain alternatives (Bar-Lev & Fox 2020). As a result, the FC reading of (1) can be derived without recursive exhaustification, as in (2b). Still, the revised operator, $\mathbf{exh}^{IE+II}$, has the property—just like other syntactically 'real' operators—that it can be inserted in various kinds of embedded environments. This predicts, correctly, that FC readings should be observed in a range of embedded positions.

Secondly, FC readings, like SIs in general, tend to be cancelled in downward entailing (DE) environments. This is illustrated by the default 'double prohibition' reading of '$\neg\Diamond\vee$'-sentences such as (3), which conveys not merely the negation of FC, but the stronger claim that Maria isn't allowed to study in either one of Tokyo or Boston, as in (3a).

(3) Maria can't study in Tokyo or Boston. $\neg\Diamond(T \vee B)$

a. $\rightsquigarrow$ *Maria can't study in Tokyo*
$\rightsquigarrow$ *Maria can't study in Boston* $\neg\Diamond(T \vee B) \Leftrightarrow \neg\Diamond T \wedge \neg\Diamond B$

This double prohibition reading is unexpected if FC was a direct semantic entailment of (1), say, hardwired via a non-standard semantics for modals and/or disjunction. For suppose that (1), based on its surface form, semantically entails FC, namely, $\Diamond T \wedge \Diamond B$. When embedded under negation, as in (3), we would then expect a 'negation of FC' reading—i.e., $\neg(\Diamond T \wedge \Diamond B) \Leftrightarrow \neg\Diamond T \vee \neg\Diamond B$—which is weaker than double prohibition. In contrast, in **exh** accounts, the cancellation of FC readings in DE environments is due to a general preference for parses with strong meanings. Roughly, when a parse with $\mathbf{exh}^{IE+II}$ leads to a weaker meaning compared to a corresponding parse without $\mathbf{exh}^{IE+II}$, as in (4a) vs. (4b), the latter is treated as the default, non-marked option.

(4) a. $[\![\neg\mathbf{exh}^{IE+II}[\Diamond[T \vee B]]]\!] = \neg\Diamond T \vee \neg\Diamond B$
b. $[\![\neg\Diamond[T \vee B]]\!] = \neg\Diamond T \wedge \neg\Diamond B$

Grammatical accounts of FC for $\Diamond\vee$-sentences can be extended to similar FC readings observed in many other kinds of configurations (Fox 2007, Chierchia 2013, Meyer 2020). However, Marty & Romoli (2020) and Romoli & Santorio (2019) have recently raised a considerable challenge to standard **exh** theories, based on the projective and filtering behavior of $\Diamond\vee$-sentences like (1) and $\neg\Diamond\vee$-sentences like (3) when embedded in two kinds of environments.

To get a feel for the challenge, consider first a $\Diamond\vee$-sentence under a negative factive, as in (5) (Marty & Romoli). On its most natural reading, (5) entails that Maria has FC, as in (5a), and that what Sam doesn't believe is that Maria can study in either Tokyo or Boston, as in (5b). Given the factivity of *unaware*, by parsing the embedded '$\Diamond\vee$'-sentence with $\mathbf{exh}^{IE+II}$—i.e., like the parse that supports FC for (1)—we predict the FC inference in (5a). Yet we also predict that what Sam doesn't believe is that Maria has FC, which is weaker than the target entailments in (5b), which say that what Sam doesn't believe is that Maria can study in either city.

(5) Sam is unaware that Maria can study in Tokyo or Boston.

a. $\rightsquigarrow$ *Maria can study in Tokyo*
$\rightsquigarrow$ *Maria can study Boston*

b. $\rightsquigarrow$ *Sam doesn't believe that Maria can study in Tokyo*
$\rightsquigarrow$ *Sam doesn't believe that Maria can study in Boston*

Consider next the disjunction in (6), which has a $\neg\Diamond\vee$-sentence as its first disjunct and the second disjunct triggers a FC presupposition (Romoli & Santorio). On its most natural reading, the first $\neg\Diamond\lor$ sentence gets its usual double prohibition interpretation. And although the second disjunct (= *Maria is the first ... that can study in Japan and the second ... that can study in the States*) presupposes that Maria has FC, i.e., can study in Japan and can study in the States, (6) as a whole doesn't inherit that FC presupposition, as captured in (6a). Given standard projection rules for disjunction, that suggests that the FC presupposition is entailed—hence filtered out—by the negation of the first $\neg\Diamond\lor$ disjunct. Yet recall that, to derive double prohibition for a $\neg\Diamond\lor$-sentence, $\mathbf{exh}^{IE+II}$ has to be dropped from under the negation. Given that parse, the negation of the $\neg\Diamond\lor$-sentence wouldn't entail—hence wouldn't filter out—the FC presupposition triggered by the second disjunct.

(6) Either Maria can't study in Tokyo or Boston, or she is the first in our family that can study in Japan and the second that can study in the States.

a. $\not\rightsquigarrow$ *Maria can study in Japan*
$\not\rightsquigarrow$ *Maria can study in the States*

Marty & Romoli and Romoli & Santorio consider various parses and supplementary stipulations, and conclude that standard **exh** accounts of FC for $\Diamond\lor$-sentences and double prohibition for $\neg\Diamond\lor$-sentences have trouble predicting the target readings of sentences like (5) and (6). This holds whether **exh** is modeled as asserting the prejacent and the negated excludable alternatives (Chierchia et al. 2012), or as also asserting (innocently) includable alternatives (Bar-Lev & Fox 2020). We will call this challenge the 'presupposed & filtering FC puzzles'.

Recent work on FC suggests two main strategies for addressing these puzzles. Grammatical theorists have proposed that **exh** can strengthen both the assertive and the presuppositional content of its prejacent (Gajewski & Sharvit 2012, Marty & Romoli 2020). Such accounts resolve part of the puzzle, but we'll argue that they don't provide a general solution. Another approach is to adopt revised Lexicalist accounts which modify the classical semantics for modals and/or disjunction (Aloni 2018, Ciardelli et al. 2018, Rothschild & Yablo 2018, Goldstein 2019). A challenge for these accounts is that the puzzles have versions involving the (negative) FC readings of '$\neg\Box\land$'-sentences like *Maria isn't required to study in Tokyo and Boston*. Yet negative FC is not directly derivable on most revised Lexicalist accounts (but see Willer 2017). The presupposed & filtering FC puzzles, then, present an intriguing challenge to most state of the art theories of FC and related phenomena.

This paper presents a novel Grammatical theory of FC which we argue resolves the presupposed & filtering FC puzzles. Current Grammatical theories differ in various ways: e.g., on how to pick out the excludable alternatives (Katzir 2007, Fox & Katzir 2011), on whether to also 'include' certain alternatives (Bar-Lev & Fox 2020), and on details about the distribution of **exh** (Magri 2011, Chierchia 2013).

Yet they share the view that the output of **exh**(ϕ) is flat or one-dimensional: if the prejacent ϕ doesn't trigger any presuppositions, then both ϕ and any of its excludable and includable alternatives are part of the assertive content. In contrast, in Bassi et al. (2021) and Del Pinal (2021) we proposed that covert exhaustification is a kind of presupposition trigger, which we called '**pex**'. Relative to how it structures assertive vs. presupposed content, **pex** is roughly the mirror image of its overt counterpart *only* (cf. Horn 1969): its prejacent is part of its assertive content, while any excludable or includable alternatives go into the non-at issue, presuppositional level. We argued that this proposal improves the predictions of Grammatical theories for basic SIs. In this paper, we show that a **pex**-based theory also substantially improves their predictions for FC and related phenomena.

The plan is as follows. In §2, we derive FC for $\Diamond\vee$ and $\neg\Box\wedge$-sentences and double prohibition for $\neg\Diamond\vee$-sentences using **pex**. We show that, on this theory, those readings are structured into an assertive component which corresponds to the classical interpretation of the prejacent, and a presuppositional component which corresponds to a homogeneity proposition. In §3-§4 we argue that our theory supports a uniform account of presupposed FC cases like (5), filtering FC cases like (6), and analogous puzzles with embedded $\neg\Box\wedge$ and $\Box\wedge$-sentences. Each solution follows from embedded, local application of **pex**, using standard assumptions about presupposition projection, filtering and accommodation to determine the behavior of the embedded homogeneity presupposition. In §5, we show that local application of **pex** also resolves various open puzzles related to FC effects in the scope of universal, existential and non-monotonic quantifiers. Taken together, our solutions of these embedded FC puzzles support the hypothesis, which we implement with **pex**, that covertly exhaustified content is a species of non-assertive, projective content.

2 Presuppositional exhaustification and basic free choice effects

Our central hypothesis is that covert exhaustification divides its output into an assertive and a non-at issue/presupposed component. Concerning its core operations, the standard view is that exhaustification asserts its prejacent and the negation of any excludable alternatives. Yet Bar-Lev & Fox (2020) show that adding an inclusion function simplifies the derivation of FC, while preserving (and sometimes improves) the predictions for simpler kinds of SIs. We will also formulate our presuppositional exhaustification operator, '**pex**', with both an exclusion and an inclusion function. In §2.1 we propose a way of adding an inclusion function to **pex**, and show that this modification preserves the main results which we used in Bassi et al. (2021) and Del Pinal (2021) to solve various puzzles for theories of SIs. In §2.2 we present a **pex** account of basic FC and double prohibition, highlighting our unique predictions concerning their presuppositional and assertive components.

2.1 Presuppositional exhaustification with innocent inclusion

We begin by defining the sets from which exhaustification picks the excludable and includable alternatives of its prejacent ϕ. Following Fox (2007), we assume that the negated alternatives are selected from the set of 'innocently excludable' alternatives:

(7) Innocently Excludable alternatives of the prejacent ϕ:
   a. Take all maximal sets of alternatives of ϕ that can be assigned 'false' consistently when conjoined with ϕ.
   b. Those alternatives that are members in all such sets form the set of the 'innocently excludable' (IE) alternatives of ϕ.

For the inclusion part, we follow Bar-Lev & Fox (2020) and assume that the set consists of those alternatives which are consistent with the conjunction of the prejacent ϕ and the negation of any IE alternatives. For our purposes, we subtract ϕ from II (the reason for this will be clear once we divide the output of exhaustification into presupposed/non-at issue and assertive components):

(8) Innocently Includable alternatives of the prejacent ϕ:
   a. Take all maximal sets of alternatives of ϕ that can be assigned 'true' consistently with ϕ and the falsity of all IE alternatives of ϕ.
   b. The set of alternatives that are members in all such sets, minus the set which includes just the prejacent ϕ, is the set of 'innocently includable' (II) alternatives of ϕ.

How should a presuppositional exhaustification operator with both IE and II be formulated? We follow our original proposal that only the prejacent, ϕ, should be included in its assertive content (Bassi et al. 2021, Del Pinal 2021). Accordingly, the prejacent goes into the assertive and the negated IE alternatives into the presupposed content, as captured in (9a) and (9b-i). It also follows that the II alternatives should go into the presupposed content. Inspired by Goldstein's (2019) insight that FC inferences involve a kind of homogeneity presupposition, we incorporate them as follows: instead of simply including each II alternative, we include the subtly weaker homogeneity proposition that the II alternatives have the same truth value, as captured in (9b-ii). The former option might seem like a more direct implementation of Bar-Lev & Fox's proposal—which is that exhaustification of ϕ asserts the falsity of its IE alternatives and the truth of its II alternatives—but we will show, based on the FC puzzles, that it is descriptively inferior to our homogeneity-based suggestion. Call this version of presuppositional exhaustification '$\mathbf{pex}^{IE+II}$':

(9) For a structure ϕ of propositional type and a local context c, $[\![\mathbf{pex}^{IE+II}(\phi)]\!]$:

a. **asserts:** $[\![\phi]\!]$
b. **presupposes:**
(i) $\bigwedge \neg [\![\psi]\!] : \psi \in IE(\phi) \wedge [\![\psi]\!] \in R_c$
(ii) $\forall \alpha((\alpha \in II(\phi) \wedge [\![\alpha]\!] \in R_c) \rightarrow [\![\alpha]\!] = 1) \vee$
$\forall \alpha((\alpha \in II(\phi) \wedge [\![\alpha]\!] \in R_c) \rightarrow [\![\alpha]\!] = 0)$
where R_c = a contextually assigned 'relevance' predicate, which minimally satisfies the following two conditions: (i) the prejacent, ϕ, is relevant (i.e., $[\![\phi]\!] \in R_c$), and (ii) any proposition that is contextually equivalent (in c) to the prejacent is also in R_c.

$\mathbf{pex}^{IE+II}$ can be seen as way of integrating three ideas about covert exhaustification: (i) it is a presupposition trigger such that its assertive content is fully determined by the prejacent (Bassi et al. 2021, Del Pinal 2021), (ii) it has both exclusion and inclusion functions (Bar-Lev & Fox 2020), and (iii) it can be used to implement and generalize the contention that basic free choice effects involve a homogeneity presupposition (Goldstein 2019), yet within a Grammatical approach to SIs which preserves the commitment to model our core semantic competence with connectives, quantifiers and modal operators using classical modal logic.

Other than sensitivity to *II*, our implementation of $\mathbf{pex}^{IE+II}$ follows Bassi et al. (2021) and Del Pinal (2021). Propositional clauses are parsed with $\mathbf{pex}^{IE+II}$ by default, and 'implicature cancelation' is usually handled by showing that the corresponding alternative is not in R_c (cf. Magri 2009, 2011). Since there are various kinds of 'non-at issue' contents (Tonhauser et al. 2013), what specific properties are we attributing to the implicatures triggered by $\mathbf{pex}^{IE+II}$? In terms of their global constraints, we assume that if the implicatures contributed by $\mathbf{pex}^{IE+II}$ are consistent yet not entailed by the common ground, they are accommodated by default. Still, like non-at issue content in general, the implicatures triggered by $\mathbf{pex}^{IE+II}$ should not be inconsistent with the common ground. Most importantly here, when $\mathbf{pex}^{IE+II}$ is embedded, any implicatures it triggers behave, with respect to projection and licensing conditions for local accommodation, like typical presuppositions.

To begin to illustrate how $\mathbf{pex}^{IE+II}$ works, we first consider its effect on simple (non-FC) scalar sentences, and show that its predictions match those obtained with $\mathbf{pex}^{IE}$ without *II*. On our approach, a simple scalar sentence like (10) is parsed by default as in (10a). Given the alternatives in (10b), the only alternative in *IE* is $\forall$ and the set of *II* alternatives is empty, as captured in (10c)-(10d). Assuming that the $\forall$-alternative is relevant, the interpretation of (10a) is then as in (10e).

