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Ranges and Paths*

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Abstract Range expressions such as *between 3 and 8*, *from 3 to 8*, and *3 through 8* resemble modified numerals such as *at least 3* and have sometimes been mentioned under that rubric. This paper shows that they are crucially different from modified numerals in their distribution, the readings available to them, and their behavior with respect to quantifiers. We also show more generally that they have an intricate grammar of their own. We distinguish three classes of readings they can receive: singleton punctual readings, on which they often give rise to ignorance inferences; set punctual readings, which arise chiefly in the scope of quantifiers; and interval readings, where the range is interpreted exhaustively. The heart of the semantics of range expressions, we suggest, is the notion of paths, generalized from its use in the analysis of locatives to extend across semantic types. This provides evidence for a close connection between the semantics of locatives and that of numerals and degrees.

Keywords: ranges, paths, measure phrases, locatives, degrees, vectors, ignorance inferences, modified numerals

1 Introduction

This paper will take up the analytical challenge of what we will call RANGE EXPRESSIONS, such as those in (1):

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This paper draws on research supported by the Social Sciences and Humanities Research Council of Canada. It differs significantly from the authors' past work in this general area. Gobeski & Morzycki (2022a) explores a variety of measure phrases composed of two elements—what we call COMPOSITE MEASURE PHRASES—but provides no semantics for ranges. Gobeski & Morzycki (2022b) is specifically about ranges, but the empirical focus and the analysis are both different.

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(1) a. Floyd saw between 10 and 15 ferrets.
b. The kids' ages ranged from 10 to 15.
c. This volume covers {from Lincoln to Taft / Lincoln through Taft}.

To provide a rough-and-ready definition for our purposes, we will take a range expression to consist of exactly two noun phrases (frequently but not necessarily involving a numeral) combined with *to, through*, or *between ... and ...*, which expresses two elements of a potentially larger ordered set.[1] These expressions are in some respects akin to modified numerals such as *at least 3*, but as we'll show, they have an interesting and subtle grammar of their own. That's especially clear in (1c), which shows that a range needn't be numeric. Any set with an appropriate ordering—such as the alphabet or American presidents—will suffice.[2]

We take range expressions to be, structurally, a special case of a more general phenomenon that in other work (Gobeski & Morzycki 2022a) we've termed COMPOSITE MEASURE PHRASES:

(2) a. The room is 10 feet by 15 feet.
b. Her odds of winning are {1 in 4 / 1 to 3 / 1 out of 4}.
c. The Padres beat the Red Sox by 3 to 2.

The distinguishing characteristic of these is that they're measure phrases with multiple measure phrases as subconstituents. The resemblance is chiefly syntactic, and we won't linger on it further here, but it's relevant to the larger point that these more complicated structural arrangements have subtle and intriguing semantic consequences.

Range expressions have sometimes been brought up as a form of modified numeral (Nouwen 2010, Schwarz et al. 2012, Rett 2014a, Buccola & Spector 2016), but there is a considerably larger story to be told. That's reflected most clearly in non-numeric examples like (1c), but also in the selectional restrictions of predicates, among other things. This suggests that, despite often containing numerals, ranges aren't a form of modified numeral; after all, they don't seem to contain any elements

1 That said, we will discuss related expressions at various points, including what may be one-sided ranges (*from 3 up*). We will return to the question of what a range is, exactly, at the end of the paper, in section 5.4.

2 We won't pursue here the question of what properties this ordering must have, but it apparently needn't actually be a true partial order at all. American presidents have served non-concurrent terms, but it's natural to build ranges based on temporal precedence. That entails that the presidential temporal precedence relation is not antisymmetric, making it only a preorder.

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one might regard as modifiers. Consequently, we will instead take the relationship to locative expressions as the starting point of our analysis in section 3. Indeed, one of the big-picture pay-offs of the enterprise is novel evidence for a close connection between locative semantics and the semantics of numerals and degrees.

Section 2 lays out the crucial empirical facts, identifying a diagnostic for range expressions that distinguishes them from ordinary modified numerals. It also divides the readings available to them into three classes, and provides a diagnostic that adjudicates among them. Section 3 addresses the locative foundations on which the grammar of range expressions rests, and introduces the crucial tool, paths, that will serve to capture the essential core of their semantics. Section 4 takes up the broader challenge of range expressions in earnest. It provides a semantics for their in individual, degree, and cardinality contexts by generalizing the notion of paths across semantic types. Along the way, evidence emerges in this section for encoding maximality in a particular way into the compositional machinery that drives range expressions. Making this assumption yields a semantics that reflects how range expressions interact scopally with quantifiers. Section 5 addresses consequences and bigger-picture issues. Section 6 concludes.

2 The data

2.1 How range expressions are built

Cast of characters and semantic selection There are at least three basic varieties of range expressions in English, each one involving its own distinct morpheme:[3]

(3) a. between 3 and 8
b. (from) 3 to 8
c. 3 through 8

It is possible to combine some of these basic types into hybrid forms, as in *from 3 through to 8*.

Certain nouns and verbs select range complements exclusively. This includes both the nominal and verbal forms of *range* and *span*, as well as verbs such as *cover*:

3 There may be a fourth type, namely, one-sided ranges. We'll discuss these in section 5.4.

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(4) a. the $\left\{\begin{array}{l}\text{range}\\\text{span}\end{array}\right\}$ $\left\{\begin{array}{l}\text{between 2 and 6}\\\text{from 2 to 6}\\\text{2 through 6}\\\text{up to 6}\\\#\text{at least 2}\\\#\text{more than 2}\\\#\text{around 4}\\\#\text{under 6}\end{array}\right\}$

b. This volume covers $\left\{\begin{array}{l}\text{between G and L}\\\text{from G to L}\\\text{G through L}\\\text{up to L}\\\#\text{at least G}\\\#\text{more than G}\\\#\text{around G}\\\#\text{under L}\end{array}\right\}$.

c. Their ages $\left\{\begin{array}{l}\text{ranged}\\\text{spanned}\\\text{varied}\end{array}\right\}$ $\left\{\begin{array}{l}\text{between 2 and 6}\\\text{from 2 to 6}\\\text{2 through 6}\\\text{up to 6}\\\#\text{at least 2}\\\#\text{more than 2}\\\#\text{around 4}\\\#\text{under 6}\end{array}\right\}$.

As (4) reflects, typical modified numerals such as *at least 2* are impossible in these positions. That's an initial important indication that it's fruitful to distinguish range expressions from arbitrary cases of numeral modification.

Predicates that select ranges are also useful as a rule-of-thumb for identifying ranges. One empirical diagnostic for ranges is the ability to occur with range-selecting predicates like *range* or *span*. This distinguishes ranges from arbitrary numeral constructions. It will serve as a preliminary definition of ranges for our purposes here, to define our aims tractably. The larger question of whether there is a natural class of phenomena here will be easier to address once we have analytical tools in place to evaluate the question, in section 5.

Predicates of ranges Some range expressions can be predicated of a particular number or, more generally, a particular element in an order. The prime example of this is *between/and*, which contrasts with *from/to* and *through* in this respect:

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(5) a. 3 is $\left\{\begin{array}{l}\text{between 1 and 5}\\ \#\text{from 1 to 5}\\ \#\text{1 through 5}\end{array}\right\}$.

b. Taft is $\left\{\begin{array}{l}\text{between Lincoln and Eisenhower}\\ \#\text{from Lincoln to Eisenhower}\\ \#\text{Lincoln through Eisenhower}\end{array}\right\}$.

c. Montreal is $\left\{\begin{array}{l}\text{between Vancouver and Toronto}\\ \#\text{from Vancouver to Toronto}\\ \#\text{Vancouver through Toronto}\end{array}\right\}$.

Ranges built with *from/to* require an extended portion of an order, that is, an interval across time, space, degrees, or numbers:

(6) a. The meeting was from 11:00 to 12:00.
b. The trip was from Vancouver to Toronto.
c. This box is (from) 8 to 9 inches wide.
d. The possible values of this variable range from 8 to 9.
e. This line is from point A to point B.
f. We saw from 5 to 10 capybaras.

There are some important further distinctions here, which we will encounter subsequently. One is worth noting immediately: in (6c) and (6f), the range of values is created by uncertainty around where a particular value is located. (We address this in 2.2.) *Through* can grammatically be inserted in the examples in (6), but bare *through* range expressions are not in general grammatical in these contexts:

(7) a. The meeting was 11:00 through 12:00.
b. ??The trip was from Vancouver through Toronto.[4]
c. #This box is 8 through 9 inches wide.
d. #The possible values of this variable range 8 through 9.
e. #This line is point A through point B.
f. #We saw from 5 through 10 capybaras.

On the other hand, *between* is possible in all of them:

(8) a. The meeting was between 11:00 and 12:00.
b. The trip was between Vancouver and Toronto.
c. This box is between 8 and 9 inches wide.
d. The possible values of this variable range between 8 and 9.

4 This seems to be possible for airline employees.

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e. This line is between point A and point B.
f. We saw between 5 and 10 capybaras.

The upshot, then, is that *from/to* ranges can be predicated of a range, whereas *between* can be predicated both of a range of values and of a single one. It's worth noting as well that this distinction emerges in the event domain, where it has aspectual consequences. *From/to* ranges are generally odd with telic predicates:

(9) a. He pushed the cart { between here and Toronto / from here to Toronto / from here through to Toronto }.

b. He exploded { between here and Toronto / #from here to Toronto / #from here through to Toronto }.

Constituency facts There is room for skepticism about whether *from/to* range expressions are actually constituents at all, rather than merely a common collocation involving distinct adjacent constituents. This matters because it bears on the compositional ingredients from which they are built. One might suspect them of being simply adjacent locative PP adjuncts. But the order of truly locative PPs can be inverted:

(10) Floyd drove { from Boston to Cleveland / to Cleveland from Boston / between Boston and Cleveland / between Cleveland and Boston }. *(locative)*

That is not the case for range expressions:

(11) This encyclopedia covers { from Boston to Cleveland / *to Cleveland from Boston }. *(range)*

Similarly, locative PPs cleft, but components of ranges don't:

(12) a. It was to Cleveland that we drove from Boston __. *(locative)*
b. *It was to Cleveland that this encyclopedia covers from Boston __. *(range)*

Range constructions are also possible in subject position:[5]

5 Thanks to Reed Steiner for helping us discover this fact.

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(13) From Boston to Cleveland is $\left\{\begin{array}{l}\text{a long way}\\ \text{too far}\end{array}\right\}$.

The analysis pursued here will indeed draw a close connection between range and locative expressions, but crucially, it will not have the consequence that they are identical. More important still, part of the interest will be in how range expressions acquire an intricate variety of distinct uses by building on their locative foundations.

Distinctness and non-zero requirements Range expressions also require that the two constituents be distinct from each other:

(14) a. #Sam saw between 3 and 3 monkeys.
b. #Floyd ate a dozen to 12 cookies.

This is unsurprising, of course, but neither is it trivial. The requirement holds even when the numbers are introduced under different descriptions:

(15) #between 3 and the number on this playing card *(odd if both are 3)*

Nor can ranges include zero except on a jokey reading:

(16) a. #?Sam saw between 0 and 3 monkeys.
b. #?We expect 0 to 3 people.
c. Floyd is $\left\{\begin{array}{l}\#0\\ 1\end{array}\right\}$ to 6 feet tall.

Of course, this restriction is present only where *0* is either independently infelicitous (*#0 feet tall*) or receives a reading similar to "no" (*Sam saw 0 monkeys*; Bylinina & Nouwen 2018). Environments in which *0* is felicitous also permit ranges that contain *0*:

(17) a. This car goes from 0 to 60 (miles per hour) in 8 seconds.
b. This toy is for ages 0 through 3.

These facts in conjunction are likely the same phenomenon as Schwarz et al. (2012)'s BOTTOM-OF-SCALE EFFECTS. They observe that an expression such as *up to* doesn't normally include 0 in the range:

(18) #Up to 1 person died in the crash.

The measurement in (18) starts at 1—and since the two range expressions must be distinct, (18) is infelicitous.

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Orders provided by context The interpretation of a range expression is sometimes constrained by a contextually-supplied order that contains its range. For example, an apparently exhaustive range expression like *houses 10 through 15* may pick out a range of houses that excludes 13 if that number has been skipped by city planners for superstitious reasons, and of course it would normally (but not necessarily!) skip fractional house numbers as well. This would be different if the context were different. In a discussion that makes the integers the salient order, skipping 13 would not be possible, but skipping fractions still would. This contextual effect is examined in more detail in section 4.6.

2.2 Three kinds of readings

We'll distinguish three kinds of readings that range expressions can receive, depending on their environment. We dub these SINGLETON PUNCTUAL READINGS, INTERVAL READINGS, and SET PUNCTUAL READINGS.[6]

Singleton punctual readings The reading that comes most readily to mind is the singleton punctual reading, on which there is a single crucial value within the range, which the speaker may or may not know:

(19) a. Floyd is {between 5 and 6 / 5 to 6} feet tall.

b. Floyd weighs {between 150 and 200 / from 150 to 200} pounds.

c. We expect {between 10 and 15 / from 10 to 15} people.

In (19a), for example, the value of interest is normally Floyd's (maximum) degree of height. The range itself is present only as an epistemic fig leaf covering the speaker's uncertainty.

