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Perspectives*

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Abstract We present a new solution to long-standing puzzles about substitution of co-referential terms. Our solution is based on a notion of perspective, where the speaker's perspective can be differentiated from the perspective of the agent whose thoughts, beliefs, etc., the speaker is reporting. Our formalization is a conservative extension of the simply-typed lambda calculus utilizing *monads*, a construction in category theory that provides a way to map a set of objects and functions into a more complex space of objects and functions. We offer a lexicalist analysis of perspective whereby certain lexical items introduce potential shifts of perspective while others do not. We show that this provides the means to give a general semantics of perspective with respect to substitutability, allowing us to capture not just the standard embedded cases of non-substitutability of distinct but co-referential terms, but also cases involving no embedding and no distinct terms. We also show that our semantics generalizes to cases outside the nominal domain, such as synonymous natural kind terms and other predicates.

Keywords: Reference, substitutability of co-referential terms, opacity, perspective, point of view, monads, category theory, linear logic, internalism

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All references are made and conceived
from a point of view.
Héctor-Neri Castañeda

1 Introduction

An important problem in the philosophy of language and the linguistic study of meaning concerns co-referential terms and substitutability in different linguistic environments. In the modern context, this problem is commonly associated with Frege (1892), and is often called *Frege's puzzle*. The puzzle can be presented in various ways, but its essence can be captured as follows: Given two co-referential linguistic expressions, why is it that in certain linguistic contexts substitution of one expression for the other is truth-preserving, while in others it is not?

For example, given that (1) is true, since *Hesperus* and *Phosphorus* are different names for the planet Venus, how is it that (2) can be true while (3) is false?

(1) Hesperus is Phosphorus.

(2) Kim believes that Hesperus is a planet.

(3) Kim believes that Phosphorus is a planet.

Alternatively, we could characterize the puzzle by observing that a sentence like the following can be true without entailing that Kim does not believe a tautology:

(4) Kim doesn't believe that Hesperus is Phosphorus.

Frege's own solution was that in addition to a reference, nominals have a sense, or 'mode of presentation', and that in certain contexts, such as those involving propositional attitudes, it is these distinct senses that block substitutability. Frege's puzzle is thus clearly related to the problem of 'referential opacity' in the study of propositional attitudes (Quine 1953, 1956, 1960). Fregean senses are not the only way to construe modes of presentation (e.g., Schiffer 1990, Fiengo & May 1998) and the notion that names, in particular, have a mode of presentation or are interpreted differently under propositional attitude verbs is not universally accepted (among many others, Kripke 1972, 1979, 1980, Recanati 1997, Richard 1990).

In this paper, we focus on linguistic aspects of substitutability/opacity. We take it for granted that there is an empirical phenomenon to be explained here — differing truth value judgements despite substitution of co-referential terms — and offer a formal mechanism for capturing and explaining it semantically. We follow Saul (1997, 2007) in observing that problems of substitutability also arise in 'simple sentences'. Our analysis captures some of these cases, too. Moreover, we also focus on cases of differential interpretation of the *same* expression (Kripke 1979, Castañeda 1989, Fiengo & May 1998). Lastly, we briefly indicate how our analysis could give insight into cases other than referential expressions, as discussed by Carnap (1947), Mates (1950), and Kripke (1979), among others. The mechanism we propose is not only formally well-founded on advances in formal logic and theoretical computer science, it also allows us to incorporate the insight of our epigraph (Castañeda 1989: 95) in the beginnings of a general formal semantics of what we might informally call *perspective*.[1]

Overview

The paper is organized as follows. In section 2 we outline the scope of the problem and set out the space of example types to be analyzed. Section 3 introduces our key formal mechanism, *monads*. Section 4 offers our analyses of particular examples. In Section 5, we compare our approach to indices to a prominent precursor and our solutions to the puzzles to the sort that could be offered by a traditional approach based on the scoping mechanism of semantic composition, which can be used to derive a *de re/de dicto* distinction. Section 6 concludes.