(10) Some students passed the exam.
a. $\mathbf{pex}^{IE+II}$[some students passed] $= \mathbf{pex}^{IE+II}(\exists)$
b. Alt($\exists$) = $\{\exists, \forall\}$

c. IE(∃) = {∀}
d. II(∃) = { }
e. $[\![(10a)]\!] = \begin{cases} \textbf{ps: } \neg\text{all students passed} & (= \neg\forall) \\ \textbf{asserts: } \text{some students passed} & (= \exists) \end{cases}$

Since *II* is inert in this case, **pex**$^{IE+II}$ has the same effect as **pex**^{IE} without *II*. This also holds for other basic (non-FC) scalar sentences. For example, it is easy to check that for exhaustification of disjunctions, such as *Mary had cake or ice-cream* (= ∨), $[\![\textbf{pex}^{IE+II}(\vee)]\!] = \vee_{\neg\wedge}$ (the subscripts of formulas indicate their presuppositions). That is equivalent to the overall reading and structuring into presupposed vs. asserted components predicted by $[\![\textbf{pex}^{IE}(\vee)]\!]$. Those correspondences are schematically captured in (11a)-(11b):

(11) a. $[\![\textbf{pex}^{IE+II}(\exists)]\!] = [\![\textbf{pex}^{IE}(\exists)]\!] = \exists_{\neg\forall}$
b. $[\![\textbf{pex}^{IE+II}(\vee)]\!] = [\![\textbf{pex}^{IE}(\vee)]\!] = \vee_{\neg\wedge}$

Moving to **pex**$^{IE+II}$, then, preserves the core characteristics of a **pex**^{IE} account of basic SIs: its output is structured into an assertive component fully determined by the prejacent and a presuppositional/non-at issue component determined by any (relevant) *IE* or *II* alternatives. In Bassi et al. (2021) and Del Pinal (2021) we argue that this perspective solves various puzzles concerning (i) how SIs project from various embedded positions, including the conditions under which they are locally accommodated, and (ii) why SIs tend to generate oddness (and are hard to globally accommodate) when they conflict with the common ground.

2.2 Derivation of basic free choice and double prohibition effects

We now use **pex**$^{IE+II}$ to derive FC for ◊∨-sentences, double prohibition for ¬◊∨-sentences, and related negative FC effects. Unlike the predictions obtained using a flat output **exh**$^{IE+II}$ operator, those readings are predicted to be structured into non-at issue and at issue components. The difference is subtle, but we will show in §3-§5 that it is the key to solve our puzzles related to embedded FC effects.

As shown in (12a)-(12d), **pex**$^{IE+II}$ issues in a simple derivation of the FC readings of ◊∨-sentences. Given the prejacent, $\Diamond(p\vee q)$, and its formal alternatives in (12b), there is only one *IE* alternative, $\Diamond(p\wedge q)$, as captured in (12c). In addition, since the disjunctive alternatives, $\Diamond p$ and $\Diamond q$, can be simultaneously and consistently conjoined with the prejacent and the negation of the *IE* alternatives—i.e., with $\Diamond(p\vee q)\wedge\neg\Diamond(p\wedge q)$—they are in *II*, as captured in (12d). Based on our formulation of **pex**$^{IE+II}$, we then have to add, as presuppositions, the negation of each *IE* alternative and the homogeneity proposition that the *II* alternatives get the same truth-value, as captured in the **ps** part of (12e).

(12) a. $\mathbf{pex}^{IE+II}[\Diamond[p \vee q]]$
b. $Alt(\Diamond[p \vee q]) = \{\Diamond[p \vee q], \Diamond p, \Diamond q, \Diamond[p \wedge q]\}$
c. $IE(\Diamond[p \vee q]) = \{\Diamond[p \wedge q]\}$
d. $II(\Diamond[p \vee q]) = \{\Diamond p, \Diamond q\}$
e. $[\![$(12a)$]\!] = \begin{cases} \textbf{ps: } (\Diamond p \leftrightarrow \Diamond q) \wedge \neg\Diamond(p \wedge q) \\ \textbf{asserts: } \Diamond(p \vee q) \end{cases}$

Note that (12e) entails FC, i.e., $\Diamond p \wedge \Diamond q$. Since the conjunctive alternative, $\Diamond[p \wedge q]$, is not relevant for our target FC puzzles, we assume from now on that it is not a member of the set of contextually supplied relevent propositions, R_c, hence by the definition in (9) its negation is pruned from the **ps** part of $[\![\mathbf{pex}^{IE+II}[\Diamond[p \vee q]]]\!]$.[1] For the FC reading of $\Diamond\vee$-sentences, then, the $\mathbf{exh}^{IE+II}$ and $\mathbf{pex}^{IE+II}$ accounts predict the same overall entailments, as captured in (13)-(14):

(13) $[\![\mathbf{exh}^{IE+II}[\Diamond[p \vee q]]]\!] = \Diamond(p \vee q) \wedge \Diamond p \wedge \Diamond q$

(14) $[\![\mathbf{pex}^{IE+II}[\Diamond[p \vee q]]]\!] = \Diamond(p \vee q)_{\Diamond p \leftrightarrow \Diamond q}$ $\qquad \models \Diamond p \wedge \Diamond q$

Yet we will show later that, for certain embedded cases like those involved in our FC puzzles, theories which output uniformly flat, assertive interpretations, as in (13), make incorrect predictions avoided by theories which output interpretations which are structured into presupposed and assertive components as in (14).

Crucially, the structure assigned to the FC reading of $\Diamond\vee$-sentences opens up an alternative and, as we will see, ultimately advantageous way of deriving double prohibition for $\neg\Diamond\vee$-sentences. We have just seen that a $\Diamond\vee$-sentence like (15), given its default parse in (15a), gets an FC interpretation which is divided into a presupposed part, $\Diamond T \leftrightarrow \Diamond B$, and an assertive part, $\Diamond(T \vee B)$.

(15) Maria can study in Tokyo or Boston.
a. $[\![\mathbf{pex}^{IE+II}[\Diamond[T \vee B]]]\!] = \Diamond(T \vee B)_{\Diamond T \leftrightarrow \Diamond B}$ $\qquad \models \Diamond T \wedge \Diamond B$

Now consider a $\neg\Diamond\vee$-sentences such as (16). According to **exh** accounts, recall, double prohibition is obtained by dropping **exh** from under negation. This follows from ‘economy’: a parse with **exh** is dispreferred if it generates a weaker reading relative to a parallel one without **exh**. The situation changes subtly with $\mathbf{pex}^{IE+II}$. Since we take parses with local $\mathbf{pex}^{IE+II}$ as the default, the preferred/unmarked parse for (16) is (16a). Yet due to the structuring of the FC interpretation, the negation only directly affects the assertive output of $\mathbf{pex}^{IE+II}$ (i.e., the $\Diamond\vee$ prejacent). As a result, (16a) doesn't violate economy, and gets the double prohibition reading.

1 $\Diamond\vee$-sentences have two FC readings (Simons 2005a): (12e) entails permission to choose any but not both options, and (14) entails permission to choose any and also both options.

(16) Maria can't study in Tokyo or Boston.

a. $[\![\neg[\mathbf{pex}^{IE+II}[\Diamond[T \vee B]]]]\!] = \neg(\Diamond(T \vee B)_{\Diamond T \leftrightarrow \Diamond B}) = \neg\Diamond(T \vee B)_{\Diamond T \leftrightarrow \Diamond B}$
$\models \neg\Diamond T \wedge \neg\Diamond B$

The key result, then, is that we can derive the double prohibition reading of $\neg\Diamond\vee$-sentences with a $\mathbf{pex}^{IE+II}$ operator below the negation.

Concerning the structure of the FC reading of $\Diamond\vee$-sentences, and how that supports double prohibition under negation despite using uniform LFs, our theory is closer to Goldstein's (2019) homogeneous alternative semantics than to standard **exh**-based Grammatical theories. Goldstein's theory builds on Lexicalist theories which hold that disjunction introduces a set of alternatives corresponding to each disjunct, and possibility modals universally quantify such that each alternative in their scope has to be possible given the set of accessible worlds (Simons 2005b, Aloni 2007). Goldstein adds the stipulation that possibility modals also have a built-in homogeneity presupposition such that all the alternatives in their scope should get the same truth-value. Combined with an appropriate notion of Strawson-entailment, the system predicts the FC and double prohibition readings for $\Diamond\vee$ and $\neg\Diamond\vee$-sentences based on surface form LFs, and assigns them an assertive and projective structure similar to that predicted by $\mathbf{pex}^{IE+II}$.[2]

Despite that similarity for $\Diamond\vee$ and $\neg\Diamond\vee$-sentences, $\mathbf{pex}^{IE+II}$ predicts—like other Grammatical accounts but unlike Goldstein's and similar Lexicalist accounts (Ciardelli et al. 2018, Aloni 2018, Rothschild & Yablo 2018)—that $\neg\Box\wedge$-sentences like (17) have an available 'negative FC' reading, as in (17a) (see Marty et al. 2021 for experimental data). As we will see, to directly solve all the presupposed & filtering FC puzzles, we need a way to derive (embedded) negative FC.

(17) Maria is not required to visit Tokyo and Boston.
$\neg\Box(T \wedge B) \Leftrightarrow \neg\Box T \vee \neg\Box B$

a. $\rightsquigarrow$ *Maria is not required to visit Tokyo*
$\rightsquigarrow$ *Maria is not required to visit Boston* $\neg\Box T \wedge \neg\Box B$

2 Tieu, Bill & Romoli (2019) test experimentally whether FC for $\Diamond\vee$ and double prohibition for $\neg\Diamond\vee$-sentences involve a homogeneity presupposition. Suppose that (15) and (16) are evaluated in a situation s_1 that is *inconsistent* with homogeneity, e.g., one in which Maria can study in Tokyo but not Boston. **Exh** theories predict that, in s_1, (15) should be judged as true but having a false implicature (for s_1 conflicts with the exhaustified but not with the bare content of the prejacent), whereas (16) should be judged as just false (since double prohibition doesn't involve exhaustification). In contrast, $\mathbf{pex}^{IE+II}$ and Goldstein's theory predict that the default FC reading of (15) and the double prohibition reading of (16) presuppose $\Diamond T \leftrightarrow \Diamond B$—so in s_1 (15) and (16) should be judged as presupposition failures. Tieu et al. interpret their results as supporting the latter prediction, although it is controversial whether they uses appropriate measures and controls for false implicatures vs. presupposition failures.

Using $\textbf{pex}^{IE+II}$, negative FC for $\neg\Box\wedge$-sentences can be derived from the parse in (18a), analogous to the one used by other Grammatical theories (see Fox 2007, Bar-Lev & Fox 2020). As captured in (18c), the only *IE* alternative is $\neg\Box[p \vee q]$. In addition, since $\neg\Box p$ and $\neg\Box q$ can together be consistently conjoined with the prejacent and the negation of the *IE* alternative—i.e., with $\neg\Box[p \wedge q] \wedge \Box[p \vee q]$—they are in *II*, as captured in (18d). Given our formulation of $\textbf{pex}^{IE+II}$, the negation of the *IE* alternative and the homogeneity presupposition that the *II* alternatives get the same truth-value go into the **ps** dimension, as in (18e).

(18) a. $\textbf{pex}^{IE+II}[\neg\Box[p \wedge q]]$
b. $Alt(\neg\Box[p \wedge q]) = \{\neg\Box[p \wedge q], \neg\Box p, \neg\Box q, \neg\Box[p \vee q]\}$
c. $IE(\neg\Box[p \wedge q]) = \{\neg\Box[p \vee q]\}$
d. $II(\neg\Box[p \wedge q]) = \{\neg\Box p, \neg\Box q\}$
e. $[\![(12a)]\!] = \begin{cases} \textbf{ps: } \neg\Box p \leftrightarrow \neg\Box q \;\wedge\; \Box(p \vee q) \\ \textbf{asserts: } \neg\Box(p \wedge q) \end{cases}$

The *IE* disjunctive alternative, $\neg\Box[p \vee q]$, is not relevant for our puzzles, so we assume that it is not in the contextually supplied set of relevant propositions, R_c, hence its negation is prunned from the **ps** part of $[\![\textbf{pex}^{IE+II}[\neg\Box[p \wedge q]]]\!]$. We can then capture the negative FC reading of (17) with the parse in (19a), and include in (19b), for comparison, the corresponding parse and interpretation using $\textbf{exh}^{IE+II}$:

(19) a. $[\![\textbf{pex}^{IE+II}[\neg\Box[T \wedge B]]]\!] = (\neg\Box T \vee \neg\Box B)_{\neg\Box T \leftrightarrow \neg\Box B} \quad \models \neg\Box T \wedge \neg\Box B$
b. $[\![\textbf{exh}^{IE+II}[\neg\Box[T \wedge B]]]\!] = (\neg\Box T \vee \neg\Box B) \wedge \neg\Box T \wedge \neg\Box B$

As before, $\textbf{pex}^{IE+II}$ and $\textbf{exh}^{IE+II}$ predict the same overall entailments for the negative FC readings of $\neg\Box\wedge$-sentences. However, $\textbf{pex}^{IE+II}$ structures that interpretation into an assertive component—i.e., the prejacent $(\neg\Box T \vee \neg\Box B)$—and a presuppositional component—i.e., homogeneity over the *II* alternatives $\neg\Box T \leftrightarrow \neg\Box B$. Finally, note that we can also derive negative FC with $\textbf{pex}^{IE+II}$ under negation, as in (20):

(20) $[\![\neg\,\textbf{pex}^{IE+II}[\Box[T \wedge B]]]\!] = (\neg\Box T \vee \neg\Box B)_{\Box T \leftrightarrow \Box B} \quad \models \neg\Box T \wedge \neg\Box B$

The embedded $\textbf{pex}^{IE+II}$ has a strong prejacent, $\Box[T \wedge B]$, which doesn't have any *IE* alternatives; but the alternatives $\Box T$ and $\Box B$ are in *II* and so we add a homogeneity presupposition $\Box T \leftrightarrow \Box B$. The matrix negation then applies directly to the prejacent of $\textbf{pex}^{IE+II}$, giving us $\neg\Box T \vee \neg\Box B$. When combined with the homogeneity presupposition, which projects from under negation, we get the strengthening to the 'negative FC' entailment $\neg\Box T \wedge \neg\Box B$. This result will also prove useful later on.

Summing up, we have seen that our $\textbf{pex}^{IE+II}$ account of basic FC and double prohibition effects has two unique features compared to standard $\textbf{exh}^{IE+II}$ accounts. First, we can derive each of those readings from LFs with local applications of

pex^{IE+II}. Secondly, although we predict the same overall entailments, **pex**$^{IE+II}$ structures the interpretations into an assertive prejacent and presupposed homogeneity component. In sections §3-§5, we show that those unique elements of our theory support a uniform solution to the presupposed, filtering and related FC puzzles: the solutions are based on applying **pex**$^{IE+II}$ locally to the embedded basic sentences, and calculating how their assertive and projective components behave given standard assumptions about presupposition projection, filtering and accommodation.[3]

3 Free choice under (negative) factives

3.1 The challenge

The presupposed FC puzzle, due to Marty & Romoli (2020), concerns the behavior of FC sentences when embedded under certain (negative) factive attitude verbs. Consider (21), which has an embedded $\Diamond\vee$-sentence. On its default reading, (21) presupposes that Olivia has FC, as in (21a), yet asserts that Noah doesn't believe Olivia can take either one of Logic or Algebra, as in (21b). The latter is stronger than saying that what Noah doesn't believe is that Olivia has FC, which is compatible with Noah believing that she can take Logic but not Algebra, or vice-versa.