Interval readings By contrast, on interval readings the whole of the range is crucial:

(20) a. Water is fluid {between 32 and 212 / from 32 to 212 / at 32 through 212} degrees Fahrenheit.

6 The use of 'punctual' in this empirical neighborhood is inspired by Rett (2014a).

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b. This volume covers American presidents $\left\{\begin{array}{l}\text{between Lincoln and Taft}\\ \text{from Lincoln to Taft}\\ \text{Lincoln through Taft}\end{array}\right\}$.

c. The snowplow cleared $\left\{\begin{array}{l}\text{between 12 and 20 Main St.}\\ \text{from 12 to 20 Main St.}\end{array}\right\}$

In (20a), we're not interested in a single temperature at which water is fluid, but rather the full interval of temperatures. Importantly, *through* seems to receive only interval readings. There is a helpful diagnostic for these readings—they alone allow exceptives:

(21) a. Water is fluid $\left\{\begin{array}{l}\text{between 32 and 212}\\ \text{from 32 to 212}\\ \text{at 32 through 212}\end{array}\right\}$ degrees Fahrenheit, except at 100 degrees, for some reason.

b. This volume covers American presidents $\left\{\begin{array}{l}\text{between Lincoln and Taft}\\ \text{from Lincoln to Taft}\\ \text{Lincoln through Taft}\end{array}\right\}$ except (for) Garfield.

c. The snowplow cleared $\left\{\begin{array}{l}\text{between 12 and 20 Main St.}\\ \text{from 12 to 20 Main St.}\end{array}\right\}$ except for 16 Main St.

Of course, (21a) is factually false, but it is perfectly well-formed, as is (21b). By contrast, range expressions that receive a singleton punctual reading don't support exceptives:

(22) a. Floyd is $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{5 to 6}\end{array}\right\}$ feet tall #except 5 foot 8.

b. Floyd weighs $\left\{\begin{array}{l}\text{between 150 and 200}\\ \text{from 150 to 200}\end{array}\right\}$ pounds #except 175.

c. We expect $\left\{\begin{array}{l}\text{between 10 and 15}\\ \text{from 10 to 15}\end{array}\right\}$ people #except 12.

Set punctual readings On these readings, there are potentially multiple values in the range that are crucial—though unlike on interval readings, these crucial values needn't exhaust the full range. Set punctual readings arise mainly in plural and quantified contexts (including modals):

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(23) a. The children were $\left\{\begin{array}{l}\text{(from) 5 to 6}\\ \text{between 5 and 6}\end{array}\right\}$ feet tall. *(plural)*

b. Every student was $\left\{\begin{array}{l}\text{5 to 6}\\ \text{between 5 and 6}\\ \text{?from 5 to 6}\end{array}\right\}$ feet tall. *(quantificational)*

c. To ride the roller coaster, you must be $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{5 to 6}\\ \text{?from 5 to 6}\end{array}\right\}$ feet tall. *(modal)*

They are also possible in temporal examples such as (24):

(24) Floyd's weight fluctuated $\left\{\begin{array}{l}\text{between 175 and 200}\\ \text{from 175 to 200}\end{array}\right\}$ pounds. *(temporal)*

These readings resemble interval readings in that they involve more than one crucial value, but the exceptive diagnostic demonstrates that they are in fact distinct—exceptives are very odd with set punctual range expressions:

(25) a. The children were $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{(from) 5 to 6}\end{array}\right\}$ feet tall, ??except (for) 5 foot 3.

b. Every student was $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{5 to 6}\end{array}\right\}$ feet tall, ??except (for) 5 foot 3.

c. To ride this roller coaster, you must be $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{5 to 6}\end{array}\right\}$ feet tall, ??except (for) 5 foot 3.

The quantificational and plural forms in (25a) and (25b), respectively, are odd at best, while the modal form in (25c) is even more deeply so. Of course, all these contexts either are or can be construed to be quantificational, so quantification seems to be the key to how these readings are achieved.

2.3 In a nutshell

We have provided a general overview of facts in this domain, partly because this doesn't seem to have been done previously. But the crucial ones that any theory should explain are:

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- In English, *between/and*, *from/to*, and *through* ranges. There is a lot of richness in this domain to cover, including contrasts like *3 is between {2 and 4 /#from 2 to 4}*.
- Their distribution, including measure phrase uses, cardinality uses, and locative uses.
- A semantics that treats range expressions as constituents rather than just adjacent adjuncts.
- The three kinds of reading: singleton punctual, interval, and set punctual.
- The selection of range expressions by heads like *cover*, *span*, and *range*.

We addressed ignorance readings of cardinal uses extensively in Gobeski & Morzycki (2022b) and touch on them here briefly in section 4.8.

3 Laying the analytical groundwork: the locative foundations of range expressions

3.1 From locatives to ranges

We will begin our analysis in what may seem a surprising place: locative adverbials.[7] As we demonstrated in 2.1, range expressions are actually consequentially different from superficially similar locative expressions. In English, this is most clearly demonstrated by the fact that range expressions have an inherent two-part syntactic structure with elements that can't be inverted—*from 5 to 10* is possible, but not **to 10 from 5*—which distinguishes them from two adjacent locative PPs.

Nevertheless, it's unmistakable that there is *some* connection between range expressions and locatives, so it doesn't seem too bold to suppose that ranges are built on the grammatical scaffolding of locatives.[8]

It's also in line with a tradition of work such as Habel (1990), Jackendoff (1983), Talmy (1996), Gawron (2005), and Zwarts (2013) who note a relationship between locatives that indicate movement (e.g., *Floyd drove from Boston to Cleveland*) and expressions that involve what Talmy (1996) calls "fictive motion", wherein the language of motion is used despite no literal movement taking place (e.g., *The Andes*

7 For convenience, we henceforth will use 'locative' to refer to expressions that are locative in a narrow sense like *above the house* as well as directional expressions like *from the house* (see Zwarts 2005 for more on the distinction and Zwarts 2017 for a recent more general appraisal of locative semantics). 'Spatial' is a more neutral term.

8 Hackl (2023) helpfully nudged us in this direction. Zwarts (2021) provides a semantics for *bên* 'between' in Hebrew in spatial uses and a bit beyond. We build on that in the following section.

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Mountains run from Venezuela to Argentina). It's certainly not obvious that range expressions must involve motion in any sense at all, fictive or otherwise. [9]

Our first step will be to commit to some representational assumptions about locative expressions. Following a range of work on locatives (Cresswell 1978, Habel 1990, Wunderlich 1991, Zwarts 1997, Zwarts & Winter 2000, Kracht 2002, Gawron 2005, Zwarts 2005, von Stechow 2006, Gehrke 2008)—and, somewhat more remotely, the approach to degree semantics of Faller (2000) and Schwarzschild (2012, 2013)—we will assume a model that includes points in space as well as PATHS in space. In section 4.1, we will commit explicitly to a particular conception of paths on which they are simply a species of ordered set. Spatial paths, on this view, are sequences of adjacent points with a direction, running from a start point to an end point.

This differs from the conception of paths assumed in Cresswell (1978) and much subsequent work, on which paths are functions from times (or numbers) to locations. Because we seek to construe paths across domains—including across sequences of numbers—the temporal component of such a conception is not helpful. We could perhaps construe our paths as functions from integers to arbitrary objects, but this would amount to an ordered set. It's not clear to us that there is any advantage in introducing this additional machinery here.

On either view, paths can take twists and turns—they aren't necessarily straight lines. It's not clear to us that this property is necessary for an account of range expressions,[10] and perhaps we would be better off using the simpler notion of vectors (directed line segments in a vector space) instead. Nevertheless, we will stick with 'path' because that is a more intuitively transparent and theory-neutral descriptive term and it helps make the connection to locative uses more robustly.

We will commit to assumptions about the ontology of paths (in our sense) in section 4.1. For the moment, the choice of a particular conception isn't crucial. The options include construing paths as sequences of locations (Nam 1995), as functions from real numbers (between zero and one inclusively) to points or locations (Kracht 2002, Zwarts 2005), and as functions from points in the running time of an event to locations (Gawron 2005). There is a range of further decisions to be made about the nature of paths empirically and the formal properties of their representations. We will defer these decisions because resolving them isn't crucial to getting started, and

9 An alternative starting point would be to build not on the similarity to locative expressions but rather the similarity to modified numerals. That is in fact a course we pursued in earlier work (Gobeski & Morzycki 2022b). We will focus on the locative approach here.

10 They are useful for spatial examples like *The trip took us from Vancouver through Winnipeg to Toronto*, but these don't seem to involve true range expressions but rather a sequence of stacked right-adjoined PPs. The PPs in this sentence can be freely reordered, as one would expect of adjuncts: *The trip took us to Toronto from Vancouver through Winnipeg.*

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it will be more profitable to return to them later with more assumptions in place. We do this in section 4.1, where the notion of paths will be generalized crosscategorially, beyond the spatial domain.

For the moment, it will suffice to assume that the type of paths is p, that path variables are p, p', p'', and so on, and that paths have start and endpoints. To build denotations, we will appeal to functions that retrieve from a path its start point and its end point: **start** and **end**, again building on Zwarts (1997), Faller (1998), Zwarts & Winter (2000), Gawron (2005), Schwarzschild (2012, 2013), among others. These are essentially like Neo-Davidsonian thematic role predicates such as **agent** that retrieve a particular participant from an event (Parsons 1990 among many others).

3.2 From/to

With this in place, we can treat locative PPs like *from Los Angeles* and *to New York* as properties of paths that, respectively, start at Los Angeles and end at New York:

(26) $[\![\textit{from Los Angeles}]\!] = \lambda p[\textbf{start}(p) = \textbf{Los-Angeles}]$
$[\![\textit{to New York}]\!] = \lambda p[\textbf{end}(p) = \textbf{New-York}]$

This brings to light another formal decision. It makes sense to think of the start point of a path as, well, a point, rather than an ordinary individual like Los Angeles, as (26) would seem to suggest. We could, of course, reflect that in the denotation, writing **location**(**Los-Angeles**) instead of just **Los-Angeles** to refer to the point at which Los Angeles is located, and likewise throughout:

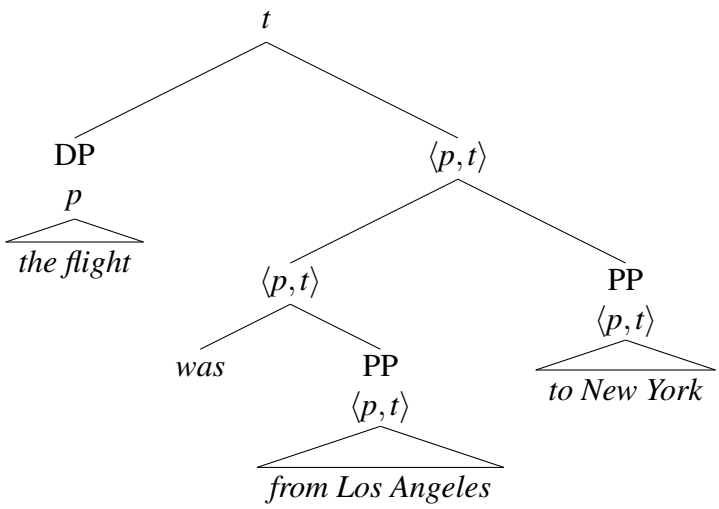
(27) $[\![\textit{from Los Angeles}]\!] = \lambda p[\textbf{start}(p) = \textbf{location}(\textbf{Los-Angeles})]$
$[\![\textit{to New York}]\!] = \lambda p[\textbf{end}(p) = \textbf{location}(\textbf{New-York})]$

But for simplicity, we'll adopt the convention that, as in (26), terms for individuals can be used to refer instead to the point at which those individuals are located.[11]

To interpret a simple locative sentence, we will need to further assume that certain nominals can refer to paths. Among them are *trip*, *flight*, *journey*, and *path* itself. A simple locative sentence could thus be interpreted as in (28):

11 This sets aside the fact that Los Angeles is located at a region rather than a point, and questions of vagueness that may arise around where its boundaries lie.

(28)



a. $[\![\textit{was from Los Angeles}]\!] = [\![\textit{from Los Angeles}]\!]$
$= \lambda p[\textbf{start}(p) = \textbf{Los-Angeles}]$

b. $[\![\textit{was from Los Angeles to New York}]\!]$ *(by Predicate Modification)*
$= \lambda p[\textbf{start}(p) = \textbf{Los-Angeles} \wedge \textbf{end}(p) = \textbf{New-York}]$

c. $[\![\textit{the flight was from Los Angeles to New York}]\!]$
$= 1$ iff $\textbf{start}(\textbf{the-flight}) = \textbf{Los-Angeles} \wedge$
$\textbf{end}(\textbf{the-flight}) = \textbf{New-York}$

We take *was* to be vacuous here. We've treated the *from* and *to* PPs here as separate constituents, with the *to* PP being an adjunct interpreted intersectively. That seems to be appropriate for locative uses, but not in range expressions, as section 2.1 noted.