The literature on substitution of co-referential terms is large and complex. We have endeavoured to provide the context for our proposals, both conceptual and formal, but we will invariably have left much out. While we believe our sample of the literature to be representative, we hope that any

1 This notion of perspective may well be related to notions of perspective or point of view in other phenomena, such as demonstratives and indexicals (e.g., Kaplan 1989, Schlenker 2003), *de se* attitudes (e.g., Lewis 1979, Chierchia 1989, Pearson 2013), logophoricity and related pronominal interpretations (e.g., Sells 1987, Hagège 1974, Kuno 1987, Oshima 2006, Sundaresan 2012), illocutionary adverbs (e.g., Austin 1975, Krifka 2001, 2014, Ernst 2009), expressivity (e.g., Potts 2005, 2007, Gutzmann 2015, Gutzmann & Gärtner 2013), and predicates of personal taste (e.g., Lasersohn 2005, Stephenson 2007a,b, MacFarlane 2014). We have not yet had the opportunity to systematically explore this, but we hope to do so in future work.

lacunae are offset by the generality, formal rigour, and relative novelty of our proposal.

2 The scope of the problem

The substitutability puzzle is standardly characterized as involving two factors: 1. embedding under a modal or propositional attitude expression, such as *believe*; and 2. co-referential but distinct terms, such as *Hesperus* and *Phosphorus*. This is just how we presented things above. However, it has been shown in the literature that neither of these factors is necessary for the substitutability puzzle or related puzzles to arise.

2.1 Simple sentences

With respect to the first factor, Saul (1997) points out that the lack of substitutability can hold even in 'simple sentences' that 'contain no attitude, modal or quotational constructions' (Saul 1997: 102, fn.1). Assuming it is common knowledge that Clark Kent is Superman's secret identity, she notes that if (5) is true, substitution of *Clark Kent* for *Superman* seems to render (6) false (Saul 1997: 102, (1) & (1*)):

(5) Clark Kent went into the phone booth, and Superman came out.

(6) Clark Kent went into the phone booth, and Clark Kent came out.

With respect to this pair, an obvious issue presents itself: Are these sentences actually semantically non-distinct, with one just being a better way to package the information (in some sense that would need to be made more precise)? That is, it seems that we could say these sentences are in fact truth-conditionally equivalent (so (6) is not false when (5) is true). This is in fact what Braun & Saul (2002) and Saul (2007) argue: namely, what is mistaken is our intuition that (5) is true while (6) is false. Braun & Saul (2002) and subsequently Saul (2007: 124ff.) offer an account of this mistaken intuition based on a psychological account of how we store and access names (and other referential expressions). We do not contest this account of these sentences and our analysis does not attempt to capture them.

Saul (2007) argues against alternative attempts to reduce the perceived distinction between (5) and (6) to either a semantic distinction — including those of Forbes (1997, 1999), Moore (1999), Pitt (2001), and Predelli (1999,

2001) — or a pragmatic distinction based on conversational implicature, such as that of Barber (2000). Saul's criticism of these accounts is two-fold:

i. These accounts necessarily make use of concepts such as *aspects* or *modes of personification* but do not adequately specify what these are. ("The Aspect Problem"; Saul 2007: 57)

ii. No aspect-dependent presupposition can be expressed unless the speaker, and perhaps the audience, is enlightened that there are multiple identities for the same individual in play. However, some anti-substitution intuitions do not depend on enlightenment. ("The Enlightenment Problem"; Saul 2007: 57)

We find much appealing about Saul's program and her (and Braun's) approach may still offer the best explanation of some simple sentences. However, we will here briefly sketch a sort of distinction that seems hard for it to explain. The same cases also provide an opportunity to demonstrate that our proposal does not depend on either enlightenment or aspects/modes of personification to capture the distinction, thus avoiding the problems above.

The following sentences display a contrast:[2]

(7) #Dr. Octopus killed Spider-Man but he didn't kill Peter Parker.

(8) Dr. Octopus murdered Spider-Man but he didn't murder Peter Parker.

Given an enlightened speaker and audience, (7) is contradictory: killing Spider-Man entails killing Peter Parker (assuming it is indeed Peter Parker who is Spider-Man at the time; i.e., there has been no passing of the mantle or any such thing). Of course, for the unenlightened the sentence would not be perceived as contradictory: It's consistent for Dr. Octopus to kill two different people. However, there is no such contrast with (8): *murder* necessarily involves intention, so (8) is not necessarily contradictory, even for the enlightened. It is not clear to us how the Saulian could capture these facts, which clearly rest on a distinction between *kill* and *murder*.