(21) Noah is unaware that Olivia can to take Logic or Algebra.

a. $\rightsquigarrow$ *Olivia can take Logic*
   $\rightsquigarrow$ *Olivia can take Algebra*

b. $\rightsquigarrow$ $\neg$*Noah believes that Olivia can take Logic*
   $\rightsquigarrow$ $\neg$*Noah believes that Olivia can take Algebra*

The embedded *Olivia can take Logic or Algebra*, then, seems to be simultaneously interpreted in two different ways: as having an enriched FC reading when determining the presuppositions triggered by the factive verb, and as having a non-enriched, classical reading when determining the content of Noah's beliefs.

Presupposed FC sentences like (21) pose a problem for standard **exh**-based Grammatical accounts of FC. We focus on **exh**$^{IE+II}$ for concreteness. Sentences

3 In Del Pinal et al. (2023) we show that we can derive the same assertive and presuppositional structure for basic FC and related interpretations using a purely IE-based **pex**. Those derivations are more complex in that they require recursive application of **pex**^{IE} and an independently motivated insertion of local accommodation. Yet the availability of a **pex**^{IE} derivation of our basic FC results should be interesting for those who think that, ultimately, we should (at least try to) derive rather than stipulate homogeneity effects. Relatedly, Fox (2020) proposes a modification of **pex**, based on exhaustivity in embedded questions, which says that the presupposition of **pex**(ϕ) is that ϕ (contextually) entails the negation of any excludable alternatives (to add inclusion, hold that **pex**(ϕ) also presupposes that ϕ contextually entails any includable alternative). As far as we can see, the results of this section could also be derived using that version of **pex**.

of the form 'x is unaware that p' are typically analyzed as presupposing that p and asserting that it is not the case that x believes that p, as captured in (22):

(22) $[\![x \text{ is unaware that } p]\!] = (\neg B_x(p))_p$

On $\mathbf{exh}^{IE+II}$ accounts, recall, double prohibition for $\neg\Diamond\vee$-sentences is obtained by applying an economy constraint which disfavors inserting $\mathbf{exh}^{IE+II}$ in positions where it would weaken overall meaning. So while the parse in (23a) supports FC, we drop $\mathbf{exh}^{IE+II}$ from under negation to get double prohibition, as in (23b):

(23) a. $[\![\mathbf{exh}^{IE+II}[\Diamond[L \vee A]]]\!] = \Diamond L \wedge \Diamond A$
b. $[\![\neg[\Diamond[L \vee A]]]\!] = \neg\Diamond L \wedge \neg\Diamond A$

To try to capture the default reading of (21), there are three parses to consider. If we parse (21) as in (24), with $\mathbf{exh}^{IE+II}$ over the embedded $\Diamond[L \vee A]$, we predict the correct presuppositions. For the factivity of *unaware* will guarantee that the FC content projects, as captured in the **ps** part of (24a). However, for the assertive part we predict that Noah doesn't believe that Olivia has FC, as captured in the **asserts** part of (24a). Yet the target reading, recall, is that Noah doesn't believe Olivia can take even one of the classes.

(24) Noah is unaware $[\mathbf{exh}^{IE+II}[\Diamond[L \vee A]]]$
a. $[\![(24)]\!] = \begin{cases} \textbf{ps: } \mathbf{exh}^{IE+II}(\Diamond(L \vee A)) = \Diamond L \wedge \Diamond A \\ \textbf{asserts: } \neg B_N(\mathbf{exh}^{IE+II}(\Diamond(L \vee A))) = \neg B_N(\Diamond L \wedge \Diamond A) \end{cases}$

If we parse (21) as in (25), where the embedded $\Diamond[L \vee A]$ isn't exhaustified, we predict the correct content for Noah's doxastic state, namely, that he doesn't believe Olivia can take even one of Logic or Algebra, as captured in the **asserts** part of (25a). However, as captured in the **ps** part of (25a), we now derive a presupposition that is too weak, namely, that Olivia is allowed to take one class but not necessarily the other one, yet the target is that Olivia has FC.

(25) Noah is unaware $[\Diamond[L \vee A]]$
a. $[\![(25)]\!] = \begin{cases} \textbf{ps: } \Diamond(L \vee A) \\ \textbf{asserts: } \neg B_N(\Diamond(L \vee A)) \end{cases}$

Finally, we can parse (21) as in (26), with matrix scope $\mathbf{exh}^{IE+II}$. In this case, having access to *II* seems to help Grammatical theories. An analogous parse with matrix recursive $\mathbf{exh}^{IE}$ would be vacuous—for the embedded $\Diamond[L \vee A]$ occurs in a DE environment, so the prejacent is already stronger than any of its alternatives, in particular, its stronger than *Noah is unaware* $\Diamond L$ and *Noah is unaware* $\Diamond A$. For the same reason, there are no *IE* alternatives for $\mathbf{exh}^{IE+II}$ in (26). Yet the alternatives

Noah is unaware $\Diamond L$ and *Noah is unaware* $\Diamond A$ are in *II*—and they presuppose, respectively, $\Diamond L$ and $\Diamond A$.

(26) $\textbf{exh}^{IE+II}$[Noah is unaware $[\Diamond[L \vee A]]$]

a. $[\![(26)]\!] = \begin{cases} \textbf{ps: } \Diamond(L \vee A) \overbrace{\wedge(\Diamond L \wedge \Diamond A)}^{\text{if } \textbf{exh}^{IE+II} \text{ is a ps hole}} \\ \textbf{asserts: } \neg B_N(\Diamond(L \vee A)) \end{cases}$

Adding those *II* alternatives has no effect on the assertive content of the prejacent, as captured in the **asserts** part of (26a). But if we assume that $\textbf{exh}^{IE+II}$ is a presupposition hole with respect to the presuppositions of its prejacent and of any of its *IE* and *II* alternatives, then those *II* alternatives would strengthen the **ps** level—by adding their presuppositions $\Diamond L$ and $\Diamond A$—which gets us the target FC inference.

Yet building on Gajewski & Sharvit (2012), Marty & Romoli reject the view that $\textbf{exh}^{IE+II}$ is a presupposition hole for the prejacent and any *IE* or *II* alternatives. One problem is that we get incorrect predictions about presuppositional level enrichments for simple SIs under factives. Consider this oddness contrast:

(27) *C*: *all students took Logic.*

a. #John is unaware that some students took Logic.
b. John is unaware that all students took Logic.

At the assertive level, it seems that there is nothing wrong with using (27a) or (27b) in *C*. The key difference is at the presuppositional level: (27a) triggers an existential proposition that is too weak given the common ground that all students took Logic. Yet this mismatch would be prevented by parsing (27a) with matrix scope $\textbf{exh}^{IE+II}$, if it was a generalized presupposition hole (cf. Spector & Sudo 2017):

(28) $\textbf{exh}^{IE+II}$[Noah is unaware $[\exists x \in S[L(x)]]$]

a. $[\![(28)]\!] = \begin{cases} \textbf{ps: } \exists x \in S(L(x)) \wedge \forall x \in S(L(x)) \\ \textbf{asserts: } \neg B_N(\exists x \in S(L(x))) \end{cases}$

The alternative, *Noah is unaware* $[\forall x \in S[L(x)]]$, is in *II*. At the assertive level, it doesn't add anything, since its assertive content is entailed by that of the prejacent. At the presuppositional level, however, it would pass on the content that all students took Logic, as captured in the **ps** part of (28a). Yet this would lead us to expect, incorrectly, that (27a) should be fine when uttered in *C*.

The standard **exh**-based account of FC, then, has difficulty deriving the default reading of sentences like (21). And as Marty & Romoli show, various other versions of the Grammatical approach are also seriously challenged by this puzzle.

3.2 A solution based on presuppositional exhaustification

In contrast, the default reading of sentences like (21) with presupposed FC under negative factives is directly captured by our $\mathbf{pex}^{IE+II}$ theory. In §2.2 we showed that the parse in (29a) supports the FC reading of basic $\Diamond\vee$-sentences. Due to the predicted division between its assertive and presuppositional components, we can maintain—unlike **exh**-theories—that $\Diamond\vee$-sentences under negation are locally exhaustified, as in (29b), and still get their default double prohibition reading:

(29) a. $[\![\mathbf{pex}^{IE+II}[\Diamond[L\vee A]]]\!] = \Diamond(L\vee A)_{\Diamond L\leftrightarrow\Diamond A}$ $\models \Diamond L\wedge\Diamond A$

b. $[\![\neg[\mathbf{pex}^{IE+II}[\Diamond[L\vee A]]]]\!] = \neg\Diamond(L\vee A)_{\Diamond L\leftrightarrow\Diamond A}$ $\models \neg\Diamond L\wedge\neg\Diamond A$

This projective component of the (locally) exhaustified reading of the embedded $\Diamond\vee$-sentences is the key to derive the default readings for various sentences in which $\Diamond\vee$ and related FC sentences appear in other DE environments, as we will now show for FC under negative factives.

To deal with FC sentences under negative factives, we have to calculate the effect of embedded $\mathbf{pex}^{IE+II}$. Since $\mathbf{pex}^{IE+II}$ is a presupposition trigger, we need to consider how presuppositions project under doxastic operators. Depending on specific operators, triggers, and contexts two projection behaviors have been observed, the 'opaque' one in (30a) and the 'transparent' one in (30b). Theories differ on which behavior they derive as the default and which as the special case: e.g., satisfaction theories derive the opaque (30a) as the default and the transparent (30b) as a special case, while DRT accounts tend to predict the opposite.

(30) a. $B_x(p'_p) = B_x(p')_{B_x(p)}$ Heim (1992)/Schlenker (2009)

b. $B_x(p'_p) = B_x(p')_p$ Geurts (1999)/Van der Sandt (1992)

With that in mind, consider again (21), repeated in (31). The default parse in our theory is (31a) (ignore for simplicity an additional matrix $\mathbf{pex}^{IE+II}$, which could operate over alternatives obtained by replacements of *Noah* or *Olivia*, among other options not relevant here):

(31) Noah is unaware that Olivia can take Logic or Algebra.

a. Noah is unaware $[\mathbf{pex}^{IE+II}[\Diamond[L\vee A]]]$

b. $[\![(31a)]\!]= \begin{cases} \textbf{ps: } \mathbf{pex}^{IE+II}(\Diamond(L\vee A)) \\ \textbf{asserts: } \neg B_N(\mathbf{pex}^{IE+II}(\Diamond(L\vee A))) \end{cases}$

c. $\mathbf{pex}^{IE+II}(\Diamond(L\vee A)) = \Diamond(L\vee A)_{\Diamond L\leftrightarrow\Diamond A}$

d. $\neg B_N(\mathbf{pex}^{IE+II}(\Diamond(L\vee A))) =$
$\neg(B_N(\Diamond(L\vee A)_{\Diamond L\leftrightarrow\Diamond A})) =$ (by 'opaque' (30a))
$\neg(B_N(\Diamond(L\vee A))_{B_N(\Diamond L\leftrightarrow\Diamond A)}) =$ (by projection from under $\neg$)
$(\neg B_N(\Diamond(L\vee A)))_{B_N(\Diamond L\leftrightarrow\Diamond A)}$

From the **ps** part of (31b), and the equivalence in (31c), we get the target result that (31) presupposes FC, i.e., that Olivia is allowed to take Logic and is allowed to take Algebra. We next need to check whether the **asserts** part of (31b), $\neg B_N(\mathbf{pex}^{IE+II}(\Diamond(L \vee A)))$, captures the target content for Noah's beliefs. Given the presupposed and assertive outputs of the embedded $\mathbf{pex}^{IE+II}$, and the opaque rule in (30a) for how belief operators interact with presuppositions in their scope, we derive the target content, as shown in (31d). What Noah doesn't believe, on this analysis, is that Olivia can take either one of Logic or Algebra (as opposed to Noah not believing merely that she has FC, which, recall, is compatible with Noah believing that Olivia can take Algebra but not Logic, or vice-versa). The parse in (31a), then, makes the correct predictions for the default interpretation of (31) concerning both its presuppositions and the content of the doxastic attribution.

Now, as captured by the last equivalence of (31d), we also predict the additional presupposition that Noah believes the homogeneity proposition that if Olivia can take either one of the classes, she can take the other one. This presupposition is perhaps unattested for (31). Yet we can avoid that prediction, without affecting the other desired parts of the derivation, by adopting, instead of (30a), the transparent rule in (30b) concerning how presuppositions project under doxastic operators. In this case, the derivation in (31d) should be replaced with the following:

(31) d′. $\neg B_N(\mathbf{pex}^{IE+II}(\Diamond(L \vee A))) =$
$\neg(B_N(\Diamond(L \vee A)_{\Diamond L \leftrightarrow \Diamond A})) =$ (by 'transparent' (30b))
$\neg(B_N(\Diamond(L \vee A))_{\Diamond L \leftrightarrow \Diamond A}) =$ (by projection under $\neg$)
$(\neg B_N(\Diamond(L \vee A)))_{\Diamond L \leftrightarrow \Diamond A}$

We now predict that $\neg B_N(\mathbf{pex}^{IE+II}(\Diamond(L \vee A))) \Leftrightarrow (\neg B_N(\Diamond(L \vee A)))_{\Diamond L \leftrightarrow \Diamond A}$. Note that the homogeneity presupposition, $\Diamond L \leftrightarrow \Diamond A$, doesn't add any presuppositional constraints to (31), since it is entailed by the content of the presupposition triggered by the factivity of *unaware*, spelled out in (31c). So this route captures exactly the target reading singled out by Marty & Romoli.

This result doesn't commit us to the view that, in general and as a default, presuppositions triggered under doxastic and epistemic operators project transparently as in (30b). Indeed, a sentence like *Noah believes that Olivia is allowed to take Logic or Algebra*, in its most salient reading, attributes to Noah the belief that Olivia has FC. Given our account, a simple derivation is via a parse with embedded $\mathbf{pex}^{IE+II}$ over the $\Diamond\vee$-sentence, yet using opaque projection as in (30a). For Noah is then represented as believing both $\Diamond(L \vee A)$ and $\Diamond L \leftrightarrow \Diamond A$, hence that Olivia has FC.[4] In this case, we don't seem to get a transparent reading in which (i) homogeneity

4 Alxatib (2023) shows that a parse with embedded $\mathbf{pex}^{IE+II}$, given an opaque projection rule, accounts for various puzzles involving ellipsis and $\Diamond\vee$/$\neg\Diamond\vee$-sentences in the scope of doxastic operators.

projects through John's belief worlds and imposes a condition on the actual world, and (ii) what John believes is that Olivia can take at least one of Logic or Algebra but need not have FC. But is it reasonable to hold that the homogeneity component of embedded FC sentences can or has the tendency to project transparently or opaquely depending on the specific type of doxastic or epistemic operator that scopes over it?