One might reasonably be skeptical about whether an eventive nominalization like *the flight* actually refers to a path. A flight can be enjoyable or boring or comfortable, but none of these could sensibly be predicated of just a set of points. Really, we should say that a flight or a trip has a PATH CORRELATE, and it's that path correlate that can be said to have a start or end point. For the moment, it will suffice to suppose that a type shift from individuals to their path correlates (type $\langle e,p\rangle$) can freely apply anywhere, and in this case it applied to the subject.[12]

The semantics in (28) makes a prediction worth noting because it will prove important. Nominals that don't denote or quantify over paths (whether inherently or as the result of a shift) cannot occur in the subject position of sentences like (28):

12 An alternative course would be to build this shift into the denotation of the range expression. Yet a third option would be to treat the NP as referring to an event, and have the predicates **start** and **end** map events directly to the location where they start and end à la Gawron (2005).

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(29) a. #Chicago is from Los Angeles to New York.
b. #Chicago is to New York.

There is a wrinkle here, though. Plurals can sometimes be shifted to path interpretations, subject to mysterious restrictions:

(30) The strikes were $\left\{\begin{array}{l}\text{from Cleveland to Cincinnati}\\ \text{\#from Cleveland}\\ \text{\#to Cincinnati}\end{array}\right\}$.

It would be possible to accommodate such uses on our approach with the aid of a distributivity operator that universally quantifies over atomic strikes and locates each of them along a path from Cleveland to Cincinnati, but that wouldn't address why the non-path subject is possible here at all.

3.3 Between

Our current aim is to sketch the locative semantics of the prepositions that are the building blocks of range expressions. The next step is to examine *between*.

Here, there is prior work to build on, principally Habel (1990) and Zwarts (2021). Zwarts (2021) examines Biblical Hebrew *bên* 'between', in its locative uses and a bit beyond. Much of the aim there is to capture an interesting property of *bên* that *between* lacks: 'between *x* and *y*' is expressed as *bên x bên y*, with each *bên* introducing an element of the 'between' expression. That suggests that each of the two *bên*-phrases need an independent interpretation and that, arguably, their meanings should be combined intersectively. This compositional challenge doesn't arise in English—at least not in the same way—and we will treat English *between* as simply taking two arguments.

Both Habel and Zwarts concern themselves explicitly with articulating various geometric configurations in which an object can be said to be between other objects. That is of course ultimately crucial to sorting out the spatial semantics of *between* and its cognates, but it will not be necessary for our purposes here.

There is another element in Zwarts that is worth highlighting. He observes that *between* has non-spatial uses too, including cases such as *a war between Rehoboam and Jeroboam* and even *discern between the unclean and the clean*. For this reason, he proceeds as far as possible in using general set-theoretical tools, of which the building blocks of spatial meanings are a special case. Although our empirical focus is different, the strategy of using general tools that operate across domains—spatial and otherwise—is one we too will ultimately pursue (in section 4.1).

Before all that can happen, we will need to establish some crucial empirical facts and a path-oriented way of construing them. The first of these is that *between* PPs

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differ from *from/to* PPs in that *between* is not normally a predicate of paths. Thus the ungrammatical examples with non-path subjects in (29) are natural with *between*:

(31) Chicago is between Los Angeles and New York.

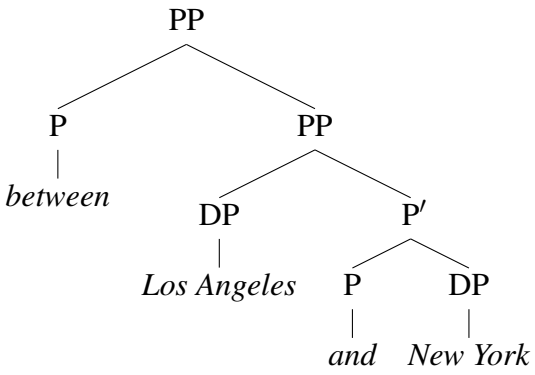
There are path uses alongside this, though (as already noted, in different terms, in section 2.1):

(32) The flight is between Los Angeles and New York.

In both uses, *between* specifies two endpoints.

Second, in order to assemble the pieces, it makes sense for the moment to construe *between* as actually having *two* internal arguments, one for each path endpoint. The natural alternative would be a single internal argument that's plural, but as we will see later in this subsection, this is only feasible for locative cases. The syntax required to achieve a structure with two internal arguments isn't straightforward and our focus here is semantic, but one might think of *between/and* as a single discontinuous preposition. This could be implemented by supposing that it's a kind of idiom with gaps for the two arguments, one of them in a specifier position of an embedded PP:

(33)



This commits to the view that there truly is an idiom with two open syntactic positions in it, and provides a denotation that isn't fully compositional. A natural fully-compositional alternative could take *between-and* to be a complex preposition occurring in the lower P position, with the *between* portion moving up to the higher P.[13] We will stick with the idiom approach here for simplicity.

With that, we can venture a denotation, one on which a *between* PP holds of an individual iff it is located on a path between the two endpoints:

13 This would make possible the fully compositional denotation in (34):

(i) $[\![\textit{between-and}]\!] = \lambda x \lambda y \lambda z\,.\,\exists p[\mathbf{start}(p) = x \wedge \mathbf{end}(p) = y \wedge z \in p]$

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(34) a. $[\![\textit{between } \alpha \textit{ and } \beta]\!] = \lambda x\,.\,\exists p[\textbf{start}(p) = [\![\alpha]\!] \wedge \textbf{end}(p) = [\![\beta]\!] \wedge x \in p]$

b. $[\![\textit{between Los Angeles and New York}]\!]$

$$= \lambda z\,.\,\exists p\begin{bmatrix}\textbf{start}(p) = \textbf{Los-Angeles} \wedge \\ \textbf{end}(p) = \textbf{New-York} \wedge z \in p\end{bmatrix}$$

Of course, given this denotation, non-path subjects are possible because z here is not a path but simply an individual located along one. To accommodate path subjects, it seems to be necessary to assume some polysemy. The denotation of the path variant would actually be simpler than (34a)—it simply requires introducing the path as an argument:

(35) $[\![\textit{between}_{\textit{path-variant}} \alpha \textit{ and } \beta]\!] = \lambda p[\textbf{start}(p) = [\![\alpha]\!] \wedge \textbf{end}(p) = [\![\beta]\!]]$

In light of the similarity between the two, one might suspect that (34a) is derived from (35) by a type shift. The type shift required would, in fact, rather resemble one that is standardly assumed in the Vector Space Semantics strand of the locatives literature. Replacing vectors with paths, the vector-space-semantics-style type shift applies to a predicate of paths and yields a predicate of individuals located at the end of those paths. What we would need here is more general, one that yields predicates of individuals located at *any point* along those paths:

(36) $[\![\text{VECTOR-SPACE-STYLE SHIFT}]\!] = \lambda P_{\langle p,t\rangle}\lambda x\,.\,\exists p[P(p) \wedge x \in p]$

(for consideration only)

But this would be a mistake. Going down that road would mean that arbitrary path predicates could be shifted this way, leaving us with no explanation for why non-path subjects like *Chicago* are impossible with path predicates like *from Los Angeles*. It truly seems to be a lexical semantic idiosyncrasy of *between* that it allows both sorts of subjects.

Another difference between *from/to* and *between/and* that's noted in some of the literature on modified numerals (Nouwen 2010, Schwarz et al. 2012, Rett 2014a, Buccola & Spector 2016) is that the former include their endpoints and the latter excludes them. We will return to this in section 4.7.

One further note about *between/and*. For strictly locative uses, the denotation we have provided won't suffice. That's because on such readings, it's possible for *between* to actually apply to only one nominal so long as it's plural:

This option is probably the more satisfying one, all things considered. For an alternative in which *between* simply takes a plural argument, see (39). That line of analysis is not available for cardinal uses: *#We saw between two numbers capybaras.*

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(37) a. Chicago is between two cities.
b. The flight is between two Delta hubs.

That suggests that the denotation we provided, which *requires* two internal arguments, can't be right in general. But it is right for at least some range expressions, which don't always allow plural arguments:[14]

(38) a. Floyd saw between $\left\{\begin{array}{l}\text{5 and 10}\\ \text{*two odd numbers}\\ \text{*some odd integers}\end{array}\right\}$ capybaras.

b. The children's ages ranged between $\left\{\begin{array}{l}\text{5 and 10}\\ \text{??two fairly young ages}\\ \text{??some odd integers}\end{array}\right\}$.

c. This encyclopedia volume covers presidents between $\left\{\begin{array}{l}\text{Cleveland and Taft}\\ \text{??(two) portly gentlemen}\end{array}\right\}$.[15]

Indeed, to the extent that the ill-formed variant in (38c) can be accommodated, it requires construing the encyclopedia to cover presidents physically located between portly gentlemen.

For this reason, we will keep the denotation provided above as our official one for the range-expression *between/and*. For the locative use, it suffices to modify *between* so that it applies to an arbitrary plural nominal and yields a property of individuals located along a path between the members of the plurality:[16]

(39) $[\![\textit{between}]\!] = \lambda x : |x| = 2 \,.\, \lambda z \,.\, \exists p[\mathbf{start}(p) \sqsubset x \wedge \mathbf{end}(p) \sqsubset x \wedge z \in p]$

Requiring as a presupposition that the plurality have precisely two members, as in (39), rather than at least two, is probably too strong. There is a prescriptive

14 One could construe (38a) as evidence for another claim: that structures like (38a) are derived by ellipsis from a form like *Floyd saw between 5 ~~capybaras~~ and 10 capybaras.* However, in the interest of space we won't pursue this further here.

15 Interestingly and mysteriously, *between two portly gentlemen named Cleveland and Taft* seems to be considerably improved.

16 We use $\sqsubset$ here to indicate the Link (1983)-style individual part relation that holds between a plurality and its members. The form in (39) slightly abuses our convention of using individual terms to refer to corresponding locations by assuming it works the other way around as well. Here's a more explicit variant:

(i) $[\![\textit{between}_{\textit{path-variant}}]\!] = \lambda x : |x| = 2 \,.\, \lambda z \,.\, \exists p \exists x' \exists x'' \left[\begin{array}{l} x' \sqsubset x \wedge x'' \sqsubset x \wedge \\ \mathbf{start}(p) = \mathbf{location}(x') \wedge \\ \mathbf{end}(p) = \mathbf{location}(x'') \wedge \\ z \in p \end{array}\right]$

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prohibition on using *between* with more than two arguments.[17] No one would bother to stigmatize the use if it didn't exist, but we will leave this denotation as it is because pursuing the issue would take us too far afield.

3.4 Through

There is a third building block of range expressions that has a locative foundation: *through*. In its locative use, it often occurs as a part of a *from/to* structure as in (40):

(40) The snowplow cleared (from) 10 Main Street through (to) Elm Street.

There are a number of fine-grained idiosyncrasies in how *through* is used in locatives and elsewhere, some of which we will sidestep. The most pressing question to address is what *through* actually contributes, both in its independent use and when it occurs inside a *from/to* structure.

To appreciate the effect of *through*, it helps to consider the circumstances under which the *through*-less variant in (41) can be true:

(41) The snowplow cleared from 10 Main Street to Elm Street.

This is certainly true if there is a stretch of Main Street that starts at 10 Main Street and ends at the intersection with Elm Street, and that stretch has been entirely cleared. But that isn't actually necessary. Plowing snow is a complicated affair, and occasionally a parked car or other obstacle requires skipping over a portion of the street. If some such obstacle meant that a proper subpart of the path in (41) had to be skipped over—say, from 16 to 20 Main Street—(41) can nevertheless be judged true.

Caution is called for. It's not completely obvious that the judgment should be understood as being that the sentence is true, or simply that it is close enough to being true for the pragmatic circumstances. This notion—that sentences that are strictly false can be regarded as close enough to being true for current purposes—is at the heart of the Lasersohn (1999) theory of loose talk and imprecision, in which every expression in a sentence has a 'halo' around its denotation consisting of elements that are more or less interchangeable with it for current purposes. Thus *6 feet* denotes a precise measure, but when discussing someone's height, one might take a microscopic fraction of an inch below 6 feet to count as close enough. Only a pedant would object that they had been lied to when a person that was claimed to

17 Prescriptivists recommend *among* instead for those cases, an injunction that if followed would lead to oddities like *#All flights among New York, Washington, and Delaware were canceled* and *#There's a town that's equidistant among New York, Washington, and Delaware.*

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be six feet tall turns out to be a thousandth of an inch shorter. Perhaps that's what's happening in (41) on the relevant reading? Perhaps if the snowplow had to skip a few houses, that *does* make the sentence strictly false after all, but it might sometimes count as close enough to true?

This matters in part because it would instantly suggest a certain analytical course. Part of the Lasersohn vision is that halos can be more or less narrow, and certain expressions he calls slack regulators can constrict the size of the halo. With respect to time, *exactly* is one such expression: 3:01 might count as *3 o'clock* in some contexts, but in most it wouldn't count as *exactly 3 o'clock*. Likewise, *through* might signal that one should use a narrower halo around a *from/to* range, tolerating fewer exceptional gaps in the range.

One reason to suspect that this isn't the right course is that known slack regulators like *exactly* don't have this effect. *#?Exactly from 10 Main Street to Elm Street* definitely doesn't mean this, and it's not altogether clear that it means anything at all. If it's well-formed, it's only barely.

In addition, if *through* were a slack regulator then we might expect that it would affect the truth conditions of a *from/to* range at the end points, similarly to how *exactly* narrows the halo around *3 o'clock*. But this doesn't seem to be true: if the snowplow starts at 12 Main Street, or if it stops one house before Elm Street, both (40) and (41) are equally false—*through* doesn't change the truth conditions in this scenario at all.