The following sentence is also potentially problematic for Saul's view:

2 We prefer to use Spider-Man in our examples, because Superman is frankly kind of boring, but also because the Peter Parker/Spider-Man case involves a different (yet still familiar) set-up: it is not as clear which is whose secret identity, since Peter Parker is as much the "main character" in those stories as Spider-Man is. This avoids the problem of Pitt's (2001) concept of *primum egos*, as discussed by Saul (2007: 31–34,140). It also avoids the problem that both Superman and Clark Kent are in fact secret identities of a third identity, Kal-El, a Kryptonian refugee (Saul 2007: 31–34).

(9) Mary Jane loves Peter Parker, but she doesn't love Spider-Man.

Let us assume that the time of evaluation is a point in the stories before Mary Jane knows that Peter Parker is Spider-Man. According to the theory presented in Saul 2007, this sentence is simply false, but that seems to entirely set aside Mary Jane's say in the matter, which strikes us as problematic. It would be very strange to insist that if Mary Jane loves Peter Parker, then she really does love Spider-Man: She certainly wouldn't agree to that. Rather, sentence (9) crucially involves Mary Jane's perspective or point of view.[3]

This is an intuition we will build on in our account below, which gives a formal analysis of such perspectives. Our conception of perspective does not suffer from the Aspect Problem, for two reasons. First, perspectives are not properties of the individuals — such as Superman/Clark or Spider-Man/Peter — under discussion (unlike aspects or modes of presentation), but are rather part of the relationship between perceivers and those individuals. For us, perspectives have to do with mental representations (as discussed further in Section 4). Not only are mental representations standardly assumed in cognitive science, but the Saulian certainly cannot object to them, as the entire explanation of substitutability in Saul 2007: 128ff. rests on psychological results about mental representations of referential terms. Second, we do adequately specify how perspectives work, by giving a formal semantics of perspectival language. The account also does not suffer from the Enlightenment Problem, because it captures distinctions, e.g., between (7) and (8), based on lexical distinctions, e.g., between *kill* and *murder*, that are independent of participants' *actual* enlightenment situation.

We should also forestall potential attempts to reduce our notion of perspectives to the notion of *guises* (Castañeda 1972, 1989, Heim 1998), despite superficial similarities to and a shared empirical base with Castañeda's theory. Our approach is distinct from one where a sentence like (9) is interpreted as simply saying that Mary Jane loves only one *guise* of the entity that corresponds to Peter Parker but not another one. First, one might object that people love entities not guises.[4] Something like this seems at play in the criticism of MacColl (1905) by Russell (1905: 491), although we suspect that a Neo-Meinongian such as Castañeda would not have found this ultimately convincing. As linguists, we feel neither prepared nor compelled to enter this

3 This seems at least superficially similar to Castañeda's story of Greta Bergman and Oscar A. A. Hecdnett, which crucially involves Hecdnett's point of view (Castañeda 1989: 21–30).

4 We thank Rob Stainton (p.c.) for this point and making us aware of its precursors in Russell 1905.

debate, but a simple guise-based theory in fact also makes false empirical predictions.

If it is indeed the case that different co-referring expressions simply pick out different guises of the same individual, then a sentence like (7) should have a non-contradictory reading, but this does not seem to be the case (again assuming it is indeed Peter Parker who is Spider-Man at the time). Killing Spider-Man simply seems to entail killing his alter ego, Peter Parker. And once again, the theory must capture the difference between, for example, *kill* and *murder*, given the contrast in (7) versus (8). Although we characterized *murder* as involving 'intention', it is not a propositional attitude verb and there is no obvious evidence of embedding. We discuss this further in Section 5.

Lastly, whatever analysis we give must not lose sight of the fact that there are incontrovertible cases of contradiction that must still be derivable, such as the following:[5]

(10) #Dr. Octopus punched Spider-Man but he didn't punch Spider-Man.

(11) #Bill shares a birthday with Spider-Man, but he doesn't share a birthday with Peter Parker.