It is fair to say that we still don't fully understand what factors affect how presuppositions project from the scope of doxastic and epistemic operators, and whether specific operator-trigger pairs generate distinctive projection and accommodation patterns (Blumberg 2023, Alxatib 2023). Crucially, however, there is arguably a class of triggers which behave analogous to what we have assumed for exhaustified $\Diamond\vee$-sentences: i.e., they tend to project opaquely under *believe* but transparently under *unaware*. Consider the projection patterns with *only* in (32), given the standard theory that *only* presupposes its prejacent and asserts the negation of its alternatives:

(32) a. John believes that only Peter came.
b. John is unaware that only Peter came.

The default reading of (32a) fits opaque projection: it presupposes that John believes Peter came, and asserts that John believes no one else (from the salient class) came. In contrast, (32b) also has a salient reading that exhibits transparent projection. Suppose Peter but no one else came to the party and that although John hopes many would, he is ignorant about the facts. (32b) is intuitively true in that situation. Yet that suggests that the presupposition triggered by the embedded *only* sentence—that Peter came—has to project transparently out of John's doxastic alternatives.[5]

3.3 Extension to presupposed negative free choice

The puzzle of presupposed FC extends to other types of embedded FC sentences, not just $\Diamond\vee$-sentences. This is important because some recent Lexicalist accounts—e.g., Goldstein (2019)—resolve the puzzle with $\Diamond\vee$ but not with $\neg\Box\wedge$-sentences under negative factives, i.e., the 'negative' FC versions of the puzzle.

5 The same point can be made using a more standard trigger like *found out*:

(i) a. John believes that Peter found out that Sue was the thief.
b. John is unaware that Peter found out that Sue was the thief.

(ia) defaults to opaque projection: it presupposes that John believes that Sue was the thief and asserts that John believes that Peter believes that Sue was the thief. Yet (ib) also has a salient transparent projection reading. Suppose John is unsure whether Sue was the thief, and Peter is a detective who, unbeknownst to John, just discovered that Sue was the thief. (ib) is intuitively true in that situation. Yet that suggests that the presupposition triggered by 'found out' (that Sue was the thief) projects transparently out of John's beliefs.

To illustrate this version, consider (33). In one of its salient readings, (33) presupposes that Olivia is not required to take Logic and is also not required to take Algebra ('negative FC'), as in (33a), and asserts that Noah doesn't believe that Olivia isn't required to take either one, as in (33b), which is stronger than not believing that Olivia doesn't have negative FC, since only the latter is compatible with believing that she is required to take Logic but not required to take Algebra, or vice-versa.

(33) Noah is unaware that Olivia is not required to take Logic and Algebra.

a. $\rightsquigarrow$ *Olivia is not required to take Logic*
$\rightsquigarrow$ *Olivia is not required to take Algebra*

b. $\rightsquigarrow$ ¬*Noah believes Olivia is not required to take Logic*
$\rightsquigarrow$ ¬*Noah believes that Olivia is not required to take Algebra*

These cases challenge any Lexicalist theory that doesn't derive negative FC for basic $\neg\Box\wedge$-sentences such as (17). And while standard **exh** theories do predict negative FC, Marty & Romoli (2020) show that in these cases they face problems analogous to those discussed in §3.1 for $\Diamond\vee$-sentences under negative factives.

In contrast, our $\textbf{pex}^{IE+II}$ account also resolves this version of the puzzle. The solution follows directly from our analysis of the negative FC reading of $\neg\Box\wedge$-sentences, presented in §2.2, combined with the same assumptions used in §3.2 to solve the original version of the puzzle.

The parse in (34a) is structurally analogous to the one we used to derive the default reading of $\Diamond\vee$-sentences under (negative) factives. As shown in the **ps** part of (34b), this predicts, as desired, that (34) presupposes negative FC. What is the prediction for Noah's doxastic state? On the target reading, Noah doesn't believe that Olivia is not required to take Logic, and also doesn't believe that Olivia is not required to take Algebra (again that state is different from not believing that Olivia has the (negative) FC to not take either class). Crucially, that is precisely what we predict for the **asserts** part, captured in (34b), given the equivalences in (34d). As before, although the **asserts** part triggers the homogeneity presupposition $\neg\Box p \leftrightarrow \neg\Box q$, that proposition is already entailed by the **ps** part, as captured in (34c), so it doesn't strengthen the overall presuppositions of (34).

(34) Noah is unaware that Olivia is not required to take Logic and Algebra

a. Noah is unaware $\textbf{pex}^{IE+II}[\neg\Box[L\wedge A]]$

b. $[\![(34a)]\!] = \begin{cases} \textbf{ps: pex}^{IE+II}(\neg\Box(L\wedge A)) \\ \textbf{asserts: } \neg B_N(\textbf{pex}^{IE+II}(\neg\Box(L\wedge A))) \end{cases}$

c. $\textbf{pex}^{IE+II}(\neg\Box(L\wedge A)) = (\neg\Box L\vee\neg\Box A)_{\neg\Box L\leftrightarrow\neg\Box A}$

d. $\neg B_N(\textbf{pex}^{IE+II}(\neg\Box(L\wedge A)))$
$= \neg(B_N((\neg\Box L\vee\neg\Box A)_{\neg\Box L\leftrightarrow\neg\Box A}))$ (by 'transparent' (30b))

$= \neg(B_N(\neg\Box L \vee \neg\Box A)_{\neg\Box L \leftrightarrow \neg\Box A})$ (by projection under $\neg$)
$= (\neg B_N(\neg\Box L \vee \neg\Box A))_{\neg\Box L \leftrightarrow \neg\Box A}$

3.4 Comparison with other revised Grammatical accounts

Marty & Romoli (2020) develop a novel Grammatical account partly to resolve the presupposed FC puzzles. Their account combines insights from Magri (2009), Gajewski & Sharvit (2012) and Spector & Sudo (2017) concerning the effect of exhaustification on assertive and presuppositional content in its scope, with Bar-Lev & Fox (2020)'s proposal that exhaustification has both *IE* and *II* functions. In this section, we introduce Marty & Romoli's theory, focusing on FC, and highlight the similarities and differences vis-à-vis our $\mathbf{pex}^{IE+II}$ account. As we will see, their theory resolves the puzzle of presupposed FC under negative factives. Yet in §4-§5 we show that it doesn't help with the other embedded FC puzzles.

Marty & Romoli call their exhaustification operator '$\mathbf{exh}^{IE+II}_{asr+prs}$'. To see what is distinctive about it, suppose that its prejacent, ϕ_p, triggers a non-trivial presupposition p. $\mathbf{exh}^{IE+II}_{asr+prs}$ is sensitive to 'assertive' (*asr*) and 'presuppositional' (*prs*) formal alternatives to ϕ_p: *asr*-alternatives are neither logically nor Strawson-entailed by ϕ_p, while *prs*-alternatives are not logically but are Strawson-entailed by ϕ_p. Marty & Romoli then define *IE* and *II* sets for each of the *asr* and *prs* alternatives, and propose that $\mathbf{exh}^{IE+II}_{asr+prs}$ performs the following operations on those sets:

(35) $$[\![\mathbf{exh}^{IE+II}_{asr+prs}(\phi_p)]\!](w) = \begin{cases} \textbf{ps: } p(w) \wedge \forall\chi_r \in (IE_{asr} \cup II_{asr} \cup II_{prs})[r(w)] \wedge \forall\psi_q \in IE_{prs}[\neg q(w)] \\ \textbf{asserts: } [\![\phi_p]\!](w) \wedge \forall\chi_r \in II_{asr}[[\![\chi_r]\!](w)] \wedge \forall\psi_q \in IE_{asr}[\neg[\![\psi_q]\!](w)] \end{cases}$$

Let us go over the main elements of this operator. At the assertive level, $\mathbf{exh}^{IE+II}_{asr+prs}$ replicates Bar-Lev & Fox's $\mathbf{exh}^{IE+II}$: it asserts the prejacent, ϕ_p, each alternative in II_{asr}, and the negation of each alternative in IE_{asr}. At the presupposition level, $\mathbf{exh}^{IE+II}_{asr+prs}$ adds any presuppositions triggered by its prejacent or by any alternative in IE_{asr}, II_{asr}, and II_{prs}, and also the *negation* of any presuppositions triggered by any alternatives in IE_{prs}.

Crucially, unlike $\mathbf{pex}^{IE+II}$, $\mathbf{exh}^{IE+II}_{asr+prs}$ is not itself a presupposition trigger: specifically, if neither its prejacent nor any of its formal alternatives are presuppositional, then the output of $\mathbf{exh}^{IE+II}_{asr+prs}$ matches the fully assertive output of $\mathbf{exh}^{IE+II}$. To illustrate, consider the derivation of FC for $\Diamond\vee$-sentences based on the parse in (36a). $\Diamond[L \wedge A]$ is the only IE_{asr}-alternative and $\Diamond L$ and $\Diamond A$ the only II_{asr}-alternatives. Applying (35), $\mathbf{exh}^{IE+II}_{asr+prs}$ will then add to the assertive level $\Diamond L \wedge \Diamond A$ and the negation of $\Diamond[L \wedge A]$. This is equivalent to what we would get for $\mathbf{exh}^{IE+II}[\Diamond[L \vee A]]$.

(36) Olivia can take Logic or Algebra.

a. $\mathbf{exh}^{IE+II}_{asr+prs}[\Diamond[L \vee A]]$
b. $Alt(\Diamond[L \vee A]) = \{\Diamond[L \vee A], \Diamond L, \Diamond A, \Diamond[L \wedge A]\}$
c. $IE_{asr}(\Diamond[L \vee A]) = \{\Diamond[L \wedge A]\}$
d. $II_{asr}(\Diamond[L \vee A]) = \{\Diamond L, \Diamond A\}$
e. $IE_{prs}(\Diamond[L \vee A]) = II_{prs}(\Diamond[L \vee A]) = \emptyset$

In addition, since neither the prejacent, $\Diamond[L \vee A]$, nor any of its formal alternatives triggers any presuppositions, both sets of potential *prs* alternatives, IE_{prs} and II_{prs}, are empty. So applying $\mathbf{exh}^{IE+II}_{asr+prs}$ to basic (non-presuppositional) $\Diamond\vee$-sentences affects only its assertive level output, by adding any II_{asr} alternatives and the negation of any IE_{asr} alternatives. Accordingly, exhaustification of basic $\Diamond\vee$-sentences with $\mathbf{exh}^{IE+II}_{asr+prs}$ has the same effect as with $\mathbf{exh}^{IE+II}$, and it has the same overall entailments, but different at-issue vs presupposed components, as with $\mathbf{pex}^{IE+II}$ (we continue to treat the conjunctive alternative as not relevant):

(37) a. $[\![\mathbf{exh}^{IE+II}_{asr+prs}[\Diamond[p \vee q]]]\!] = [\![\mathbf{exh}^{IE+II}[\Diamond[p \vee q]]]\!] = \Diamond(p \vee q) \wedge \Diamond p \wedge \Diamond q$
b. $[\![\mathbf{pex}^{IE+II}[\Diamond[p \vee q]]]\!] = \Diamond(p \vee q)_{\Diamond p \leftrightarrow \Diamond q}$

How does $\mathbf{exh}^{IE+II}_{asr+prs}$ help solve the presupposed FC puzzle? The goal, recall, is to derive the reading of sentences like (21) captured in (21a)-(21b). Given the result in (37a), a parse as in (38)—parallel to the one that predicts the target reading with $\mathbf{pex}^{IE+II}$—gets the interpretation in (38a), which doesn't fully capture the target reading. From the factivity of 'unaware' and the embedded $\mathbf{exh}^{IE+II}_{asr+prs}(\Diamond(L \vee A))$ we get the desired FC entailment, but at the assertion level we predict the too weak reading that what Noah doesn't believe is that Olivia has FC.

(38) Noah is unaware $[\mathbf{exh}^{IE+II}_{asr+prs}[\Diamond[L \vee A]]]$

a. $[\![(38)]\!] = \begin{cases} \textbf{ps: } \mathbf{exh}^{IE+II}_{asr+prs}(\Diamond(L \vee A)) = \Diamond L \wedge \Diamond A \\ \textbf{asserts: } \neg B_N(\mathbf{exh}^{IE+II}_{asr+prs}(\Diamond(L \vee A))) = \neg B_N(\Diamond L \wedge \Diamond A) \end{cases}$

The parse in (39), with matrix $\mathbf{exh}^{IE+II}_{asr+prs}$, is more promising. For due to the factive presupposition triggered by *unaware*, the prejacent and all its formal alternatives, in (40), are presuppositional, so the novel operations of $\mathbf{exh}^{IE+II}_{asr+prs}$ can kick in.

(39) $\mathbf{exh}^{IE+II}_{asr+prs}$[Noah is unaware $[\Diamond[L \vee A]]$]

(40) $Alt(\text{Noah is unaware } [\Diamond[L \vee A]]) = \begin{cases} \text{Noah is unaware } [\Diamond[L \vee A]] \\ \text{Noah is unaware } [\Diamond[L \wedge A]] \\ \text{Noah is unaware } [\Diamond L] \\ \text{Noah is unaware } [\Diamond A] \end{cases}$

We continue to treat the conjunctive alternative as irrelevant. Of the remaining options in (40), none are *asr* alternatives. Yet the two disjunctive alternatives are *prs* alternatives (they can each be undefined when the prejacent is true), and while neither is in IE_{prs}, they are both in II_{prs}:

(41) $$II_{prs}(\text{Noah is unaware } [\Diamond[L \vee A]]) = \begin{cases} \text{Noah is unaware } [\Diamond L] \\ \text{Noah is unaware } [\Diamond A] \end{cases}$$

Based on (35), the effect of $\mathbf{exh}^{IE+II}_{asr+prs}$ in (39) is captured in (42). At the assertive level, it outputs the assertive content of the prejacent, $\neg B_N(\Diamond(L \vee A))$, and at the presuppositional level, it outputs the presupposition triggered by the prejacent, $\Diamond(L \vee A)$, and by each of the alternatives in II_{prs}, $\Diamond L$ and $\Diamond A$.

(42) $$[\![\mathbf{exh}^{IE+II}_{asr+prs}[\text{Noah is unaware } [\Diamond[L \vee A]]]]\!] = \begin{cases} \textbf{ps: } \Diamond(L \vee A) \wedge \Diamond L \wedge \Diamond A \\ \textbf{asserts: } \neg B_N(\Diamond(L \vee A)) \end{cases}$$

This captures the target reading of (21): it asserts that Noah doesn't believe that Olivia can take even one of Logic or Algebra, and presupposes that Olivia has FC.