Another indication that *through* isn't a slack regulator is its position. It can occur just left of *to*, or in place of it, as (40) shows. But it can't occur on the extreme left, where we'd normally expect a slack regulator to occur in English:

(42) *The snowplow cleared through (from) 10 Main Street (to) Elm Street.

It is possible on the left only in one-sided ranges with an endpoint, where it can still occur to the left of *to*:

(43) The snowplow cleared $\left\{\begin{array}{c}\text{through (to) Elm Street}\\ \text{*through from Elm Street}\end{array}\right\}$.

In the absence of *to*, the *through* is required for grammaticality, which may also be an indirect sign that it isn't fundamentally a slack-regulating modifier.

There is an alternative course. What *through* seems to do is indicate a relation between a path and another path, provided by context, that contains it. It requires that the path include a continuous stretch of the superpath, from one point to another. In the plow case, it makes sense to think of a master list of addresses along a street that are to be plowed. A path from 10 Main Street to 30 Main Street could be said to

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be *through* these addresses if it doesn't skip over any addresses on the master list. And conversely, if it does skip over any, it's a *from/to* path but not a *through* path.

Of course, the master list can take various forms, and accordingly, what counts as a *through* path can change. If the master list is not of addresses but, say, of yard-long stretches of the street, a *through* path would need include every yard of the street from its start point to its end point.

Putting all this together, the crucial part of the snowplow example in (40) would have the denotation in (44), where p' is the contextually provided implicit superpath argument of *through*:[18]

(44) $[\![\textit{from 10 Main Street through}_{p'}\textit{ to Elm Street}]\!]$

$$= \lambda p \left[\begin{array}{l} \textbf{start}(p) = \textbf{10-Main-Street} \wedge \textbf{end}(p) = \textbf{Elm-Street} \wedge \\ p \subseteq p' \wedge \forall x[\textbf{start}(p) \geq_{p'} x \geq_{p'} \textbf{end}(p) \rightarrow x \in p] \end{array} \right]$$

The first point to highlight is the start and end, which are provided by *from/to*. *Through* provides everything else. It includes a conjunct that requires that the path be a part of the contextually-provided superpath p'. That bit might well be a presupposition, though we set that aside here for simplicity. The crucial conjunct is the next one. It requires the *through* path be a continuous stretch of the superpath—one that leaves nothing out, one that has no gaps. Everything in the superpath from the start of the *through* path to its end must be included in the subpath.

Compositionally, this makes it possible to provide a denotation for *through*:

(45) $$[\![\textit{through}_{p'}]\!] = \lambda x \lambda p \left[\begin{array}{l} \textbf{end}(p) = x \wedge p \subseteq p' \wedge \\ \forall x[\textbf{start}(p) \geq_{p'} x \geq_{p'} \textbf{end}(p) \rightarrow x \in p] \end{array} \right]$$

This actually incorporates the meaning of locative *to* directly into *through*. One could instead treat *through* as an intersective modifier that contributes only the information about the contextually-supplied superpath, but its distribution seems to argue against that. *Through* can't combine with just any property of paths. If it could, we might expect it to modify a *from* PP (**through from Cleveland*) or a *between* PP (**through between Cincinnati and Cleveland*). We take the *to* that it can optionally occur with to be vacuous, its meaning now completely absorbed into *through*:

18 Because there are in principle two orders involved here—the one for p and for p'—we indicate which path the order is associated with with a subscript, as in $\geq_{p'}$. Some additional assumptions about subpaths: we will write $p \subseteq p'$ to mean p is a subpath of p'; this requires that the elements of p be a subset of the elements of p' and that the ordering relation associated with p and p' agree on any element they both contain. The $p \subseteq p'$ portion of the denotation in (44) may well be a presupposition, but we avoid reflecting that explicitly for the sake of simplicity.

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(46) $[\![\textit{through}_{p'} \textit{ (to) Elm Street}]\!]$
$$= \lambda p \left[\begin{array}{l} \textbf{end}(p) = \textbf{Elm-Street} \wedge \\ p \subseteq p' \wedge \forall x[\textbf{start}(p) \geq_{p'} x \geq_{p'} \textbf{end}(p) \rightarrow x \in p] \end{array} \right]$$

This PP in turn can combine with a *from* PP in a variety of ways. In most of the locative examples we have so far considered, the full *from/to* structure is not itself a constituent but rather two adjacent PPs. In these cases, a property of paths VP meaning could combine intersectively with a *from* PP and then with (46). In principle, though, there is nothing preventing (46) from combining intersectively directly with a *from* PP, since both denote properties of paths.

3.5 Summary so far

This section has proposed a semantics for locative uses of *from/to*, *between*, and *through* framed in terms of paths. For some of these cases, there are alternatives in principle. Kracht (2002), for example, analyzes a class of prepositions that includes *between* in terms of regions rather than paths. The preceding discussion has, we hope, demonstrated that this conclusion is not unavoidable. And given our current goals, it's especially useful to avoid it, because it's more straightforward to generalize paths beyond the locative domain than it is for regions. The range expressions that these prepositions build are fundamentally about navigating paths crosscategorially, across semantic types and syntactic categories.

This discussion leaves a number of important larger issues unaddressed. These include how path descriptions can serve as arguments and how they are interpreted when adjoined to VP, where it's not clear that the VP itself must denote a path description. We set both aside for the moment because our ultimate goal is a semantics for ranges, not for locative uses in particular. The question of how they serve as arguments will reemerge in section 4.2.

The next challenge, then, is to provide a semantics for non-locative uses of range expressions, grappling with some of the special challenges they raise.

4 Paths beyond locations

4.1 Generalizing paths

Having laid the locative foundations of the semantics of ranges, it's now possible to provide a semantics for range expressions more broadly.

The first move will be to generalize points and paths. Just as a path can chart a course in a two- or three-dimensional space, it can chart a course in any domain that is (or can be construed to be) one-dimensional. That's where non-locative uses of

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range expressions are especially useful. The prototypical uses are numerical, where the path is, intuitively, on a number line. But of course paths can be across times as well: another one-dimensional domain. Likewise for degrees—although the whole domain of degrees isn't totally ordered, any particular degree participates in a scale that does have a total order. There the generalization is straightforward. Things are perhaps slightly trickier in cases where individuals themselves are ordered, as in for example a line of people arranged alphabetically. But for the most part, a linear order is a linear order, and individuals linearly ordered aren't crucially different—for the purposes of defining ranges—from points in space, times, or degrees. This line of thinking accords other work on non-locative paths, including Talmy (1996) on fictive motion and Gawron (2005) on extent predicates, which also notes that paths needn't be locative to be considered as such, so long as some form of order is followed. Gawron (2005) calls this order an 'axis' rather than a scale, as we've done, but the principle is the same.

For the sake of explicitness, a few additional words about types. We'll assume that the model includes a domain of paths, D_p. In order to distinguish them from purely locative paths, we will call these RANGE PATHS. These are not an atomic type. A range path is simply a linearly ordered set composed of members of an atomic type. In the locative case, it's simply points or locations in space (members of D_l), but it can also be degrees (D_d), including numbers, which we take to be a special case of degrees. It can also be times (D_i) or individuals (D_e). In (47) this is stated more formally ($\mathscr{P}$ is the power set symbol):[19]

(47) a set S with an ordering $\preceq$ is a member of D_p iff
$S \in \mathscr{P}(D_l) \cup \mathscr{P}(D_d) \cup \mathscr{P}(D_i) \cup \mathscr{P}(D_e)$ and $\preceq$ is a total order

A path, on this definition, is any linearly ordered set of elements that are all locations, all degrees, and so on. This is in addition to the standard definitions of atomic and functional types. As mentioned in 2.1, one-membered ranges don't seem to be possible, and one could build that into (47), but it might be better to treat this effect as pragmatic instead.

Such an approach is mostly just a generalization of locative paths, but not exactly. Kracht (2002) and Zwarts (2005) among others (following a standard definition in topology) define paths as functions from a real number from 0 to 1 inclusively to locations in space. We avoid this primarily because it is less natural for non-locative uses, where simply an ordered set suffices. It seems odd, for example, to commit oneself to representing the set of US presidents by assigning to each of them a

19 As noted previously, it's probably too strong to require a partial order at all, rather than a preorder. American presidents have served nonconsecutive terms, so temporal precedence of presidents is just a preorder, yet we do need it to count as a path. We abstract away from this issue here.

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fractional real number. One could generalize the notion of a path instead to allow any real number to be used, and treat some paths as partial functions whose domain is the integers. That may ultimately be necessary, but for our purposes, ordered sets will suffice. Indeed, Nam (1995) takes such an approach to paths in space as well.

The definition in (47) also bars mixed paths, ones that are composed of some combination of, for example, individuals and numbers. That seems sensible a priori, and we can find no linguistic evidence for such paths. The ill-formedness of e.g. *#from 7 to Abraham Lincoln* militates against it as well. One might imagine various far-fetched possibilities that superficially seem to require mixed paths—perhaps a list of index cards with concept descriptions on each of them—but examples like this might be better construed as involving the individual correlates of elements of other types.

It seems to be possible to use range expressions across a number of syntactic categories (Morzycki & Gobeski 2023). That may suggest that it's too restrictive to build paths only out of members of an atomic type. A PP example is in (48a); an AP example, in (48b); and a degree word example in (48c):

(48) a. There was plankton at all levels of the ocean, from just under the surface to far below it.

b. The volunteers ran the gamut from slightly annoying to utterly intolerable.

c. Floyd was somewhere between barely and profoundly skeptical.

These examples seem to be strong evidence in favor of generalizing the definition of paths, but we will not pursue this here.

There is another respect in which this system may be too restrictive. It's not actually altogether clear that a linear (i.e., total) order is required. It's quite natural to define ranges among American presidents, for example. But American presidents are not totally ordered, or indeed in the strict sense ordered at all, because the ordering relation among them is not antisymmetric. That's because, for instance, Grover Cleveland served nonconsecutive terms. He both preceded and followed Benjamin Harrison despite being, if historical sources are accurate, not identical to him. That may be evidence for taking a functional view of paths after all. But when range expressions are used in such contexts, it seems reasonable to suppose that the speaker actually has in mind a linear order, irrespective of what the facts may be. But if one is asked to list all the presidents through Cleveland, it's not actually obvious whether to include Harrison or not. This doesn't seem to be a matter of ambiguity; it's not that the instruction is ambiguous between excluding and including Harrison. Rather, once the complication is pointed out, one has the sense of resignation normally associated

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with a sentence that one suddenly discovers to have a false presupposition.[20] A similar point could be made about alphabetized names, where confusion around prefixes like *de* or *von* can lead to errors in which a name both precedes and follows itself. We can refer to ranges across such a list, but only by thinking in terms of items in the list rather than names.

4.2 Predicates of ranges

On this view, the analysis of range-selecting predicates like *cover*, *range over*, and *range* itself is straightforward. For range-selecting nouns like *range* and *span*, it is sufficient to assume that they require a complement that denotes a property of paths and have no other effect:

(49) $[\![range]\!] = \lambda f_{\langle p,t\rangle} \lambda p \,.\, f(p)$

Verbal cases like *cover* make a less trivial contribution, but are otherwise similar (resorting in (50b) to ordinary English for the sake of clarity):

(50) a. This volume covers from aardvarks to alligators.
b. $[\![cover]\!] = \lambda f_{\langle p,t\rangle} \lambda x \,.\, x$ includes in its coverage all elements of p

Thus (50a) will be true iff this volume includes in its coverage all elements of the ordered set that begins with aardvarks and ends with alligators.

More interestingly, as section 2.1 observed, *from/to* range expressions are themselves predicates of ranges and can't be predicated of a single range element:

(51) a. #7 is from 5 to 10.
b. #Chester A. Arthur is from Lincoln to Kennedy.

This seems to be a case of a fact already encountered for locatives in section 3.1:

(52) a. #Chicago is from Los Angeles to New York.
b. #Chicago is to New York.

What needs to be done now is to spell out the denotation more explicitly, and to add one empirical wrinkle.

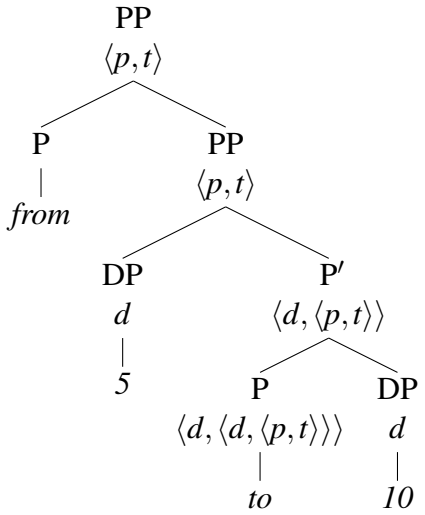
In locative uses, there are typically two distinct PPs, one headed by *from* and the other by *to*. With range expressions, there seems to be a single *from/to* structure with

20 There is even a convention for imposing a linear order on US presidents. They are conventionally numbered, and Cleveland is taken to be both 22nd and 24th.

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a fixed internal order, as section 2.1 noted.[21] Its syntax, we suggest, involves a head preposition, *from*, that takes a PP complement.[22] That PP complement requires an overt specifier for type reasons, as in (53):

(53)



The specifier is necessary because otherwise the type of the matrix PP would be $\langle d, \langle p, t \rangle \rangle$, which would lead to a type clash in most PP positions.[23] We don't have strong evidence to determine how to distribute the semantics across the two prepositions, but we will build it into this version of *to* and leave *from* uninterpreted. (It can, after all, often be omitted.)