Marty & Romoli show that their account also solves the presupposed negative FC version of the puzzle, illustrated in (33). The key is to again use a parse with matrix $\mathbf{exh}^{IE+II}_{asr+prs}$, as in (39). Matrix $\mathbf{exh}^{IE+II}_{asr+prs}$ also captures the contrast in (27). For in (27a) the alternative *John is unaware that all students took Logic* is in IE_{psr}, and so the negation of its presupposition (i.e., 'not all students took Logic') is added to the presuppositions of (27a), which conflicts with the common ground C and explains why it is odd. Despite these good results, we show in §4-§5 that $\mathbf{exh}^{IE+II}_{asr+prs}$ doesn't help Grammatical theories solve the filtering FC and related puzzles concerning FC effects in the scope of universal, existential and non-monotonic quantifiers.

3.5 Summary

The presupposed FC puzzle concerns the intricate behavior of $\Diamond\vee$ and $\neg\Box\wedge$-sentences when embedded under negative factives, as in (21) and (33). This puzzle challenges many influential theories of FC. Standard $\mathbf{exh}^{IE/IE+II}$ theories, such as Fox (2007) and Bar-Lev & Fox (2020), have problems with all versions of the puzzle, while Lexicalist theories which do not directly predict negative FC for $\neg\Box\wedge$-sentences, such as Ciardelli et al. (2018), Aloni (2018), Rothschild & Yablo (2018) and Goldstein (2019), have trouble with the presupposed negative FC cases. In contrast, our $\mathbf{pex}^{IE+II}$-based theory issues in a uniform solution to the puzzle, which follows directly from our account of (negative) FC for basic $\Diamond\vee$ and $\neg\Box\wedge$-sentences, and double prohibition for $\neg\Diamond\vee$-sentences, given standard assumptions about presupposition projection. Finally, we also saw that the Grammatical $\mathbf{exh}^{IE+II}_{asr+prs}$ theory developed by Marty & Romoli (2020) can also deal with this puzzle.

4 Filtering free choice

4.1 The challenge

The filtering FC puzzle, due to Romoli & Santorio (2019), concerns an intricate pattern of projection and filtering of FC effects in certain complex sentences. Consider the most salient reading of (43). The first main disjunct (*Maria can't study in Tokyo or Boston*) has the double prohibition reading usually assigned to $\neg\Diamond\vee$-sentences. In addition, although the second main disjunct (*she is the first/second in our family who can study in Japan/States*) triggers the FC presupposition that Maria can study in Japan and can study in the States, (43) as a whole doesn't inherit that FC presupposition, as captured in (43a). Given certain standard views of filtering in disjunctions, it's as if the negation of the first main disjunct somehow filters out the FC presupposition triggered by the second main disjunct.[6]

(43) Either Maria can't study in Tokyo or Boston, or she is the first in our family who can study in Japan and the second who can study in the States.

a. $\not\rightsquigarrow$ *Maria can study in Japan*
$\not\rightsquigarrow$ *Maria can study in the States*

Following Romoli & Santorio, we schematically represent (43) as in (44), where A^+/B^+ asymmetrically entails A/B, and $C_{\Diamond A\wedge\Diamond B}$ says that C is asserted while $\Diamond A \wedge \Diamond B$ is presupposed.

(44) Either $\neg\Diamond(A^+ \vee B^+) \vee C_{\Diamond A\wedge\Diamond B}$ $\qquad \not\rightsquigarrow \Diamond A; \not\rightsquigarrow \Diamond B$

In addition, we adopt the standard view that disjunctions with a presupposition in the second disjunct, $p \vee q_r$, project a conditional presupposition, as in (45) (Heim 1982, Chierchia 1995, Beaver 2001). It follows that r is filtered if $\neg p \models r$, which explains why a sentence like *Either Maria didn't study in Tokyo, or she is the first in our school who studied in Japan* doesn't presuppose that Maria studied in Japan. We also assume that presuppositions triggered in the first disjunct tend to project

6 To check that, on the preferred reading of (43), the first $\neg\Diamond\vee$ disjunct (*Maria can't study in Tokyo or Boston*) is interpreted as a double prohibition, consider a world w_1 in which Maria can study in Tokyo but can't study in Boston. On its preferred reading, (43) is false in w_1. Yet while the second disjunct is false in w_1, the first disjunct is false if it is interpreted as a double prohibition, but true if interpreted as the negation of FC. The reading of (43) in which *Maria can't study in Tokyo or Boston* entails the negation of FC isn't a problem for most theories of FC, but is also dispreferred.

unconditionally, as in (46) (since $\neg p_r \to q$ presupposes r, e.g., *if John doesn't find out that Mary came, she will feel sad* presupposes that Mary came).

(45) $p \vee q_r = \begin{cases} \textbf{ps: } \neg p \to r \\ \textbf{asserts: } p \vee q \end{cases}$

(46) $p_r \vee q = \begin{cases} \textbf{ps: } r \\ \textbf{asserts: } p \vee q \end{cases}$

Why then does (43) present a challenge to **exh** accounts of FC? On the one hand, to get double prohibition for the first disjunct of (43) (*Maria can't study in Tokyo or Boston*), we need to parse it without $\textbf{exh}^{IE+II}$ under negation, as in (47a). Yet we would then predict an incorrect presupposition for (43), captured in the **ps** part of (47b). For as shown in (47c), the conditional presupposition in the **ps** part doesn't filter out $\Diamond A$ and $\Diamond B$ (i.e., that Maria can study in Japan and in the States), since $\Diamond(A^+ \vee B^+) \not\models \Diamond A \wedge \Diamond B$.

(47) a. $\neg\Diamond[A^+ \vee B^+] \vee C_{\Diamond A \wedge \Diamond B}$

b. $[\![(47a)]\!] = \begin{cases} \textbf{ps: } \neg\neg\Diamond(A^+ \vee B^+) \to (\Diamond A \wedge \Diamond B) \\ \textbf{asserts: } \neg\Diamond(A^+ \vee B^+) \vee C \end{cases}$

c. $\neg\neg\Diamond(A^+ \vee B^+) \to (\Diamond A \wedge \Diamond B)$
$= \Diamond(A^+ \vee B^+) \to (\Diamond A \wedge \Diamond B)$ $\quad \neq \top$

On the other hand, consider a parse for (43) with $\textbf{exh}^{IE+II}$ under negation in the first main disjunct, as in (48a). As captured in the **ps** part of (48b), and given the equivalence in (48c), this would correctly filter out the presupposition, triggered in the second main disjunct, that Maria can study in Japan and can study in the States. However, we now loose double prohibition for the first disjunct, *Maria can't study in Tokyo or Boston*, and get instead the unattested and weaker 'negation of FC' reading, as captured in the **asserts** part of (48b).

(48) a. $\neg\textbf{exh}^{IE+II}[\Diamond[A^+ \vee B^+]] \vee C_{\Diamond A \wedge \Diamond B}$

b. $[\![(48a)]\!] = \begin{cases} \textbf{ps: } \neg\neg\textbf{exh}^{IE+II}(\Diamond(A^+ \vee B^+) \to (\Diamond A \wedge \Diamond B)) \\ \textbf{asserts: } \neg\textbf{exh}^{IE+II}(\Diamond(A^+ \vee B^+)) \vee C \end{cases}$

c. $\neg\neg\textbf{exh}^{IE+II}(\Diamond A^+ \vee \Diamond B^+) \to (\Diamond A \wedge \Diamond B)$
$= (\Diamond A^+ \wedge \Diamond B^+) \to (\Diamond A \wedge \Diamond B)$ $\quad = \top$

A parse like (48a), but with $\textbf{exh}^{IE+II}$ above the negation, also doesn't help: for since $\textbf{exh}^{IE+II}(\neg\Diamond(A^+ \vee B^+)) = \neg\Diamond(A^+ \vee B^+)$, we get the same result as with (47a).

Romoli & Santorio show that this puzzle also challenges various multidimensional theories which allow exhaustification to have an effect on the assertive and presuppositional content of its prejacent (e.g., Magri 2009, Gajewski & Sharvit 2012). As we saw in §3.4, Marty & Romoli (2020)'s $\textbf{exh}^{IE+II}_{asr+psr}$ theory integrates exclusion and inclusion functions with multidimensional exhaustification, improves the predictions of previous Grammatical accounts, and solves the presupposed FC puzzles. So we need to determine if it can also deal with the filtering FC puzzles.

It turns out that $\mathbf{exh}^{IE+II}_{asr+prs}$ doesn't help with this puzzle. Recall two facts about $\mathbf{exh}^{IE+II}_{asr+prs}$ established in §3.4. First, when applied to basic $\Diamond\vee$-sentences, $\mathbf{exh}^{IE+II}$ and $\mathbf{exh}^{IE+II}_{asr+prs}$ have the same effect, namely, FC at the assertive level and no presuppositions. For unlike $\mathbf{pex}^{IE+II}$, $\mathbf{exh}^{IE+II}_{asr+prs}$ is not a presupposition trigger, and when neither its prejacent nor any of its formal alternatives are presuppositional, $\mathbf{exh}^{IE+II}_{asr+prs}$ outputs the same fully assertive content as $\mathbf{exh}^{IE+II}$. Secondly, when its prejacent is presuppositional, $\mathbf{exh}^{IE+II}_{asr+prs}$ can only strengthen the presuppositions of its output: for it not only passes on the presuppositions of its prejacent, but also adds those of any alternatives in IE_{asr}, II_{asr}, and II_{prs}, plus the negation of any presuppositions in IE_{prs}. Just like the function of $\mathbf{exh}^{IE+II}$, at the assertive level, is to strengthen the content of its prejacent, $\mathbf{exh}^{IE+II}_{asr+prs}$ is designed to strengthen—not weaken—the presuppositional content of its prejacent.

With that in mind, consider possible parses, beginning with (49). Given the first fact—and in particular that $\mathbf{exh}^{IE+II}_{asr+prs}(\Diamond(A^+\vee B^+)) = \Diamond A^+ \wedge \Diamond B^+$—(49) predicts the same reading as the one predicted by the parallel parse with $\mathbf{exh}^{IE+II}$. That is, we get the filtering FC effect, but not double prohibition for the first main disjunct, since $\neg\mathbf{exh}^{IE+II}_{asr+prs}(\Diamond(A^+\vee B^+))$ amounts to the weaker 'denial of FC' reading.

(49) $\neg\mathbf{exh}^{IE+II}_{asr+prs}[\Diamond[A^+\vee B^+]]\vee C_{\Diamond A\wedge\Diamond B}$

Next, dropping the $\mathbf{exh}^{IE+II}_{asr+prs}$ from under negation in (49) again doesn't help: for as shown in (47a)-(47c) above, although that predicts double prohibition for the first disjunct, it fails to filter out the FC presupposition triggered by the second main disjunct. Another possibility is to apply $\mathbf{exh}^{IE+II}_{asr+prs}$ over the entire disjunction, as in (50). This might seem promising, since the expressive power of $\mathbf{exh}^{IE+II}_{asr+prs}$ really comes out when its prejacent or its alternatives are presuppositional.

(50) $\mathbf{exh}^{IE+II}_{asr+prs}[\neg\Diamond[A^+\vee B^+]\vee C_{\Diamond A\wedge\Diamond B}]$

(50) captures the desired double prohibition for the first main disjunct. Yet recall the second fact about $\mathbf{exh}^{IE+II}_{asr+prs}$, i.e., that it can only strengthen the presuppositional content of its prejacent. In (50), the prejacent of $\mathbf{exh}^{IE+II}_{asr+prs}$ is $\neg\Diamond[A^+\vee B^+]\vee C_{\Diamond A\wedge\Diamond B}$, which doesn't filter the FC presupposition $\Diamond A\wedge\Diamond B$. In addition, the conjunctive alternative, $\neg\Diamond[A^+\vee B^+]\wedge C_{\Diamond A\wedge\Diamond B}$, which is in $IE_{asr}(\neg\Diamond[A^+\vee B^+]\vee C_{\Diamond A\wedge\Diamond B})$, also fails to filter out FC. For as shown in (51), the first conjunct doesn't entail the FC presupposition triggered by the second:

(51) $\neg\Diamond(A^+ \vee B^+) \wedge C_{\Diamond A \wedge \Diamond B}$ $\quad \neg\Diamond A^+ \wedge \neg\Diamond B^+ \not\models \Diamond A \wedge \Diamond B$

Since the presuppositional level output of the matrix $\mathbf{exh}^{IE+II}_{asr+prs}$ in (50) has to be at least as strong as the presupposition of its prejacent and of its IE_{asr} conjunctive alternative, we don't predict filtering of the FC presupposition for (43).

The filtering FC puzzle, then, challenges all current **exh**-based theories of FC.

4.2 A solution based on presuppositional exhaustification

In contrast, our $\mathbf{pex}^{IE+II}$-based account directly resolves the filtering FC puzzle. A key difference between **exh** and $\mathbf{pex}^{IE+II}$ accounts, recall, is that while the former assign a fully assertive structure to the FC reading of $\Diamond\vee$-sentences, as in (52), $\mathbf{pex}^{IE+II}$ structures FC into presupposed and assertive components, as in (53):

(52) $\mathbf{exh}^{IE+II}(\Diamond(p \vee q)) = \mathbf{exh}^{IE+II}_{asr+prs}(\Diamond(p \vee q)) = \Diamond p \wedge \Diamond q$

(53) $\mathbf{pex}^{IE+II}(\Diamond(p \vee q)) = \Diamond(p \vee q)_{\Diamond p \leftrightarrow \Diamond q}$ $\quad \models \Diamond p \wedge \Diamond q$

Both accounts predict a FC reading for $\Diamond\vee$-sentences and double prohibition for $\neg\Diamond\vee$-sentences. Yet due to the unique way in which it structures FC into presupposed and assertive components, only the $\mathbf{pex}^{IE+II}$ account can derive both readings while locally exhaustifying the $\Diamond\vee$-sentence (see §2.2). This fact plays a crucial role when, for a sentence like (43), we want to derive double prohibition for its first disjunct (= *Maria can't study in Tokyo or Boston*) and yet also FC for the negation of that same disjunct when determining whether the FC presupposition of the second disjunct ('Maria can study in Japan and can study in the States') is filtered out.

The filtering FC sentence (43) is repeated in (54). Given our $\mathbf{pex}^{IE+II}$ account, the parse in (54a) is a default option—and as we now show, it generates the target reading. Assuming projection rule (45), (54a) presupposes $\mathbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+)) \rightarrow (\Diamond A \wedge \Diamond B)$, as shown in the **ps** part of (54b). The antecedent of this conditional is just an FC interpretation of a $\Diamond\vee$-sentence, as shown in (54c), which entails $\Diamond A^+ \wedge \Diamond B^+$ and thus correctly filters out $\Diamond A$ and $\Diamond B$. Consider next the interpretation of *Maria can't study in Tokyo or Boston*, which is parsed as $\neg\,\mathbf{pex}^{IE+II}[\Diamond[A^+ \vee B^+]]$. As shown in (54d), due to the presupposed vs. assertive structure generated by $\mathbf{pex}^{IE+II}$, the homogeneity presupposition projects from under negation. The latter then applies directly to $\Diamond(A^+ \vee B^+)$, which results in double prohibition.

(54) Either Maria can't study in Tokyo or Boston, or she is the first in our family who can study in Japan and the second who can study in the States.

a. $\neg\,\mathbf{pex}^{IE+II}[\Diamond[A^+ \vee B^+]] \vee C_{\Diamond A \wedge \Diamond B}$

b. $[\![\text{(54a)}]\!] = \begin{cases} \textbf{ps: } \mathbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+)) \rightarrow (\Diamond A \wedge \Diamond B) \\ \textbf{asserts: } \neg\,\mathbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+)) \vee C \end{cases}$

c. $\textbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+)) \rightarrow (\Diamond A \wedge \Diamond B)$
$= \Diamond(A^+ \vee B^+)_{\Diamond A^+ \leftrightarrow \Diamond B^+} \rightarrow (\Diamond A \wedge \Diamond B)$ $= \top$

d. $\neg\,\textbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+))$
$= \neg(\Diamond(A^+ \vee B^+)_{\Diamond A^+ \leftrightarrow \Diamond B^+})$ (ps projects under $\neg$)
$= (\neg\Diamond(A^+ \vee B^+))_{\Diamond A^+ \leftrightarrow \Diamond B^+}$

Unlike **exh**-based accounts of FC, then, our $\textbf{pex}^{IE+II}$ account predicts a reading for (54) that doesn't presuppose that Maria can study in Japan and in the States, yet assigns to the embedded *Maria can't study in Tokyo or Boston* its usual double prohibition reading. That captures the default reading, pointed out by Romoli & Santorio (2019), of filtering FC sentences like (54). Importantly, those predictions follow directly from our $\textbf{pex}^{IE+II}$ account of FC for basic $\Diamond\vee$-sentences and double prohibition for $\neg\Diamond\vee$-sentences, given standard assumptions about presupposition projection and filtering in disjunctions.[7]

4.3 Extension to filtering negative free choice

The filtering FC puzzle is also observed with embedded FC conjunctions, as in (55). The second disjunct of (55) (*[Maria is] the first in her family who is not required to study in Tokyo and the second who's not required to study in Boston*) presupposes the negative FC proposition that Maria is not required to study in Tokyo and is not required to study in Boston. Yet that doesn't project as a presupposition of (55), as captured in (55a). Again, it's as if the negative FC presupposition triggered by the second disjunct is entailed and so filtered out by the (negative FC reading of) the negation of the first disjunct ($\neg$*Maria is required to study in Japan and the States*).

(55) Either Maria is required to study in Japan and the States, or she's the first in her family who is not required to study in Tokyo and the second who's not required to study in Boston.

a. $\not\rightsquigarrow$ *Maria is not required study in Tokyo*
$\not\rightsquigarrow$ *Maria is not required study in Boston*

7 The filtering FC puzzle supports our proposal that $\textbf{pex}^{IE+II}$ should presuppose homogeneity over, rather than the conjunction of, the *II*-alternatives. The latter option might seem closer to Bar-Lev & Fox's proposal, but it fails to derive the target reading of (54). Again, to filter out the $\Diamond A \wedge \Diamond B$ presupposition triggered in the second disjunct of (54), we need a parse as in (54a). If $\textbf{pex}^{IE+II}$ presupposed the conjunction of the *II* alternatives, $\textbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+))$ would presuppose $\Diamond A^+ \wedge \Diamond B^+$. Due to projection from under negation, the first disjunct of (54), $\neg\,\textbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+))$, would then entail $\Diamond A^+ \wedge \Diamond B^+$, which would clash with its assertive content, $\neg(\Diamond A^+ \vee \Diamond B^+)$, or project out as a presupposition of (54) itself. Either way, we fail to get the target reading.

Following Romoli & Santorio, let us schematically represent the target reading of (55) as in (56), where A^+/B^+ asymmetrically entails A/B:

(56) $\Box(A \wedge B) \vee C_{\neg\Box A^+ \wedge \neg\Box B^+}$ $\quad\not\rightsquigarrow \neg\Box A^+, \not\rightsquigarrow \neg\Box B^+$

Based on the projection rule for disjunctions in (45), the presupposition triggered in the second main disjunct of (56), $\neg\Box A^+ \wedge \neg\Box B^+$, is filtered out if it is entailed by the negation of the first main disjunct, $\neg\Box(A \wedge B)$. It follows that, given an LF which closely matches the surface form of (56), we would not predict the target filtering effect, since $\neg\Box(A \wedge B) \not\models \neg\Box A^+ \wedge \neg\Box B^+$.

This version of the filtering FC puzzle is interesting because, while recent Lexicalist accounts such as Aloni (2018), Ciardelli et al. (2018), Rothschild & Yablo (2018), and Goldstein (2019) predict the main desiderata for sentences like (43), they don't directly help with variants like (55). The problem parallels the one posed by presupposed (negative) FC under negative factives (see §3.3). The presupposition triggered in the second main disjunct of (55) is filtered out if it is entailed by the negation of the first main disjunct, i.e., by ¬*Maria is required to study in Japan and the States*. So we would get the target filtering effect if we could directly derive a negative FC reading for the latter. However, those Lexicalist accounts do not predict a negative FC for $\neg\Box\wedge$-sentences (an exception is Willer 2017).

Going back to standard Grammatical accounts, one may hope that the parse in (57a) predicts the target filtering effect. However, the effect of $\mathbf{exh}^{IE+II}$ in the first disjunct is vacuous, since no *IE* alternatives can be negated, and the *II* alternatives $\Box A$ and $\Box B$ are entailed by the prejacent. Since $\mathbf{exh}^{IE+II}(\Box(A \wedge B)) = \Box(A \wedge B)$, the effect is as if we had a parse without $\mathbf{exh}^{IE+II}$ in the first disjunct. As a result, the $\neg\Box A^+$ and $\neg\Box B^+$ presuppositions of the second disjunct are not filtered out, since $\neg\mathbf{exh}^{IE+II}(\Box(A \wedge B))$ entails neither $\neg\Box A^+$ nor $\neg\Box B^+$, as shown in (57c).

(57) a. $\mathbf{exh}^{IE+II}[\Box[A \wedge B]] \vee C_{\neg\Box A^+ \wedge \neg\Box B^+}$

b. $[\![(57a)]\!] = \begin{cases} \textbf{ps: } \neg\mathbf{exh}^{IE+II}(\Box(A \wedge B)) \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+) \\ \textbf{asserts: } \Box(A \wedge B) \vee C \end{cases}$

c. $\neg\mathbf{exh}^{IE+II}(\Box(A \wedge B)) \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$
$= \neg(\Box(A \wedge B)) \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$
$= (\neg\Box A \vee \neg\Box B) \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$ $\quad \neq \top$

Switching to $\mathbf{exh}^{IE+II}_{asr+prs}$ again doesn't help. Given a parse parallel to (57a), we face the same problem as with $\mathbf{exh}^{IE+II}$, since it also holds that $\mathbf{exh}^{IE+II}_{asr+prs}(\Box(A \wedge B)) = \Box(A \wedge B)$. Given a parse with matrix scope $\mathbf{exh}^{IE+II}_{asr+prs}$, we don't predict filtering, since $\mathbf{exh}^{IE+II}_{asr+prs}$ can only strengthen any presuppositional content triggered in its prejacent (see §4.1). To filter out the presupposition of the second disjunct of (55), we need to get—when calculating its presuppositions based on rule (45)—$\mathbf{exh}^{IE+II}$ (or

$\mathbf{exh}^{IE+II}_{asr+prs}$) to scope over the negation of the first disjunct, since $\mathbf{exh}^{IE+II}(\neg\Box(A \wedge B))$ entails $\neg\Box A^+ \wedge \neg\Box B^+$. The problem is that we can't get that scoping effect, at least without stipulating ad hoc syntactic operations.

In contrast, our $\mathbf{pex}^{IE+II}$ account captures the target reading of (55) without any additional stipulations. Consider the parse in (58a), which is structurally analogous to the one in (57a). $\mathbf{pex}^{IE+II}(\Box(A \wedge B))$ doesn't exclude anything. However, the conjunctive alternatives of the prejacent, $\Box A$ and $\Box B$, are in II, since taken together they can be consistently conjoined with the prejacent and negation of any IE alternatives. It follows that $\mathbf{pex}^{IE+II}(\Box(A \wedge B)) = (\Box(A \wedge B))_{\Box A \leftrightarrow \Box B}$. Then when determining the presupposition of (58a), shown in (58b), the homogeneity presupposition projects out of the negation in the antecedent, as shown in (58c).

(58) a. $\mathbf{pex}^{IE+II}[\Box[A \wedge B]] \vee C_{\neg\Box A^+ \wedge \neg\Box B^+}$

b. $[\![(58a)]\!] = \begin{cases} \textbf{ps: } \neg\,\mathbf{pex}^{IE+II}(\Box(A \wedge B)) \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+) \\ \textbf{asserts: } \Box(A \wedge B) \vee C \end{cases}$

c. $\neg\,\mathbf{pex}^{IE+II}(\Box(A \wedge B)) \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$
$= \neg(\Box(A \wedge B))_{\Box A \leftrightarrow \Box B} \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$
$= (\neg\Box A \vee \neg\Box B)_{\Box A \leftrightarrow \Box B} \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+) \qquad = \top$

The antecedent of (58c) entails the consequent, since $(\neg\Box A \vee \neg\Box B)_{\Box A \leftrightarrow \Box B} \models \neg\Box A \wedge \neg\Box B$, and $\neg\Box A \wedge \neg\Box B \models \neg\Box A^+ \wedge \neg\Box B^+$ (i.e., that Maria is not required to study in Japan and not required to study in the States entails that she is not required to study in Tokyo and not required to study in Boston). Accordingly, the $\neg\Box A^+ \wedge \neg\Box B^+$ presupposition triggered in the second disjunct of (58a) is filtered out (by the negation of the first main disjunct), so is not inherited as a presupposition of (58a) as a whole. This is precisely the target result.

4.4 Homogeneity in enemy territory

Our $\mathbf{pex}^{IE+II}$ account of filtering FC sentences such as (54) uses the parse in (54a) to predict the target reading: i.e., double prohibition for the first disjunct and filtering of the FC presupposition of the second disjunct. Yet as shown in (54a)-(54d), (54a) also predicts that (54) inherits the homogeneity presupposition that Maria can study in Tokyo iff she can study in Boston. An analogous point holds for our analysis of filtering (negative) FC sentences like (55). Romoli & Santorio argue that this raises an issue: for it seems that S can felicitously assert (54)—with the target reading—in a context that entails that S doesn't believe the homogeneity proposition:

(59) Maria applied to Tokyo or Boston. I have no idea whether she was admitted to only one, both, or neither, but ...

a. Either she can't go study in Tokyo or Boston, or she is the first in our family who can study in Japan and the second who can study in the States.

Can we handle these cases? The presuppositions triggered by $\mathbf{pex}^{IE+II}$ tend to be globally accommodated when they are consistent with the common ground (see Bassi et al. 2021, Del Pinal 2021). So why not appeal to global accommodation? The problem is that, in (59), S explicitly claims ignorance of homogeneity. As the example is intended, S doesn't, in the middle of the discourse, acquire or remember any relevant new information. So interlocutors can't reasonably globally accommodate homogeneity when they process (59a): for they would then represent S as simultaneously agnostic and believing in the homogeneity proposition.

What we need is to block the projection of homogeneity from out of the first main disjunct of (59a), without affecting the derivation of the target reading. Could we just apply local accommodation—via an operator, ACC, such that $\text{ACC}(p_q) = q \wedge p$—over the first main disjunct? Consider the parse in (60a), which is like the one that supports the target predictions (in neutral contexts) for the original filtering FC examples, but with ACC over the first disjunct to block the projection of homogeneity. This preserves double prohibition for the first disjunct, as captured in the **asserts** part in (60b) given the equivalence in (60c). Yet at at the presuppositional level, we now no longer filter out $\Diamond A$ and $\Diamond B$, as captured in the **ps** part of (60b). For based on (60c), we can see that $\neg\text{ACC}(\neg\mathbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+))) = \neg(\neg\Diamond A^+ \wedge \neg\Diamond B^+) = \Diamond A^+ \vee \Diamond B^+$, and obviously $\Diamond A^+ \vee \Diamond B^+ \not\models \Diamond A \wedge \Diamond B$.

(60) a. $\text{ACC}[\neg\mathbf{pex}^{IE+II}[\Diamond[A^+ \vee B^+]]] \vee C_{\Diamond A \wedge \Diamond B}$

b. $[\![\text{(60a)}]\!] = \begin{cases} \textbf{ps: } \neg\text{ACC}(\neg\mathbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+))) \rightarrow \Diamond A \wedge \Diamond B \\ \textbf{asserts: } \text{ACC}(\neg\mathbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+))) \vee C \end{cases}$

c. $\text{ACC}(\neg\mathbf{pex}^{IE+II}(\Diamond(A^+ \vee B^+)))$
$= (\Diamond A^+ \leftrightarrow \Diamond B^+) \wedge \neg\Diamond(A^+ \vee B^+) \quad \Leftrightarrow \neg\Diamond A^+ \wedge \neg\Diamond B^+$

Could we go for a 'direct' solution and apply ACC over each disjunct, as in (61)?

(61) $\text{ACC}_1[\neg\mathbf{pex}^{IE+II}[\Diamond[A^+ \vee B^+]]] \vee \text{ACC}_2[C_{\Diamond A \wedge \Diamond B}]$

ACC_1 blocks the projection of homogeneity from the first disjunct, without altering its double prohibition reading. ACC_2 blocks the projection of $\Diamond A$ and $\Diamond B$ from out of the second disjunct. So the parse in (61) captures the two desiderata of the target reading of (59a). Yet are there reasonable licensing conditions for ACC which permit, in contexts like (59), generating parses like (61)?

Local accommodation is usually thought to have strict licensing conditions. A standard hypothesis is that ACC is only licensed when it is marked with specific intonation patterns or the corresponding parse without ACC would result in incoherent or defective contents or discourses (Gazdar 1979, Heim 1983). The extension to discourses is needed to apply the local accommodation-based account (of the coherence) of sentences like (62a) to parallel discourses like (62b), as seems natural:

(62) a. The king of France isn't bald, since there is no king of France!
b. The kind of France isn't bald. For there is no king of France!

Based on those licensing conditions, we can show that the parse in (61) is licensed when a speaker *S* asserts (59a) after (59), or in any context which entails that *S* doesn't believe the homogeneity proposition. Again, ACC_1 is required to avoid attributing to *S* the incoherent attitude of being both agnostic towards and believing in the homogeneity proposition $\Diamond A^+ \leftrightarrow \Diamond B^+$. ACC_2 is required to avoid representing *S* as incapable of drawing basic implications of *S*'s own doxastic states. For without ACC_2, *S* would be represented as holding both of the following beliefs:

(B_1) Maria can study in Japan and can study in the States.