(54) $[\![to_{range}]\!] = \lambda d \lambda d' \lambda p [\mathbf{start}(p) = d' \wedge \mathbf{end}(p) = d]$

The impossibility of subjects that fail to denote range paths—such as *7* in (51a)—follows from the fact that the PP is defined only for path subjects, or in any case subjects that have path correlates.

In locative uses, potential subjects were expressions like *the trip*, which could be construed as inherently denoting paths. Something similar is possible in non-locative cases with nouns like *the range* or *the interval*:

(55) The $\left\{ \begin{matrix} \text{range} \\ \text{interval} \end{matrix} \right\}$ was from 5 to 10.

21 See also Zwarts (2013), who observes that *from N to N* constructions are a schematic idiom, rather than being built up from a *from* preposition and a *to* preposition.

22 Prepositions with PP complements are not unusual in English: *from under the bed*, *out of the window*.

23 One could engineer this even more effectively by supposing that *from* actually denotes an identity function of type $\langle \langle p, t \rangle, \langle p, t \rangle \rangle$, thereby creating a type clash irrespective of the position of the PP.

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But these simple cases are just the starting point. There are also cases in which it's less clear that the subject directly denotes a path:

(56) a. The children's ages were from 5 to 10.
b. The estimated weights were from 5 to 10.

It's certainly possible in principle to construe *the children's ages* as a path: it could simply be a set of degrees of oldness, such as {2 years old, 4 years old, 5 years old}, and one can treat *the estimated weights* likewise. But such representations do seem to lose the underlying complexity of degree nominalizations. There are good reasons to doubt that degree nominalizations refer directly to a degree (see Moltmann 2009 and references there) even in the singular case. In (56b), for example, it seems odd to suppose that a particular degree of weight—say, 150 pounds—has the property of being estimated. In both examples in (56), an additional component of the meaning has to come from plural morphology. So one way or another, the picture of how these nominalizations come to refer to paths has to be more complicated.

Indeed, as we noted before in section 3.2, even the example we treated as a path-denoting nominal—*the trip*—is not obviously only path-referring. One can enjoy a trip or regret a trip, but one can't enjoy simply a path in space or regret a path, so something more must be said about how nominals come to denote paths. We will conceptualize these operations as the result of type shifts. In the case of expressions like *age*, much hinges on what one assumes to be the underlying semantics of the degree nominalization itself. Addressing that in detail would take us too far afield, but for the sake of explicitness, it's worth sketching how things might work. The starting point is to suppose, for the sake of simplicity alone, that *age* is a predicate of degrees. The plural *ages* might be a predicate of a plurality of degrees, a notion for which there is independent evidence from other domains (Dotlačil & Nouwen 2016). The full nominal *the children's ages* would thus refer to the maximal plurality of degrees that consists of (contextually relevant) children's ages. The mapping to a path, then, is simple: it's that plurality of degrees represented as a set, ordered by the inherent ordering provided by the scale of which these degrees are members.

In the case of eventive nouns like *trip*, the crucial element of the shift to path reference is provided by an appropriate trace function, in the sense of Link (1998): either the temporal trace function τ that maps an event to its running time, or its locative counterpart.

That's a lot of type shifting, and as it turns out there is more in store. In both of these cases, though, the shift is a simple and natural one. And in both cases, it is one whose precise character involves far more than the grammar of paths: in one case, the thorny puzzle of what degree nominalizations denote, and in the other the grammar of events. These shifts simply allow us to evade these distinct issues. There

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is, of course, an alternative, as there often is with type shifts: instead of the shift, one might use a functional head. We don't have evidence against such a strategy.

There is some reason, though, to avoid another conceivable analytical path: building the type shifts directly into the denotations of the range expressions. One might, for example, take *from Cincinnati to Cleveland* to denote not a property of paths that start at Cincinnati and end at Cleveland, but rather a property of *events* that do so. The trouble is that this strategy would require building into the range expression at least two different type shifts: the eventive one, and the degree one considered above. The result would be that every range expression would be polysemous between being a predicate of events and of degrees. Indeed, times are also a possibility. Moreover, the different flavors of range expression (*from/to*, *between/and*, *through*) would all need to be polysemous in exactly the same way. That's not inconceivable, of course, but it is not the most parsimonious course.

4.3 Cardinality and singleton punctual readings

As we have seen, many uses of range expressions are about cardinality:

(57) Floyd saw five to ten capybaras.

It's now possible to make sense of this phenomenon in terms of the analysis so far.

First, some assumptions about cardinality ascriptions. Following a long tradition, we take cardinal numerals as measure phrase arguments of an implicit adjective (Bresnan 1972, Hackl 2000; see Morzycki 2016 for further references and discussion). This is standardly taken to be a form of *many*, so that *five capybaras* is underlyingly *five* MANY *capybaras*. The implicit adjective denotes a relation between a degree provided by the numeral and a plural individual ($|\cdot|$ maps an individual to its cardinality):

(58) a. $[\![\text{MANY}]\!] = \lambda d \lambda x[|x| = d]$
b. $[\![\textit{five}]\!] = \mathbf{5}$
c. $[\![\textit{five}\ \text{MANY}\ \textit{capybaras}]\!] = \lambda x[|x| = \mathbf{5} \wedge \mathbf{capybaras}(x)]$

Thus, as in (58b), *five* MANY denotes a property of individuals that can be interpreted intersectively with *capybaras*. This can be existentially closed by an implicit indefinite determiner.

To keep the compositional moving parts to a minimum, we will combine MANY with such an implicit indefinite determiner meaning as in (59), yielding a single existentially-quantifying determiner that we'll call $\text{MANY}_{\exists}$:

(59) $[\![\text{MANY}_{\exists}]\!] = \lambda d \lambda P_{\langle e,t\rangle} \lambda Q_{\langle e,t\rangle}\ .\ \exists x[|x| = d \wedge P(x) \wedge Q(x)]$

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This step is not crucial to the analysis.

For the sake of explicitness, (60) illustrates how the pieces come together in a simple cardinal numeral case:[24]

(60) [[Five MANY$_\exists$] capybaras] saw Floyd.

a. $[\![\textit{[five MANY}_\exists\textit{] capybaras}]\!] = [\![\text{MANY}_\exists]\!]([\![\textit{five}]\!])([\![\textit{capybaras}]\!])$
$= \lambda Q_{\langle e,t\rangle} \,.\, \exists x[|x| = \mathbf{5} \wedge \mathbf{capybaras}(x) \wedge Q(x)]$

b. $[\![\textit{[[five MANY}_\exists\textit{] capybaras]}]\!]([\![\textit{saw Floyd}]\!])$
$= \exists x[|x| = \mathbf{5} \wedge \mathbf{capybaras}(x) \wedge \mathbf{saw}(x, \mathbf{Floyd})]$

With this in place, it's now possible to ask what happens with a range expression in place of the numeral:

(61) a. Five to ten capybaras saw Floyd.

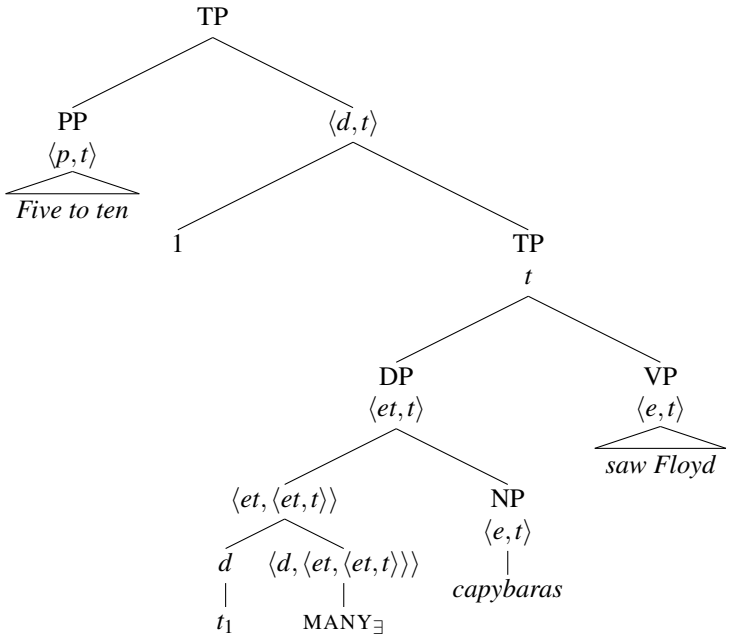
b. **LF:** [[[Five to ten] MANY$_\exists$] capybaras] saw Floyd

The short answer is a type clash. Our standing assumptions treat *(from) five to ten* as a property of paths. That clashes with MANY$_\exists$, which expects a degree. A standard response to a type clash is QR, and we propose to use it here too. In order to dodge the type clash, the displaced range expression needs to leave behind a trace that denotes a degree. That gives rise to the logical form in (62):[25]

24 We have changed our pet example to embed the numeral inside the subject, thereby making it possible to avoid spelling out the complicating effect of quantifier raising (QR). That's useful because, in a moment, there will be a need for an independent case of quantifier raising that would otherwise complicate the picture needlessly.

25 We represent the lambda abstraction triggered by movement in the Heim & Kratzer 1998 style as a numeric index directly in the tree.

(62)



The movement of the range expression triggers lambda abstraction in the usual way (Heim & Kratzer 1998), and in this case, because the trace denotes a degree, the lambda must bind a degree. This leaves us in an interesting position. The lambda expression now denotes a property of degrees:

(63) $[\![1\ [[t_1\ \text{MANY}_\exists]\ capybaras]\ saw\ Floyd]\!]$
$= \lambda d\ .\ \exists x[|x| = d \wedge \textbf{capybaras}(x) \wedge \textbf{saw}(x, \textbf{Floyd})]$

More precisely, it's the property of being a particular cardinality of capybaras that saw Floyd.

This remedies the type clash at the bottom of the tree, but there remain two elements at the top of the tree that still can't combine straightforwardly. One is the range expression, a property of paths. The other is a property of degrees. To reconcile these pieces, it will take a few additional steps, at least one of which is similar to the type shift we already entertained in section 4.2.

The truth conditions of the sentence require that the range include the maximal number of capybaras that saw Floyd. In general terms, the maximal degree in the degree property must be a member of the path. The natural way to implement this is with a type shift:[26]

26 We assume here a standard definition of maximality in which the maximality operator yields the greatest degree in a set:

(i) $\textbf{max}(D) \stackrel{\text{def}}{=} \iota d[\forall d'[d \geq d']]$

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(64) $[\![\text{PATH-SHIFT}]\!] = \lambda D_{\langle d,t\rangle} \lambda p \,.\, \mathbf{max}(D) \in p$

In this case, it combines with the degree property as in (65):

(65) $[\![\text{PATH-SHIFT}]\!]([\![\textit{1 [[}t_1 \text{ MANY}_\exists\textit{] capybaras] saw Floyd}]\!])$

$$= \lambda p \,.\, \mathbf{max}\left(\lambda d \,.\, \exists x \left[\begin{array}{l} |x| = d \wedge \mathbf{capybaras}(x) \wedge \\ \mathbf{saw}(x, \mathbf{Floyd}) \end{array} \right] \right) \in p$$

This yields a property of paths that contain the crucial maximal degree. That in turn means that this can combine intersectively with the property of paths expressed by the range expression:

(66) a. $[\![\textit{five to ten}]\!] = \lambda p \,.\, \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10}$

b. $[\![\textit{[five to ten]} \text{ PATH-SHIFT } \textit{1 [[}t_1 \text{ MANY}_\exists\textit{] capybaras] saw Floyd}]\!]$

$$= \lambda p \left[\begin{array}{l} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \mathbf{max}\left(\lambda d \,.\, \exists x \left[\begin{array}{l} |x| = d \wedge \mathbf{capybaras}(x) \wedge \\ \mathbf{saw}(x, \mathbf{Floyd}) \end{array} \right] \right) \in p \end{array} \right]$$

At this stage, a truth value is required because we have reached the top of the tree, but what we have in hand is a property of paths. The natural move, and one widely independently motivated from other domains, is existential closure. The sentence is true iff there exists a path that satisfies (66b). The closure operation is spelled out in its path-based form in (67), though one could also view this as a special case of a more general cross-categorial existential closure operation:

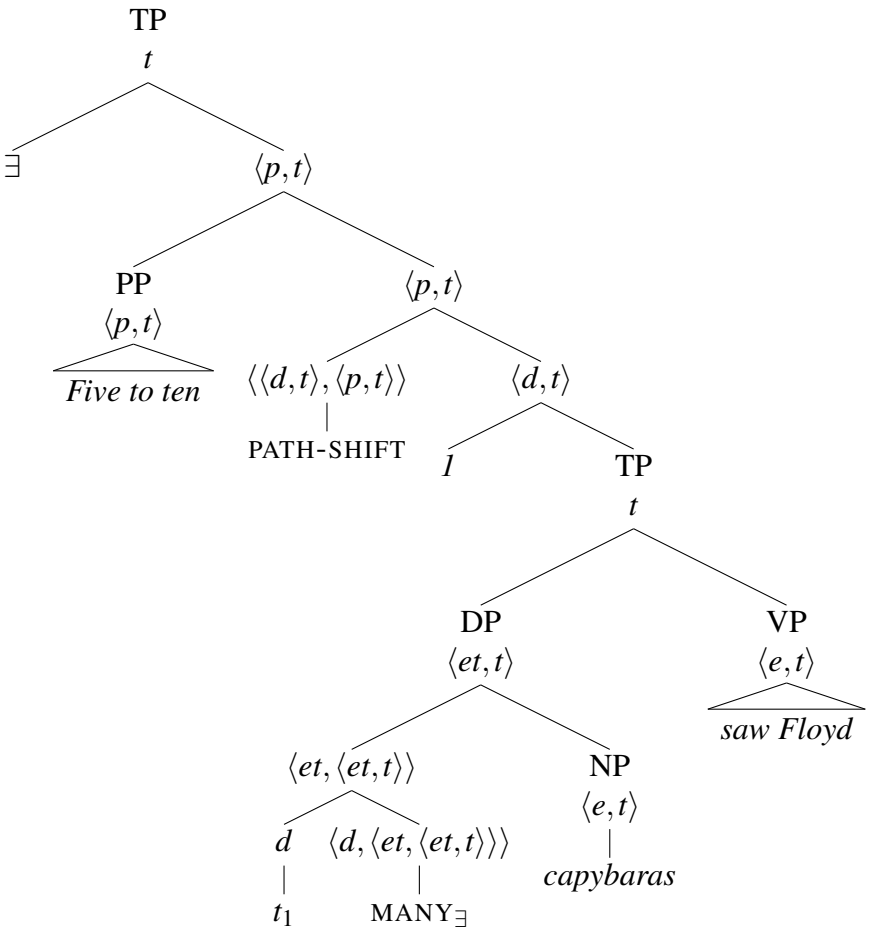
(67) $[\![\exists]\!] = \lambda P_{\langle p,t\rangle} \,.\, \exists p[P(p)]$

The tree in (68) is updated to include the two type shifts, with the denotation in (69):

We briefly consider an alternative assumption about maximality in section 5.3, and an alternative way to conceptualize this type shift that doesn't involve maximality in section 4.9.

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(68)



(69) $[\![\exists$ *[five to ten]* PATH-SHIFT *1 [[*t_1 MANY$_\exists$*] capybaras] saw Floyd* $]\!]$

$$= \exists p \left[\begin{array}{l} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \mathbf{max}\Big(\lambda d \,.\, \exists x \Big[\begin{array}{l} |x| = d \wedge \mathbf{capybaras}(x) \wedge \\ \mathbf{saw}(x, \mathbf{Floyd}) \end{array} \Big] \Big) \in p \end{array} \right]$$

This yields the correct truth conditions: the sentence is true if there's a path that starts at 5 and ends at 10 and contains the maximal number of capybaras that saw Floyd. This, of course, could be the case only if the number of capybaras was no smaller than 5 and no greater than 10.

This also reflects why maximality is crucial. It isn't sufficient for there to simply be some number of capybaras in the range that saw Floyd. It must actually be the case that all the capybaras that saw him number no more than 10.

We implemented this analysis with a simple type shift and a separate existential closure operation. That isn't necessary. We could have combined the two into a single more complicated type shift:

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(70) $[\![\text{PATH-SHIFT}]\!] = \lambda D_{\langle d,t\rangle} \lambda P_{\langle p,t\rangle} \,.\, \exists p[P(p) \wedge \mathbf{max}(D) \in p]$
(alternative not adopted)

Why separate the two? Because existential closure is an operation that's widely independently motivated (Heim 1982, Diesing 1990, Kratzer 1995 among countless others), and the original path shift that remains truly seems like a type shift. Its contribution is minimal. Indeed, it's so minimal that it makes sense to think of it as simply expressing an aspect of the grammatical logic of paths.

4.4 Quantification, scope, and set punctual readings

An additional and rather hairy compositional challenge awaits. The analysis in the previous section articulated how cardinality and range expressions interact in simple sentences. Along the way, it was necessary to suppose that range expressions undergo quantifier raising. But what does that predict about other quantifiers?

In particular, there needs to be an explanation of how, in quantified contexts, what we earlier called set punctual readings arise. As a brief reminder, the crucial observation is that in the scope of a quantifier as in (71), readings of range expressions arise in which there are multiple crucial values in the range:

(71) Every tourist saw five to ten capybaras.

In this case, each tourist may have seen a different number of capybaras, so long as it's the case for every tourist that their count of capybaras falls in the range.

This will turn out to be a matter of quantifier scope. The computation has a number of moving parts (some of which literally move syntactically). First, in (71), the range expression occurs in object position, so the indefinite *five to ten capybaras* will need to scope out on its own. Second, there is the crucial matter of how *every tourist* scopes. Third, there is the independent movement of the range expression.

Leaving the subject in situ for the moment, (71) would have the LF in (72):

(72) $\exists$ [five to ten]
[PATH-SHIFT [1 [t_1 MANY$_\exists$ capybaras] [2 [every tourist saw t_2]]]]

This scopes out the indefinite, and then scopes the range expression outside of it. The semantics is as in (73):

(73) a. $[\![2 \textit{ every tourist saw } t_2]\!] = \lambda x \,.\, \forall y[\mathbf{tourist}(y) \rightarrow \mathbf{saw}(y,x)]$
b. $[\![t_1 \text{ MANY}_\exists \textit{ capybaras}]\!] = \lambda Q_{\langle e,t\rangle} \,.\, \exists x[|x| = [\![t_1]\!] \wedge \mathbf{capybaras}(x) \wedge Q(x)]$

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c. $[\![t_1 \text{ MANY}_{\exists} \textit{ capybaras}]\!] ([\![\textit{2 every tourist saw } t_2]\!])$
$= \exists x[\, |x| = [\![t_1]\!] \wedge \mathbf{capybaras}(x) \wedge \forall y[\mathbf{tourist}(y) \rightarrow \mathbf{saw}(y,x)]\,]$

d. $[\![\textit{1 [}t_1 \text{ MANY}_{\exists} \textit{ capybaras] [2 [every tourist saw } t_2\textit{]]}]\!]$
$= \lambda d\,.\, \exists x[\, |x| = d \wedge \mathbf{capybaras}(x) \wedge \forall y[\mathbf{tourist}(y) \rightarrow \mathbf{saw}(y,x)]\,]$

e. $[\![\text{PATH-SHIFT}]\!] ([\![\textit{1 [}t_1 \text{ MANY}_{\exists} \textit{ capybaras] [2 [every tourist saw } t_2\textit{]]}]\!])$
$= \lambda p\,.\, \mathbf{max}\left(\lambda d\,.\, \exists x \begin{bmatrix} |x| = d \wedge \mathbf{capybaras}(x) \wedge \\ \forall y[\mathbf{tourist}(y) \rightarrow \mathbf{saw}(y,x)] \end{bmatrix} \right) \in p$

f. $[\![\textit{five to ten}]\!] = \lambda p[\mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10}]$

g. $\left[\!\!\left[\begin{array}{l} \exists \textit{ [five to ten] [}\text{PATH-SHIFT } \textit{[1 [}t_1 \text{ MANY}_{\exists} \textit{ capybaras]} \\ \textit{[2 [every tourist saw } t_2\textit{]]]]} \end{array} \right]\!\!\right]$
$= \exists p \begin{bmatrix} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \mathbf{max}\left(\lambda d\,.\, \exists x \begin{bmatrix} |x| = d \wedge \mathbf{capybaras}(x) \wedge \\ \forall y[\mathbf{tourist}(y) \rightarrow \mathbf{saw}(y,x)] \end{bmatrix} \right) \in p \end{bmatrix}$

This yields a reading under which there's a particular cardinality of capybaras that all the tourists saw, and it's contained in a path from 5 to 10. Each individual tourist may have seen a greater number of capybaras than 10, but the number of capybaras seen by all the tourists falls within the path. That seems to be a possible reading, but it's not the most interesting or natural one—nor is it a set punctual reading.

The set punctual reading requires scoping out the universal quantifier too, to give it wide scope over the existential quantifier contributed by $\text{MANY}_{\exists}$, as in the LF in (74):

(74) [every tourist] [3 [$\exists$ [five to ten]
[PATH-SHIFT [1 [t_1 $\text{MANY}_{\exists}$ capybaras] [2 [t_3 saw t_2]]]]]]

The computation is in (75), abbreviated for the sake of sanity:

(75) a. $\left[\!\!\left[\begin{array}{l} \textit{3 } \exists \textit{ [five to ten] [}\text{PATH-SHIFT } \textit{[1 [}t_1 \text{ MANY}_{\exists} \textit{ capybaras]} \\ \textit{[2 [}t_3 \textit{ saw } t_2\textit{]]]]} \end{array} \right]\!\!\right]$
$= \lambda y\,.\, \exists p \begin{bmatrix} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \mathbf{max}\left(\lambda d\,.\, \exists x \begin{bmatrix} |x| = d \wedge \\ \mathbf{capybaras}(x) \wedge \\ \mathbf{saw}(y,x) \end{bmatrix} \right) \in p \end{bmatrix}$

b. $[\![\textit{every tourist}]\!] = \lambda P_{\langle e,t \rangle} \lambda Q_{\langle e,t \rangle}\,.\, \forall y[P(y) \rightarrow Q(y)]$

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c. $\left[\!\!\left[\begin{array}{l} \textit{every tourist [3 } \exists \textit{ [five to ten] [PATH-SHIFT [1 [}t_1 \textit{ MANY}_\exists \textit{ capybaras]} \\ \textit{[2 [}t_3 \textit{ saw } t_2\textit{]]]]]} \end{array} \right]\!\!\right]$

$$= \forall y \left[\mathbf{tourist}(y) \rightarrow \exists p \left[\begin{array}{l} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \mathbf{max}\left(\lambda d \,.\, \exists x \left[\begin{array}{l} |x| = d \wedge \\ \mathbf{capybaras}(x) \wedge \\ \mathbf{saw}(y,x) \end{array} \right] \right) \in p \end{array} \right] \right]$$

The result is a reading in which for every tourist, there is a potentially different exact number of capybaras seen, and each such number is contained in the range from 5 to 10.[27]

It's worth acknowledging that a different way of rendering the crucial reading is conceivable that would represent it more intuitively:[28]

(76) $$\exists p \left[\begin{array}{l} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \forall y \left[\mathbf{tourist}(y) \rightarrow \mathbf{max}\left(\lambda d \,.\, \exists x \left[\begin{array}{l} |x| = d \wedge \mathbf{capybaras}(x) \wedge \\ \mathbf{saw}(y,x) \end{array} \right] \right) \in p \right] \end{array} \right]$$

This would also capture the crucial facts: that there is a single path from 5 to 10, and that the maximal number of capybaras every tourist saw is a member of that path. Apart from looking more straightforward, it's not clear to us that this representation has any advantages over (75). But it does have a clear disadvantage: it seems to be a deep prediction of our compositional assumptions that the LF necessary for (76) couldn't be generated. That's because this reading would require the *every tourist* to scope immediately over the PATH-SHIFT operator—but that doesn't work type-theoretically, because PATH-SHIFT yields a property of paths, which is not compatible with the usual generalized quantifier semantics for quantified nominals. Even if that configuration could be achieved, the structure would then have to return to being a property of paths immediately above *every tourist* so that the range expression could be interpreted. Of course, all this may be an indication that our compositional assumptions should be substantially refined. But as it stands, there doesn't seem to be compelling evidence that this is necessary.

This subsection provides a theory of set punctual readings in general. These arise as simply the consequence of wide-scope universal quantifiers. That seems a simple explanation, and it's consistent with much of the data. It does have the interesting side-effect, though, that something must be said about how these readings arise with plural subjects in the absence of an overt quantifier. A natural answer is available,

27 This may seem to allow a different path for each tourist, but each of these paths would have to start at 5 and end at 10—and therefore be identical in all relevant ways after all.

28 Thanks to an S&P reviewer for pushing us on this point.

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though: an implicit quantifier, such as a distributivity operator (e.g. Beck 2013). Pursuing this, however, is beyond the scope of this paper.

4.5 Measure phrases and singleton punctual readings with overt adjectives

We now have an analysis of both singleton and set punctual readings of cardinality expressions. But what about other forms of measure phrase? A basic example of a range expression serving as a measure phrase to an overt adjective is in (77):[29]

(77) Floyd is from five feet to ten feet tall.

The most common analysis of gradable adjectives treats them as relations between individuals and degrees, as in (78a), yielding sentence denotations like (78b):

(78) a. $[\![\textit{tall}]\!] = \lambda d \lambda x \,.\, \textbf{tall}(x,d)$

b. $[\![\textit{Floyd is six feet tall}]\!] = \textbf{tall}(\textbf{Floyd})(\textbf{6ft})$

The measure phrase position can be occupied by a range expression. Because the adjective expects to find a degree in the measure phrase position, range expressions can't be directly interpreted there because they denote properties of paths. Here, the story largely recapitulates what we have already encountered for cardinality cases. The range expression must QR to a position where it can be interpreted with the aid of PATH-SHIFT as in the logical form in (79):

(79) **LF:** $\exists$ [from five feet to ten feet] PATH-SHIFT 1 Floyd is t_1 tall

This means that the clause from which the range expression emerged will denote a property of degrees, as in (80a), and the matrix clause will have the denotation in (80b):

(80) a. $[\![\textit{1 Floyd is } t_1 \textit{ tall}]\!] = \lambda d \,.\, \textbf{tall}(\textbf{Floyd}, d)$

b. $[\![\exists \textit{ [from five feet to ten feet]} \text{ PATH-SHIFT } \textit{1 Floyd is } t_1 \textit{ tall}]\!]$

$= \exists p [\, \textbf{start}(p) = \textbf{5ft} \wedge \textbf{end}(p) = \textbf{10ft} \wedge \textbf{max}(\, \lambda d \,.\, \textbf{tall}(\textbf{Floyd}, d)\,) \in p \,]$

The resulting denotation requires that the maximal degree to which Floyd is tall is an element of a path from five feet to ten feet.