(B_2) (Only) if Maria can study in Tokyo and in Boston, she is the first in her family who can study in Japan and the second who can study in the States.[8]

Given B_1 and *S*'s agnosticism about whether Maria can study in Tokyo or Boston, it is hard to see why *S* would believe B_2. For we would be attributing to *S* the strange belief that whether anyone in Maria's family before her was allowed to study in Japan/States depends on whether she can now study specifically in Tokyo (and not elsewhere in Japan) and in Boston (and not elsewhere in the States).

This result suggests that, in contexts that entail that *S* is agnostic with respect to the homogeneity proposition, both ACC operators in (61) are licensed to avoid attributing to *S*, if not strictly incoherent, at least very strange beliefs. That is consistent with holding that parallel ACC operators over the main disjuncts are *not* licensed when sentences like (54) are asserted in contexts which are compatible with (hence allows for global accommodation of) the homogeneity presupposition.

4.5 Summary

The FC filtering effects of embedded $\neg\Diamond\vee$ and $\Box\wedge$-sentences in cases like (43) and (55) pose a challenge to various Grammatical theories, including versions with $\textbf{exh}^{IE/IE+II}$ (Fox 2007, Bar-Lev & Fox 2020) and also with more powerful operators

8 The conditional belief attributed to *S* is likely exhaustive (i.e., 'only if'): for the main *or* in *either p or q* sentences like (59a) is usually enriched to exclusive-'or'. This can be captured by adding a matrix $\textbf{pex}^{IE+II}$ to the parse in (61) which associates with the main $\vee$.

like $\mathbf{exh}^{IE+II}_{asr+prs}$ which can enrich both the assertive and presuppositional content of its prejacent (Gajewski & Sharvit 2012, Marty & Romoli 2020). This puzzle is also challenging for any Lexicalist theories, such as Ciardelli et al. (2018), Aloni (2018), Rothschild & Yablo (2018) and Goldstein (2019), which do not predict negative FC, and so can directly solve only half of the cases. In contrast, our $\mathbf{pex}^{IE+II}$ theory supports a uniform solution to all the filtering FC puzzles, which follows directly from the default parses for the embedded $\neg\Diamond\vee$ and $\Box\wedge$-sentences, given standard assumptions about presupposition projection, filtering and accommodation.

5 Extensions: FC effects in the scope of universal, existential and non-monotonic quantifiers

Our $\mathbf{pex}^{IE+II}$ approach to the presupposed & filtering FC puzzles follows a simple strategy: for each puzzle, apply $\mathbf{pex}^{IE+II}$ locally to the embedded $\Diamond\vee$, $\neg\Diamond\vee$, $\neg\Box\wedge$, or $\Box\wedge$-sentence. This works because $\mathbf{pex}^{IE+II}$ structures its output such that the prejacent is treated as the the assertive content and any excludable/includable alternatives as non-at issue, projective content. That embedded $\mathbf{pex}^{IE+II}$ solves those puzzles is not just a happy accident—rather, it is part of a more systematic observation about the projective behavior of embedded exhaustive inferences. To further support this conjecture, we now apply this strategy to various open FC puzzles with $\Diamond\vee$, $\neg\Diamond\vee$, $\neg\Box\wedge$ and $\Box\wedge$-sentences under universal, existential and non-monotonic quantifiers. These puzzles present a serious challenge to all **exh**-based theories. In contrast, we will show that LFs with embedded $\mathbf{pex}^{IE+II}$ issue in uniform and simple solutions—and for exactly parallel reasons related to the projection of exhaustive content, which in all these cases includes homogeneity propositions.[9]

5.1 Universal and existential FC and VP-ellipsis puzzles

Let us begin with FC effects under universal quantifiers. Chemla (2009b) presents evidence that a '$\forall\Diamond\vee$'-sentence like (63) (a $\Diamond\vee$-sentence in the scope of a universal quantifier) has a universal FC reading, captured in (63a). Similarly, a $\neg\exists\Box\wedge$-sentence like (64) (a $\Box\wedge$-sentence in the scope of a negative universal quantifier) has a universal negative FC reading, captured in (64a).

(63) Every student is allowed to eat cake or ice cream. $\forall x \in S(\Diamond(Cx \vee ICx))$

a. $\rightsquigarrow \forall x \in S(\Diamond Cx) \wedge \forall x \in S(\Diamond ICx)$

(64) No student is required to solve (both) problem A and problem B.

9 In Bassi et al. (2021) we defend an analogous point about embedded SIs: when scalar items appear under DE and non-monotonic operators, analyses with local, embedded $\mathbf{pex}^{IE+II}$ lead to better predictions than accounts with flat **exh** because of how embedded SIs are predicted to project.

$\neg\exists x \in S(\Box(Ax \wedge Bx))$

a. $\rightsquigarrow \neg\exists x \in S(\Box Ax) \wedge \neg\exists x \in S(\Box Bx)$

This pattern is tricky for $\mathbf{exh}^{IE}$ theories, and Bar-Lev & Fox (2020) use it to motivate the move to $\mathbf{exh}^{IE+II}$. The FC reading of (63) can be derived from an LF with embedded recursive $\mathbf{exh}^{IE}$ over $\Diamond(Cx \vee ICx)$ in the scope of *every student*, as in (65a). But a parallel parse for (64) with embedded recursive $\mathbf{exh}^{IE}$, as in (65b), doesn't predict its target reading, for the $\Box(Ax \wedge Bx)$ sentence is already the strongest of its alternatives.

(65) a. $\forall x \in S[\mathbf{exh}^{IE}[\mathbf{exh}^{IE}[\Diamond[Cx \vee ICx]]]]$
b. $\neg\exists x \in S[\mathbf{exh}^{IE}[\mathbf{exh}^{IE}[\Box[Ax \wedge Bx]]]]$

Replacing recursive $\mathbf{exh}^{IE}$ in (65b) with $\mathbf{exh}^{IE+II}$ doesn't help. For $\mathbf{exh}^{IE+II}$ can include some alternatives (e.g., $\Box Ax$ and $\Box Bx$), yet since they go into the assertive level and are entailed by the prejacent, $\Box(Ax \wedge Bx)$, the effect is vacuous. So we again only get the reading that no student is required to solve both problems, which is weaker than the target that no one is required to solve A and no one is required to solve B. Bar-Lev & Fox show, however, that we can get the target FC readings for (63) and (64) with matrix scope $\mathbf{exh}^{IE+II}$, as in (66a) and (66b):

(66) a. $\mathbf{exh}^{IE+II}[\forall x \in S[\Diamond[Cx \vee ICx]]]$
b. $\mathbf{exh}^{IE+II}[\neg\exists x \in S[\Box[Ax \wedge Bx]]]$

The key observation, for (66a), is that the universal disjunctive alternatives, $\forall x \in S(\Box Cx)$ and $\forall x \in S(\Box ICx)$, are both in II. Similarly for (66b): the negative universal alternatives, $\neg\exists x \in S(\Box Ax)$ and $\neg\exists x \in S(\Box Bx)$, are both in II. By adding those II alternatives we get, in each case, the target universal FC enrichment.

Moving to a $\mathbf{pex}^{IE+II}$ theory, the first interesting result, which as we will see in a moment turn out to be useful, is that we can also derive the universal FC readings for (63) and (64) based on LFs with embedded $\mathbf{pex}^{IE+II}$, as in (67a) and (67b):

(67) a. $\forall x \in S[\mathbf{pex}^{IE+II}[\Diamond[Cx \vee ICx]]]$
b. $\neg\exists x \in S[\mathbf{pex}^{IE+II}[\Box[Ax \wedge Bx]]]$

Consider first (63) given the LF in (67a). $\mathbf{pex}^{IE+II}(\Diamond(Cx \vee ICx))$ triggers the homogeneity presupposition $\Diamond Cx \leftrightarrow \Diamond ICx$ in the scope of *every student*. Presuppositions triggered in the scope of a universal quantifier tend to project universally (Chemla 2009a, Fox 2013, Mayr & Sauerland 2015). The universally quantified homogeneity presupposition, combined with the assertive content, entails universal FC:

(68) $\forall x \in S(\Diamond Cx \leftrightarrow \Diamond ICx) \wedge \forall x \in S(\Diamond(Cx \vee ICx))$

$$\models \forall x \in S(\Diamond Cx) \wedge \forall x \in S(\Diamond ICx)$$

Consider next (64) given the LF in (67b). $\mathbf{pex}^{IE+II}(\Box(Ax \wedge Bx))$ triggers the homogeneity presupposition $\Box Ax \leftrightarrow \Box Bx$ in the scope of *no student*. Assuming again universal projection from the scope of a universal (negative) quantifier, when the resulting homogeneity presupposition is combined with the assertive content, we get the target universal (negative) FC proposition:

(69) $\forall x \in S(\Box Ax \leftrightarrow \Box Bx) \wedge \neg\exists x \in S(\Box(Ax \wedge Bx))$
$$\models \neg\exists x \in S(\Box Ax) \wedge \neg\exists x \in S(\Box Bx)$$

Bar-Lev & Fox argue that, even for $\forall\Diamond\vee$-sentences like (63), there should be a derivation of their FC reading with matrix exhaustification. They appeal to FC and VP ellipsis puzzles like (70). On the target reading, the first $\forall\Diamond\vee$-sentence gets the universal FC reading in (70a). In addition, it licenses VP ellipsis in the second $\neg\exists\Diamond\vee$-sentence, which in turn gets a universal double prohibition reading as in (70b). That is, the elided $\Diamond\vee$-sentence occurs in a DE environment, and seems to get its un-enriched (classical) interpretation:

(70) Every student in section A is allowed to eat cake or ice cream on their birthday. Weirdly, no student in section B is ~~allowed to eat cake or ice cream on their birthday~~.

a. $\rightsquigarrow \forall x \in S_A(\Diamond Cx) \wedge \forall x \in S_A(\Diamond ICx)$
b. $\rightsquigarrow \neg\exists x \in S_B(\Diamond Cx) \wedge \neg\exists x \in S_B(\Diamond ICx)$

Due to the bound variable *their*, the material in the scope of *Every student in A* should be part of the parallelism domain for ellipsis (Rooth 1992, Heim 1996). Accordingly, if we get FC for the first sentence by local $\mathbf{exh}^{IE+II}$ over the $\Diamond\vee$-sentence in the scope of *Every student in A*, we also need to apply $\mathbf{exh}^{IE+II}$ in the scope of *No student in B* in the second sentence, which results in the unattested too weak reading that no student has FC. However, if we derive universal FC for the first sentence using matrix $\mathbf{exh}^{IE+II}$, we don't need to apply matrix or local $\mathbf{exh}^{IE+II}$ over the ellided $\Diamond\vee$-sentence in the scope of *No student*, and can get the target universal double prohibition.

Yet the FC and VP ellipsis puzzle in (70) can also be resolved using embedded $\mathbf{pex}^{IE+II}$. Consider, for both sentences, LFs with embedded $\mathbf{pex}^{IE+II}$ immediately over the $\Diamond(Cx \vee ICx)$ clause in the scope of the quantifiers:

(71) a. $\forall x \in S_A[\mathbf{pex}^{IE+II}[\Diamond[Cx \vee ICx]]]$
b. $\neg\exists x \in S_B[\mathbf{pex}^{IE+II}[\Diamond[Cx \vee ICx]]]$

These LFs respect the parallelism constraint. As shown in (68) above, (71a) predicts universal FC for the first $\forall\Diamond\vee$-sentence. In addition, (71b) predicts universal double prohibition for the second $\neg\exists\Diamond\vee$-sentence. As shown in (72), this follows directly from the way in which the universally projected homogeneity presupposition, triggered by the $\mathbf{pex}^{IE+II}(\Diamond(Cx \vee ICx))$ in the scope of *No student in B*, interacts with the assertive content of the $\neg\exists\Diamond\vee$-sentence:

(72) $$\forall x \in S_B(\Diamond Cx \leftrightarrow \Diamond ICx) \wedge \neg\exists x \in S_B(\Diamond(Cx \vee ICx)) \models \neg\exists x \in S_B(\Diamond Cx) \wedge \neg\exists x \in S_B(\Diamond ICx)$$

Thus far, we can handle universal FC and double prohibition, and the FC and VP-ellipsis puzzle, using either matrix scope $\mathbf{exh}^{IE+II}$ or embedded $\mathbf{pex}^{IE+II}$. It turns out, however, that the approach with local $\mathbf{pex}^{IE+II}$ has a substantial advantage.

As Bar-Lev & Fox point out, we can construct a version of the FC and VP-ellipsis puzzle using an '$\exists\Diamond\vee$'-sentence like the first one in (73). That sentence, note, can get the 'existential FC' reading in (73a)—that some students have FC—while the second $\neg\exists\Diamond\vee$-sentence still gets the universal double prohibition reading.

(73) Some students in section A are allowed to eat cake or ice-cream on their birthday. Weirdly, no student in section B is ~~allowed to eat cake or ice cream on their birthday~~.

a. $\rightsquigarrow \exists x \in S_A(\Diamond Cx \wedge \Diamond ICx)$

b. $\rightsquigarrow \neg\exists x \in S_B(\Diamond Cx) \wedge \neg\exists x \in S_B(\Diamond ICx)$

Yet the existential FC reading of $\exists\Diamond\vee$-sentences can't be derived—at least without additional stipulations—using matrix scope $\mathbf{exh}^{IE+II}$. For although the existential disjunctive alternatives, $\exists x \in S_A(\Diamond Cx)$ and $\exists x \in S_A(\Diamond ICx)$, are in *II* for the matrix $\mathbf{exh}^{IE+II}$, adding them only gets us the (weaker) inferences that some students are allowed cake and that some are allowed ice-cream, which is compatible with no student having FC (since there need be no overlap between the two sets of students). To be sure, we can get existential FC with local $\mathbf{exh}^{IE+II}$ over the embedded $\Diamond[Cx \vee ICx]$ clause in the scope of the existential quantifier. Yet due to the parallelism constraint, we would also have to insert $\mathbf{exh}^{IE+II}$ in the same embedded position for the second $\neg\exists\Diamond\vee$-sentence, and would thus only get a 'no student has FC' reading, and not the target (stronger) universal double prohibition.