29 It is, of course, more natural to avoid repetition of *feet*:

(i) Floyd is from five to ten feet tall.

Providing a semantics for this, though, requires making assumptions about the internal grammar of measure phrases without shedding much additional light on range expressions, the topic at hand.

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4.6 Interval ranges and exceptives

We have now provided an analysis of singleton punctual readings and set punctual readings, with the difference being that set punctual readings arise automatically in the presence of quantifiers with wide scope. Next on the agenda are interval readings.

As a reminder, interval readings involve cases in which all of a range seems to be crucial. They are normally expressed in English with *through*, and unlike punctual readings they are compatible with exceptives:

(81) This volume covers U.S. presidents from Kennedy through to Carter (except Nixon).

We have in hand a denotation for *through* in locative cases from section 3.4. What needs to be done is to extend it to ranges more generally.

The idea for locative *through* is that it makes crucial reference to a contextually-provided superpath, represented as p' in the denotation, repeated here from section 3.4:

(82) $$[\![through_{p'}]\!] = \lambda x \lambda p \left[\begin{array}{l} \textbf{end}(p) = x \wedge p \subseteq p' \wedge \\ \forall x [\textbf{start}(p) \geq_{p'} x \geq_{p'} \textbf{end}(p) \rightarrow x \in p] \end{array} \right]$$

This denotation requires that every element of the superpath that's between the endpoints of its subpath be included in the subpath. No eligible elements of the superpath can be skipped. That's what makes it possible for a plow to clear snow from 10 Main Street through 20 Main Street as long as it skips over no addresses in the list of addresses along Main Street. The list, in this example, provides the superpath. Importantly, if the list is the superpath, it would allow the plow to skip any addresses that are not in the list itself, perhaps because there is no house with a particular number.

To extend this beyond locative uses, we need to loosen an assumption. The variable x in (82) must be able to range over any types of which a path may be composed, whether individuals, degrees, or something else. For the most part, that suffices to achieve what we need. In the American president case, for example, this ensures that presidents *Kennedy through to Carter* be a range that's part of the larger ordering of American presidents, and that it skip over none.

What remains to be explained is how the exceptive diagnostic works. Why is it that exceptives are possible with interval readings but not with punctual readings?

The first part of an answer comes from a fact about the distribution of exceptives. As von Fintel (1993) shows, exceptives are systematically possible in the scope of a universal quantifier. Because the exceptives at issue target elements of ranges, this needs to be understood to mean universal quantification over range elements.

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Von Fintel makes his exceptive denotations remove elements from contextually-provided quantificational domain restrictions. To make this work for ranges, it will be necessary to provide *through* with a domain restriction C:

(83) $$[\![through_{p',C}]\!] = \lambda x \lambda p \left[\begin{array}{l} \textbf{end}(p) = x \wedge p \subseteq p' \wedge \\ \forall x \in C[\textbf{start}(p) \geq_{p'} x \geq_{p'} \textbf{end}(p) \rightarrow x \in p] \end{array} \right]$$

This means that *through* is now doubly context-sensitive. In addition to the contextually-provided superpath p', it is sensitive to a set of salient elements C of the superpath that count toward evaluating whether the subpath includes them.[30] This introduces a kind of exception tolerance. To return to the snowplow example, if the snowplow plowed addresses 20 through 30 Main Street, it must normally be the case that every address from 20 to 30 has been included. But suppose that 25 Main Street actually includes a small apartment, numbered $25\frac{1}{2}$, to which no stretch of the street belongs. It would be natural to exclude that in assessing which parts of the street were plowed. Likewise, if one address is in the process of renovation and has temporary dumpsters set up in the street, one might imagine that even a very conscientious plow operator may skip over those, and that these too would be disregarded in evaluating whether all addresses in the range have been plowed.

This exception-tolerance is crucial to expressing the contribution of exceptives. On the von Fintel view, exceptives exclude elements from the contextual domain restriction. That's what the denotation in (84) does:

(84) a. $[\![except_{p',C}]\!] = \lambda x \lambda p : x \in p' \,.\, x \notin C$

b. $[\![except_{p',C}\ Nixon]\!] = \lambda p : \textbf{Nixon} \in p' \,.\, \textbf{Nixon} \notin C$

More precisely, this supposes that *except* is sensitive to a shared domain restriction with *through*, and that it is also sensitive to the superpath. The only reason to include sensitivity to the superpath is that it makes it possible to express the presupposition that *except* expects its complement to be a member of the superpath. That provides an explanation for why e.g. *??US presidents from Kennedy through to Carter except Donald Duck* is odd, because Donald Duck is not (as of this writing) a member of the ordered set of US presidents. What is crucial to the immediate point, though, is just that the complement of *except* is excluded from the domain restriction. In the case at hand, it means that Nixon is excluded from C, and that Nixon is disregarded in determining whether the path p is a subpath of the path of US presidents p'.

The exceptive PP can combine intersectively with a range expression it modifies, as in (85):

30 Strictly, nothing in (83) requires that all elements of C be elements of the superpath, but any that aren't are simply disregarded.

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(85) a. $[\![\textit{Kennedy through}_{p',C}\ \textit{to Carter}]\!]$
$$= \lambda p \left[\begin{array}{l} \textbf{start}(p) = \textbf{Kennedy} \wedge \textbf{end}(p) = \textbf{Carter} \wedge p \subseteq p' \wedge \\ \forall x \in C[\textbf{start}(p) \geq_{p'} x \geq_{p'} \textbf{end}(p) \rightarrow x \in p] \end{array} \right]$$

b. $[\![\textit{Kennedy through}_{p',C}\ \textit{to Carter except}_{p',C}\ \textit{Nixon}]\!]$
$$= \lambda p \left[\begin{array}{l} \textbf{start}(p) = \textbf{Kennedy} \wedge \textbf{end}(p) = \textbf{Carter} \wedge p \subseteq p' \wedge \\ \forall x \in C[\textbf{start}(p) \geq_{p'} x \geq_{p'} \textbf{end}(p) \rightarrow x \in p] \wedge \textbf{Nixon} \notin C \end{array} \right]$$

The result is a property of paths starting at Kennedy, ending at Carter, all part of a larger superpath, here most naturally understood to consist of American presidents skipping Nixon, because Nixon is excluded from C.[31]

4.7 Inclusive and exclusive ranges

It is sometimes suggested that one difference between *from/to* and *between/and* is that *between/and* excludes its endpoints but *from/to* includes them. That is, *between 10 and 20* involves a range that includes values above 10 and under 20, and *from 10 to 20* involves a range that includes 10 and 20 themselves (Nouwen 2010, Schwarz et al. 2012, Rett 2014a, Buccola & Spector 2016, Gobeski & Morzycki 2022b). That hasn't played a crucial role for us here, but some words on the topic are in order.

The first step will be to include some additional evidence. Whatever the default orientation of range expressions, it's possible to change it with adverbials:

(86) Floyd listed all the integers $\left\{ \begin{array}{l} \text{from 10 to 20 exclusive(ly)} \\ \text{between 10 and 20 inclusive(ly)} \end{array} \right\}$.

It's also worth pointing out that this really does seem to be a matter of defaults. Psycholinguistic experiments often invite sentences such as (87):

(87) Participants answered on a Likert scale between 1 and 5.

The intended reading is normally that 1 and 5 are possible ratings. Likewise, if one describes an encyclopedia volume as in (88), one would be surprised to find that 1700 is contained in the previous volume:

(88) This volume covers between 1700 and 1800.

An inclusive reading is also natural in (89):

31 The truth conditions predicted here may be too weak in that Nixon isn't actually required to be omitted from p. This concern is a special case of a more general issue in the analysis of exceptives, and not specifically a fact about ranges, so we will not investigate this further here (Hoeksema 1990, von Fintel 1993).

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(89) We invited between 9 and 10 people.

If (89) were strictly exclusive, it would require inviting fractional guests. And of course, if I ask you to keep some confidential information *just between you and me*, I don't intend that neither of us should know about it.

That all suggests that rather than hard-wiring the distinction into *between/and* and *from/to*, it should be left flexible and underspecified. It seems likely that *between* normally defaults to exclusive readings, but that may well be a statistical tendency having to do with frequency of use rather than lexical semantics.

The question then becomes how to represent the underspecification, and how to allow modifiers to manipulate it. The crucial step has been with us all along. The foundation of the paths approach to range expression has been the **start** and **end** predicates. We have never actually been explicit about whether a path that starts at an element must necessarily include that element, and likewise for where it ends. The facts here suggest that this is how it should remain. These predicates don't presuppose that paths must include their start points, so paths that start just after their start point are perfectly possible.

This also suggests a direction for adverbial endpoint modifiers. When a speaker asks that a range expression be construed inclusively, they normally intend just that its start- and endpoints be included in the range. That can be expressed naturally in the language of paths:

(90) a. $[\![\textit{inclusive(ly)}]\!] = \lambda p \,.\, \mathbf{start}(p) \in p \wedge \mathbf{end}(p) \in p$

b. $[\![\textit{exclusive(ly)}]\!] = \lambda p \,.\, \mathbf{start}(p) \notin p \wedge \mathbf{end}(p) \notin p$

These elements are simply properties of paths, inclusive or exclusive. They can be interpreted intersectively:

(91) $[\![\textit{five to ten exclusively}]\!] = \lambda p \begin{bmatrix} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \mathbf{start}(p) \notin p \wedge \mathbf{end}(p) \notin p \end{bmatrix}$

The path-modifying variant of *between* would work similarly.

4.8 Ignorance inferences

A brief word about ignorance inferences. These are inferences that a speaker doesn't know a particular numerical value exactly, and is using a range expression or modified numeral to signal the fact:

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(92) Floyd saw { at least 3 / at most 3 / maximally 3 / up to 3 / 3 to 6 } capybaras.

This topic has occupied the modified numerals literature extensively (Büring 2007, Coppock & Brochhagen 2013, Mayr 2013, Westera & Brasoveanu 2014, Rett 2014b, Kennedy 2015, Schwarz 2016a,b, Ciardelli et al. 2018, Cremers et al. 2021), and its interactions with range expressions were a major focus of an earlier and very different version of this work (Gobeski & Morzycki 2022b). We will for the most part set the issue aside here, because the other empirical properties of range expressions this paper has focused on so far are probably more revealing about how range expressions themselves work. Nevertheless, it's worth highlighting one potentially interesting point about how ignorance inferences interact with the inclusive/exclusive distinction raised in the previous subsection.

One of the essential puzzles about ignorance inferences is why some expressions give rise to them and others don't seem to. The modified numerals in (92) do, but the ones in (93) don't:

(93) Floyd saw { fewer than 3 / more than 3 / less than 3 } capybaras.

It would be odd to go on to say (92) if one knows exactly how many capybaras Floyd saw, but it wouldn't be to say (93).

A leading idea about the source of the distinction (Kennedy 2015) is that it has to do with whether the ordering relation involved is strict ($<$ or $>$) or not ($\leq$ or $\geq$). Strict orders don't systematically give rise to ignorance inferences, the observation goes, but non-strict ones do. This is said to follow ultimately from Gricean inferences and a particular conception of what alternatives these inferences compare.

It is sometimes said that range expressions fit into this picture. *From/to* ranges are said to include their endpoints, which predicts—given this theory—that they shouldn't give rise to ignorance inferences. *Between/and* ranges are said to exclude them, which predicts that they should. Given the observations in section 4.7 immediately above, though, the picture changes. If range expressions are in general underspecified with respect to this distinction, it's less clear where ignorance readings should arise. More important, though, is that endpoint modifiers like *inclusive* and *exclusive* explicitly articulate precisely what's at issue here—strict vs non-strict—and therefore should be a straightforward means of turning ignorance

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inferences on or off. It's not obvious that this predicted behavior is found (example sentence adapted from Nouwen 2010):

(94) I know exactly how many capybaras Floyd saw. It was $\left\{\begin{array}{l}\text{from 3 to 6}\\ \text{between 3 and 6}\end{array}\right\}$ $\left\{\begin{array}{l}\text{inclusive(ly)}\\ \text{exclusive(ly)}\end{array}\right\}$.

The judgment is not perfectly clear, but without the endpoint modifiers, *between* is probably more natural, as predicted. But it's not at all clear that ignorance inferences are affected by the endpoint modifiers here. This suggests the strict/non-strict distinction can't be the whole story.

4.9 The source of maximality: an alternative

The approach we have taken so far is to rely on a type shift that encodes maximality directly. This is an expression of the hypothesis that range expressions implicitly bring with them the compositional requirement for this kind of maximality. That's certainly coherent, but one might wonder whether assuming range expressions invoke maximality in this way is actually necessary or merely analytically convenient.