In contrast, we have seen that we can derive universal double prohibition for $\neg\exists\Diamond\vee$-sentences even with embedded $\mathbf{pex}^{IE+II}$ (see the discussion around (72) above). The only thing left to show, then, is that an analogous LF with embedded $\mathbf{pex}^{IE+II}$, as in (74), supports an existential FC interpretation:

(74) $$\exists x \in S_A[\mathbf{pex}^{IE+II}[\Diamond[Cx \vee ICx]]]$$

With existential quantifiers, there are two cases to consider, since there is disagreement concerning whether presuppositions in their scope project universally or existentially (Sudo et al. 2012). If the homogeneity presupposition projects universally, then we straightforwardly predict the target existential FC reading:

(75) $\forall x \in S_A(\Diamond Cx \leftrightarrow \Diamond ICx) \wedge \exists x \in S_A(\Diamond(Cx \vee ICx))$
$\models \exists x \in S_A(\Diamond Cx \wedge \Diamond ICx)$

What if the homogeneity presupposition in the scope of the existential quantifier projects existentially? Theories which predict this need to be paired, for independent reasons, with a theory of dynamic binding (to ensure that in basic cases like *some students stopped smoking* the presupposition that 'x used to smoke' and the assertive proposition that 'x doesn't smoke now' get bound by the same existential DP). Paired with any such suitable binding theory (Heim 1982, Fox 2013, Sudo 2016), the parse in (74) also gets the existential FC entailments, since the existential quantifier will bind any free variables in its scope, which we can represent as in (76):

(76) $\exists x \in S_A((\Diamond Cx \leftrightarrow \Diamond ICx) \wedge \Diamond(Cx \vee ICx))$
$\models \exists x \in S_A(\Diamond Cx \wedge \Diamond ICx)$

5.2 FC in non-monotonic environments

Gotzner, Romoli & Santorio (2020) present experimental evidence that (77) has a salient reading that entails that one student has FC, while all the other students have double prohibition, as captured in (77a):

(77) Exactly one student can take Logic or Calculus.

a. ⇝ *Exactly one student can take Logic and can take Calculus*
⇝ *Each of the other students can take neither Logic nor Calculus*

They also present evidence that (78) has a salient reading that entails that one student has double prohibition, while all other students have FC, as captured in (78a):

(78) Exactly one student can't take Logic or Calculus.

a. ⇝*Exactly one student can take neither Logic nor Calculus*
⇝ *Each of the other students can take Logic and can take Calculus*

Gotzner et al. show that $\mathbf{exh}^{IE}$ theories predict the 'all others double prohibition' reading of (77), but not the 'all others FC' reading of (78). In addition, while switching to $\mathbf{exh}^{IE+II}$ helps derive the target readings, the required auxiliary assumptions are rather problematic.

To see why, let us focus on (78), the case that resists an $\mathbf{exh}^{IE}$ analysis. We need an LF with matrix $\mathbf{exh}^{IE+II}$ as in (79), and the alternatives for the prejacent in (79a). As we note in (79a), all the disjunctive alternatives are II, so the result of exhaustification is as in (79b) (assume for simplicity that the IE conjunctive alternatives are irrelevant). From the disjunctive alternatives, $\exists x^{|x|=1} \in S[\neg\Diamond Lx]$ and $\exists x^{|x|=1} \in S[\neg\Diamond Cx]$, we get the inferences that exactly one student can't take Logic and exactly one student can't take Algebra. Since the prejacent says that exactly one student has double prohibition, it follows that one and the same student has to witness all three conditions, and so each of the others has FC.

(79) $\mathbf{exh}^{IE+II}[\exists x^{|x|=1} \in S[\neg\Diamond[Lx \vee Cx]]]$

a. $$Alt(\exists x^{|x|=1} \in S[\neg\Diamond[Lx \vee Cx]]) = \left\{\begin{array}{ll} \exists x^{|x|=1} \in S[\neg\Diamond[Lx \vee Cx]] & (\in II) \\ \exists x^{|x|=1} \in S[\neg\Diamond Lx] & (\in II) \\ \exists x^{|x|=1} \in S[\neg\Diamond Cx] & (\in II) \\ \exists x^{|x|=1} \in S[\neg\Diamond[Lx \wedge Cx]] & (\in IE) \\ \exists x \in S[\Diamond[Lx \vee Cx]] & (\in II) \\ \exists x \in S[\Diamond Lx] & (\in II) \\ \exists x \in S[\Diamond Cx] & (\in II) \\ \exists x \in S[\Diamond[Lx \wedge Cx]] & (\in IE) \end{array}\right.$$

b. $$[\![(79)]\!] = \left\{\begin{array}{l} (|\{x \in S : \neg\Diamond Lx \wedge \neg\Diamond Cx\}| = 1) \wedge \\ (|\{x \in S : \neg\Diamond Lx\}| = 1) \wedge \\ (|\{x \in S : \neg\Diamond Cx\}| = 1) \end{array}\right.$$

Importantly, the assumption that 'exactly one' sentences like (78) have the 'some' alternatives, and specifically the disjunctive ones without negation—i.e., $\exists x \in S[\Diamond Lx]$ and $\exists x \in S[\Diamond Cx]$—is needed to get the target result. Without those alternatives, $\exists x^{|x|=1} \in S[\neg\Diamond Lx]$ and $\exists x^{|x|=1} \in S[\neg\Diamond Cx]$ would both be in IE, and the resulting interpretation wouldn't capture the 'all others FC' reading, e.g., it would be true if there are three students (Jimmy, Sue, Beth), Jimmy has double prohibition, Sue can't take Logic but can take Calculus, and Beth can't take Calculus but can take Logic. Yet consider what happens if we have the existential disjunctive alternatives. The prejacent says that exactly one student has double prohibition, and if we combine that with $\neg\exists x^{|x|=1} \in S[\neg\Diamond Lx]$, we get that more than one student can't take Logic and hence that at least one student can take Calculus (since only one has double prohibition). This is captured in (80). We get an analogous result if we combine the prejacent with $\neg\exists x^{|x|=1} \in S[\neg\Diamond Cx]$, as captured in (81).

(80) $(|\{x \in S : \neg\Diamond Lx \wedge \neg\Diamond Cx\}| = 1) \wedge \neg(|\{x \in S : \neg\Diamond Lx\}| = 1) \models \exists x \in S(\Diamond Cx)$

(81) $(|\{x \in S : \neg\Diamond Lx \wedge \neg\Diamond Cx\}| = 1) \wedge \neg(|\{x \in S : \neg\Diamond Cx\}| = 1) \models \exists x \in S(\Diamond Lx)$

If the conjunction of the prejacent and the negation of an alternative entail another alternative not entailed by the prejacent alone, then those alternatives are in symmetry, since their joint negation can't be part of a maximally consistent set together with the prejacent. As a result, they can't be in IE. So from (80) and (81), we can conclude that $\exists x^{|x|=1} \in S[\neg\Diamond Lx]$ and $\exists x^{|x|=1} \in S[\neg\Diamond Cx]$ aren't in IE, which is why they are then available for II when computing the output of $\mathbf{exh}^{IE+II}$.[10] Yet the stipulation that (78) has existential disjunctive alternatives without negation is problematic. For example, an analogous procedure for generating alternatives—i.e., which allows deletion of negation—would create problems when using matrix scope $\mathbf{exh}^{IE+II}$ to derive simple indirect SIs for $\neg\forall$-sentences (e.g., *John didn't eat all of the cookies* $\rightsquigarrow$ *John ate some of the cookies*): for if we assume that $\neg\forall$-sentences have not just a $\neg\exists$ but also an $\exists$ alternative, we get a symmetry effect hence neither can be in IE, and we fail to derive the indirect SI.

In contrast, uniform parses with embedded $\mathbf{pex}^{IE+II}$ straightforwardly predict the target readings, and we need not make any additional, controversial stipulations about the alternatives at play. Specifically, we can capture the 'all others double prohibition' reading of (77) via the LF in (82), and the 'all others FC' reading of (78) via the LF in (83). As before, those locally exhaustified LFs for the embedded $\Diamond\vee$ and $\neg\Diamond\vee$-sentences correspond to the ones we used to capture their default FC and double prohibition readings in unembedded cases.

(82) $\exists x^{|x|=1} \in S[\mathbf{pex}^{IE+II}[\Diamond[Lx \vee Cx]]]$

(83) $\exists x^{|x|=1} \in S[\neg[\mathbf{pex}^{IE+II}[\Diamond[Lx \vee Cx]]]]$

In both (82) and (83), $\mathbf{pex}^{IE+II}[\Diamond[Lx \vee Cx]]$ triggers the homogeneity presupposition $\Diamond Lx \leftrightarrow \Diamond Cx$ in the scope of the non-monotonic quantifier. Given standard assumptions about projection from the scope of non-monotonic quantifiers, (82) and (83) presuppose a universally quantified homogeneity proposition, $\forall x \in S(\Diamond Lx \leftrightarrow \Diamond Cx)$. In (82), its assertive part says that exactly one student is allowed to take Logic or Calculus—and when conjoined with universal homogeneity, that entails that exactly one has FC and all the others can't take either one, which captures the target 'all other double prohibition':

(84)
$$\begin{aligned}&\forall x \in S(\Diamond Lx \leftrightarrow \Diamond Cx)\\&\wedge(|\{x \in S : \Diamond Lx \vee \Diamond Cx\}| = 1)\\&\qquad\qquad\models (|\{x \in S : \Diamond Lx \wedge \Diamond Cx\}| = 1)\\&\qquad\qquad\quad \wedge \forall x \in S(\neg(\Diamond Lx \vee \Diamond Cx) \rightarrow (\neg\Diamond Lx \wedge \neg\Diamond Cx))\end{aligned}$$

10 Gotzner et al. (2020) argue that the $\mathbf{exh}^{IE+II}$ account doesn't generalize well to 'all others FC/double prohibition' readings for arbitrary sentences of the form 'exactly n'. It seems to us that the $\mathbf{exh}^{IE+II}$ analysis works in general, but only using analogous stipulations as those criticized above.

In the case of (83), its assertive part says that exactly one student can take neither Logic nor Calculus—and when conjoined with universal homogeneity, that entails that all the others can take Logic and can take Calculus, which captures the 'all others FC' reading:

(85) $$\forall x \in S(\Diamond Lx \leftrightarrow \Diamond Cx) \\ \wedge(|\{x \in S : \neg(\Diamond Lx \vee \Diamond Cx)\}| = 1) \\ \models (|\{x \in S : \neg\Diamond Lx \wedge \neg\Diamond Cx\}| = 1) \\ \wedge\ \forall x \in S(\Diamond Lx \vee \Diamond Cx) \rightarrow (\Diamond Lx \wedge \Diamond Cx))$$

Finally, it is easy to check that our analysis captures the 'all others double prohibition' reading for any n in sentences of the form *Exactly n students can take Logic or Calculus*, and the target 'all others FC' reading for any n in sentences of the form *Exactly n students can't take Logic or Calculus*. It can also be extended, in a fully analogous way, to versions of (77)-(78) with embedded $\Box\wedge$ and $\neg\Box\wedge$-sentences, which as pointed out by Gotzner et al., pose special problems for Lexicalist accounts which do not directly predict negative FC.

6 Conclusion

Grammatical theories support models of our core semantic competence based on classical modal logic, and derive FC effects via covert exhaustification operators which act as approximate grammaticalizations of quantity-based, information maximization pragmatic enrichment procedures. The presupposed & filtering FC puzzles, however, seriously challenge those theories. Specifically, the projection properties of $\Diamond\vee$, $\neg\Diamond\vee$, $\Box\wedge$ and $\neg\Box\wedge$-sentences, when embedded in environments like (21), (33), (43) and (55), undermine the widely held assumption that the output of **exh** operators—when the prejacent itself doesn't trigger any presuppositions—consists of flat, fully assertive contents (Fox 2007, Chierchia et al. 2012, Bar-Lev & Fox 2020, Marty & Romoli 2020). That conclusion is further supported by certain FC effects in the scope existential, universal, and non-monotonic quantifiers.

In this paper we developed a novel Grammatical theory of FC based on an exhaustification operator, $\mathbf{pex}^{IE+II}$, which asserts its prejacent but is a presupposition trigger with respect to any of its excludable or includable alternatives. When $\mathbf{pex}^{IE+II}$ is locally applied to $\Diamond\vee$, $\neg\Box\wedge$, $\neg\Diamond\vee$ and $\Box\wedge$-sentences, it structures their interpretation into assertive and projective components such that—combined with standard views on projection, accommodation and filtering—we get a uniform solution to the embedded FC puzzles. This result complements earlier work in which we argue that $\mathbf{pex}^{IE+II}$ improves the predictions of Grammatical theories of SIs (Bassi et al. 2021, Del Pinal 2021). This approach also simplifies Grammatical theories.

For it supports analyses with local application of $\mathbf{pex}^{IE+II}$ of embedded FC effects which previous theorists were forced to try to solve using matrix scope **exh** in ways that require stipulating ever more complex operations, both with respect to the sets of alternatives which have to be generated and kept track of (Bar-Lev & Fox 2020), and the way in which exclusion and inclusion works for the assertive and presupposed content of the prejacent and its alternatives (Marty & Romoli 2020).

A $\mathbf{pex}^{IE+II}$-based Grammatical theory opens up various projects, incl. extensions to homogeneity effects (cf. Bar-Lev 2021)—currently being developed by Guerrini & Wehbe (2023) for plurals and by Paillé (2023) for summatives—and to exceptives and polarity sensitive items (cf. Gajewski 2008, Chierchia 2013, Nicolae 2012, 2017). In addition, certain overt operators and constructions seem to call for a $\mathbf{pex}^{IE+II}$-like analysis, e.g., Indian English post-positional scalar *only* (Ghoshal 2023) and it-clefts (Velleman et al. 2012, Büring & Kriz 2013, Onea 2019). These applications promise to advance our understanding of the taxonomy and distribution of exhaustification operators. Finally, we have argued that exhaustive inferences project like presuppositions, yet can be globally informative when consistent with the common ground. Do the 'non-assertive' outputs of other exchaustifiers and exclusives behave in similar ways? Are there other ways of modelling non-at issue contents with that kind of profile?

This paper focused on Grammatical accounts, yet future research should compare $\mathbf{pex}^{IE+II}$ with recent non-**exh**-based theories of FC. We have seen that Goldstein (2019)'s homogeneous alternative semantics is similar to our account in the way it structures the interpretation of $\Diamond\vee$ and $\neg\Diamond\vee$-sentences. Yet the source of the associated homogeneity effects is different. We derive them via general meaning-enrichment procedures triggered by $\mathbf{pex}^{IE+II}$, rather than on the basis of specific lexical stipulations, and as a result directly predict negative FC. Still, since negative FC seems to be less robust than basic FC, it is an open question whether unified accounts are preferable to hybrid ones (see Marty & Romoli 2020, Marty et al. 2023). Aloni (2022) also develops an important non-**exh** theory. It uses a bilateral state-based modal logic to model literal and enriched interpretations—and when combined with the hypothesis that humans tend to neglect empty models, it predicts an impressive range of FC effects. Comparisons between this and Grammatical theories may ultimately depend on developmental, processing and robustness patterns for various FC and SI effects (Chemla & Bott 2014, Tieu et al. 2016, van Tiel & Schaeken 2017, Marty et al. 2021, 2023).

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Guillermo Del Pinal
E305 South College,
150 Hicks Way
Amherst, MA 01003
gdelpinal@umass.edu

Itai Bassi
Leibniz-ZAS
Pariser Str. 1
10719 Berlin, Germany
bassi@leibniz-zas.de

Uli Sauerland
Leibniz-ZAS
Pariser Str. 1
10719 Berlin, Germany
sauerland@leibniz-zas.de