In earlier incarnations of this work, we took a different route: taking the relevant maximality to arise from distinct and independently motivated sources. That makes it possible for the type shift to simply require that the degree property to which it applies be part of the path:

(95) $[\![\text{PATH-SHIFT}]\!] = \lambda D_{\langle d,t\rangle}\lambda p\,.\,D \subseteq p$ *(alternative approach)*

This is a simpler compositional move than the version of PATH-SHIFT we adopt above. Perhaps in that respect it makes for a more theoretically appealing type shift.

The question, then, becomes what the alternative source of maximality is. For cardinal uses, one could adopt an 'exactly' semantics for cardinal numerals—and in particular, that of Nouwen (2010), which builds it into a version of what we called MANY$_\exists$ above. For measure phrases with gradable adjectives, one could adopt a semantics that incorporates maximality into the adjective, as in Kennedy (1997) among others.

Ultimately, one might wish to choose between these two approaches on empirical grounds. We leave the question open here.

5 Appraisal and loose ends

The previous section laid out the analysis. In this one we will briefly highlight loose ends, consider analytical alternatives, and explore bigger-picture issues.

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5.1 Linear order restriction

The linear order of the constituents of a range expression is generally required to accord with the order on the underlying scale:

(96) a. Sam saw $\left\{\begin{array}{c}\text{between 3 and 6}\\ \text{\#between 6 and 3}\end{array}\right\}$ monkeys.

b. We expect $\left\{\begin{array}{c}\text{(from) 3 to 6}\\ \text{\#(from) 6 to 3}\end{array}\right\}$ people.

c. This class is for ages $\left\{\begin{array}{c}\text{3 through 6}\\ \text{\#6 through 3}\end{array}\right\}$.

This restriction is also present for non-numeric ranges. It's also present in measure phrases with negative adjectives such as *short*. That's important because, on some theories of polar antonyms, the scale of a negative adjective is taken to be the same as for a positive adjective but with the ordering inverted (see, among others, Kennedy 2001, Heim 2006). Thus one might expect order of elements in a range expression to invert as well:

(97) Floyd is $\left\{\begin{array}{c}\text{4 to 7}\\ \text{\#7 to 4}\end{array}\right\}$ inches shorter than Clyde.

In this respect, range expressions may provide a useful proving ground for theories of scale structure and polar antonyms, though we will not elaborate on this here.

The ill-formedness in (96) and (97) is striking, but in other cases the requirement seems to be lifted and we don't know why. In our recurring snowplow example, it's perfectly possible to go in either direction on the house numbering scale, depending on the direction of the snow plow:

(98) The snow plow cleared from $\left\{\begin{array}{c}\text{10 to 30}\\ \text{30 to 10}\end{array}\right\}$ Main Street.

Likewise, one might imagine a chemist saying something like (99) (this example is a version of one suggested by Vera Hohaus, p.c.):

(99) The transition from 90 to 60 degrees is crucial to the reaction.

Again, it seems to be important that there is a direction of travel forward in time, and that moving forward in time involves reversing the direction of the scale. Perhaps the difficulty in some of (96–97) is that they involve an ignorance reading, where there is no question of following an order along a particular dimension in time.

As it stands, our analysis doesn't predict the ill-formedness of any of these cases. We aren't sure if this is a bug or a feature. It seems plausible that what goes wrong in

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(96–97) is pragmatic: it's simply too uncooperative (in the Gricean sense) to reverse a scale in a context in which there is no plausible implicature that this could convey. But perhaps such a pragmatic account doesn't suffice to explain why these examples seem not merely infelicitous but actually ungrammatical.

5.2 Anaphora

Anaphora to ranges is possible:

(100) Floyd saw 5 to 10 million bees and Clyde saw that many bees too.

Crucially, *that many* can refer to the full range, not just to one of its subcomponents, so *that many* doesn't necessarily refer to 5 million or 10 million. More interestingly, it also doesn't refer to the exact number of bees Clyde saw—Clyde and Floyd may have seen different numbers of bees.

Anaphora is also possible via degree nominalizations:

(101) This group is for kids 10 through 14, and that one is for kids that age too.

Accounting for these facts would take us too far afield because it would need to wrestle with the details of how anaphoric expressions like *that many* and *that age* work (though about nominalizations like the latter, Bochnak et al. (2025) is relevant). But the groundwork for an analysis has probably been laid. Ranges are, on this analysis, simply a type of object in the model—a path—so there is every reason to expect that anaphoric reference to them should be possible.

5.3 Maximal informativity

There is a known difficulty with maximality in the semantics of adjectives and degree constructions: in some cases, what appears to be necessary is a *minimality* requirement instead (Beck & Rullmann 1999, Beck 2013, von Fintel et al. 2014, Buccola & Spector 2016, Hohaus & Bade 2022). The crucial examples take various forms, but the one in (102) is especially clear-cut:

(102) I have more flour than is sufficient to bake this cake.

The trouble starts in the *than* clause. On a standard semantics, one would interpret this to pick the maximal amount of flour sufficient to bake this cake. But there is no such maximum. Adding more flour can never make the amount insufficient. What seems to be required in this case is actually the minimum amount of flour that is sufficient. The principal line of analysis for these sentences is to construe a

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maximality requirement as rather a requirement of MAXIMAL INFORMATIVITY: that is, to adapt Beck (2013)'s formulation, the maximally informative element selected from a set is the one that can't be inferred from any other. In the cake scenario, that's the smallest amount of flour. For greater amounts, their sufficiency follows from the sufficiency of the smaller ones. It's only the smallest whose sufficiency doesn't follow from any other amount.

The question, then, is whether ranges work similarly. They seem to. Given our semantics, (103a) should have the denotation in (103b):

(103) a. Two to three cups of flour is sufficient to bake this cake.

b. $[\![\exists$ *[two to three cups [*PATH-SHIFT *[1* t_1 *is sufficient to bake this cake]]]* $]\!]$

$$= \exists p \left[\begin{array}{l} \textbf{start}(p) = \textbf{2cups} \wedge \textbf{end}(p) = \textbf{3cups} \wedge \\ \textbf{max}(\, \lambda d \,.\, \textbf{sufficient-to-bake-this-cake}(d)\,) \end{array} \right]$$

So ranges give rise to the same effect. It does strongly suggest an ignorance interpretation, because on any other reading, if two to three cups is the minimum necessary, then two cups is. These facts do seem to be an argument for recasting our maximality operator as a maximal informativity one instead. We will not do that here for the sake of simplicity and familiarity. It appears to be the case that maximality in general must sometimes be construed as maximal informativity, in various linguistic contexts that include at least degree constructions, questions, and definite descriptions. But there is nothing in particular about ranges that especially requires this kind of treatment. It's a general problem we inherit, and to the extent that maximal informativity corrects the problem, we would inherit the solution as well.

5.4 What counts as a range and the challenge of one-sided ranges

It makes sense to pause at this point to reflect on what counts as a range empirically. We have stipulated that it includes *from/to*, *between*, and *through* PPs, but of course that doesn't generalize beyond English. Across languages, range expressions should include any constructions that crucially express properties of ordered sets, typically ones drawn from some larger conceptually-provided order. That includes as a special case any numeric expressions that pick out intervals on the number line, for example, but it would also include various locative and temporal expressions. The latter haven't played a major role here, but they don't seem to be unusual across languages and may represent fertile ground for future research. They also include expressions that characterize intervals of a degree scale. In all cases, there is a notion of a larger domain from which the range is drawn. That strikes us as likely to be a conceptual fact about language use rather than a feature hard-wired into the semantics, and our

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definition of paths—which represent ranges in the model—doesn't include this as a requirement.

One way to probe a definition is to seek out edge cases. Happily, such cases arise for ranges in English. There seem to be range expressions that are bounded only on one end:

(104) a. from 3 up
b. up to 8
c. from 27 BC on

What makes these range expressions? Well, they occur as the arguments of range-selecting predicates like *range*, *span*, and *interval*:

(105) a. the range from 3 up
b. the interval up to 8
c. the span

They also occur as predicates of ranges:

(106) a. The kids' ages were from 3 up.
b. The final volume of this encyclopedia covers from 'xylophone' on.

Interestingly, *up* or *on* seems to be crucial. Expressions like *from 3* and *to 8* are of course fully grammatical in principle, but they don't occur in range expressions:

(107) a. *the range from 3
b. *the interval to 3
c. *?the span from 27 BC

(108) a. *The kids' ages were from 3.
b. *The final volume of this encyclopedia covers from 'xylophone'.

Cases like *up to* are discussed extensively in Schwarz et al. (2012).

These are consistent with the conceptual definition we have broached. They are also substantively compatible with our analysis. Each of them involves an ordered set, in some cases bounded by 0 or a contextually-provided value (*up to 8*) and in others not bounded at all (*from 3 up*). We leave to future research the challenge of explaining why e.g. *from 3 up* denotes a range but *from 3* does not.

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5.5 Paths are not metaphors

We return now to a question we brought up in section 2.1: what, if any, is the difference between range expressions and locatives? We've seen that range expressions overlap with locatives with regard to paths, so it wouldn't be unreasonable to think that non-locatives constitute a metaphorical extension of locative uses. But this, we argue, is not a necessary assumption. It's not that paths are being extended metaphorically to non-locative uses, but rather that paths are defined for any order, not just locative ones. This means that there's nothing additional that needs to be said to make paths work with range expressions, because ranges are just another instance of an order. It's worth noting that this isn't a novel argument; this is broadly the approach that Talmy (1996), and Gawron (2005) take in their analyses of fictive motion and similar phenomena, and Zwarts (2013) in his analysis of *from N to N* constructions. The difference, however, is that those works tend to treat these as an extension of the locative system. Our approach takes orderings to be foundational to paths in themselves.

This has broader implications for more than just ranges. Since paths can be constructed out of any ordering, they're not limited to locative uses. But it also means that the locative uses themselves can be reanalyzed. Instead of paths being a function of any real number from 0 to 1, we can instead of think of them as applying to ordered sets. This would include dense measurements such as physical distances and numeric scales but also sets such as the order of, e.g., American presidents or World Cup champions.

This approach can also help explain certain locative puzzles. Habel (1990), for instance, points out that sentences such as (109) can vary truth-conditionally depending on whether you're talking about above-ground space or London Underground tube lines:

(109) Tower Hill lies between Monument Station and Liverpool St. Station.

This statement isn't really true for the above-ground interpretation, because the set of locations between Monument Station and Liverpool St. doesn't include Tower Hill, which is to the southeast:

(110) {Monument, Lombard St., Cornhill, Threadneedle St., Wormwood St., Liverpool St.}[32]

But this is perfectly fine for the set of Circle line stations included in the path of stations from Monument to Liverpool St.:

32 This is a very coarse list, consisting of just the major streets from Monument Station to Liverpool St. The actual set would be infinitely more fine-grained.

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(111) {Monument, Tower Hill, Aldgate, Liverpool St.}

Here, (109) is simply ambiguous as to which relevant set it's picking out. Crucially, whether the path is formed from the locative set in (110) or the ordered set of tube stations in (111), the path object itself doesn't change.

6 Concluding remarks

The aim here has been to provide a description of the behavior of range expressions in English and an analysis that derives their particular constellation of properties ultimately from the locative foundation on which their semantics is built. More precisely, we proposed an analysis of their locative uses based in notion of paths, an idea amply independently motivated in the locative literature. Generalizing paths across semantic types, it was possible to explain their uses both in degree and cardinality contexts. Interestingly, doing this offers a glimpse into the role maximality may play in the semantics of both cardinal numerals and ordinary measure phrases. On our implementation, it demonstrates that ranges rely on adding maximality as an ingredient in a type shift. An alternative theory is that this maximality arises instead from the semantics of cardinality and of adjectives. We have no data to decide between these, but both of these are interesting results. To implement this analysis, we relied on a type shift that relates degrees and paths in a simple but particular way and the idea that range expressions are scope-taking. The same approach sheds light on individual-based paths as well, and it may lay the groundwork for the explanation of how range expressions operate across syntactic categories more broadly. Addressing that in earnest, though, will need to await future work.

Early on, we noted that range expressions have three kinds of readings: singleton punctual, set punctual, and interval. The paths approach has also yielded an explanation for all three. Singleton punctual readings, in which as single value is crucial, arise straightforwardly in the absence of appropriate quantification. Set punctual readings arise automatically when appropriate quantifiers scope over range expressions. Interval readings are associated with when a path is related to a contextually-provided superpath in the right way, without excluding any points within the crucial range.

It's also worth highlighting that the range facts constitute a new line of evidence for a close connection between locative semantics and the semantics of numerals and degrees.

The focus here has been resolutely on English, in part because it provides more than enough empirical richness to suffice for a single paper, but range expressions are common across the languages of the world, including in some languages in which the mathematical vocabulary has been systematically impoverished by a history of schooling in a dominant language. One case in point is Ktunaxa, an isolate spoken

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in British Columbia and parts of the American northwest, where Sandoval (2024), Sandoval & Morzycki (to appear) shows ranges can be expressed via an interesting combination of conjunction and locative demonstratives. Indeed, Ktunaxa lacks prepositions entirely, so its strategies for expressing ranges have to take a form very different from the data discussed here. While many other languages do use prepositional strategies, as English does, there's every reason to expect interesting diversity in this domain.

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