What unites *murder* and *love* versus *kill*, *punch*, and *share a birthday with* is the fact that, for the former pair, the subject/agent's perspective is part of the interpretation.[6]

2.2 Non-distinct terms but distinct beliefs

Kripke (1979) presents a puzzle that is closely related to the substitutability puzzle, but which relates to the second factor mentioned above: whether the terms involved must be distinct. He considers the case of 'phonetically identical tokens of a single name'. He provides the example of an individual, Peter, who has learned that *Paderewski* was the name of an accomplished Polish pianist. The following then seems true:

(12) Peter believes that Paderewski had musical talent.

Peter then hears of a Polish politician named *Paderewski*, and concludes that this is a different person, since he has no reason to believe that politicians

5 We thank an anonymous reviewer for the second example.

6 A proponent of a guise-based theory may contend that our proposal is essentially similar to guises, but unless they can flesh out the comparison, with the same level of formal rigour as we provide, this is just a weak analogy that does not strike us as particularly insightful.

make good musicians. Given that the same Paderewski was in fact both a politician and a pianist, is the following true or not?

(13) Peter believes that Paderewski had no musical talent.

Kripke (1979) argues that this is a true paradox and we can neither conclude that (13) is true nor false, given the situation.

Fiengo & May (1998) deny this conclusion on the basis of a theory of reference that crucially holds that names do not directly refer but only do so once part of linguistic expressions, which bear distinguishing indices, such as '[$_{\text{NP}_1}$ Paderewski]' and that what the speaker believes is characterized by statements of the following form (Fiengo & May 1998: 388):

(14) $\ulcorner$'[$_{\text{NP}_i}$ X]' has the value $\text{NP}_i\urcorner$

They also propose the following principle:

(15) **Singularity Principle**
If cospelled expressions are covalued, they are coindexed.

For Fiengo & May, then, there are two distinct Paderewski indexations at play for Peter, which means that the two "cospelled" instances of Paderewski are not covalued, given the Singularity Principle.

Fiengo & May (1998: 399) ask us to consider a version of the Paderewski puzzle in which the speaker believes that John believes that there are two people named *Paderewski*, but the speaker herself believes that there is only one (contextually relevant) person named *Paderewski*. The speaker may then say, without contradiction, (16a), which has the Fiengo & May logical form (16b), and (17a), which has the logical form (17b).[7]

(16) a. John believes that Paderewski is a genius.

b. John believes that [Paderewski_1 is a genius and 'Paderewski_1' has the value Paderewski_1]

(17) a. John does not believe that Paderewski is a genius.

b. John does not believe that [Paderewski_2 is a genius and 'Paderewski_2' has the value Paderewski_2]

Thus, the beliefs of John are distinguished by the indexation. Similarly, so long as Peter believes there are two Paderewskis, he can simultaneously

7 Fiengo & May's notation is unfortunately ambiguous; we have added the bracketing to make it clearer.

believe that one had musical talent while the other did not. If and when he realizes that these two are the same person, then the Singularity Principle requires that the two *Paderewski* expressions bear the same index and Peter could no longer believe both without contradiction.

The informal theory that Fiengo & May (1998) put forward is closely related to the Interpreted Logical Form theory of Larson & Ludlow (1993), who provide the following memorable, but ultimately unconvincing, example (Larson & Ludlow 1993: 336):

> **Context:** Jason is from New York and does not know how the name *Harvard* is pronounced in a Boston accent.

(18) Jason believes [Harvard is a fine school].

Using [harvard] to indicate Jason's pronunciation of *Harvard* and [hahvahd] to indicate the Boston pronunciation, Larson & Ludlow point out that, given this context, (19) is true, while (20), is false:

(19) Jason believes that [[harvard] is a fine school].

(20) Jason believes that [[hahvahd] is a fine school].

Why do we find this unconvincing? It seems to us that, for Jason, [harvard] and [hahvahd] are just different words. The fact that they are different pronunciations of the same word is etymological knowledge that is irrelevant to Jason's synchronic knowledge of language. Coincidence of spelling is similarly irrelevant — a criticism that applies to Fiengo & May's Singularity Principle, too (cf. 'cospelled expressions' in (15) above). Kripke in fact characterized things much more aptly when he wrote of 'phonetically identical tokens of a single name': homo*phony* is what's at stake, not homo*graphy*.[8]

In our opinion, a more satisfactory analysis of these kinds of linguistic puzzles rests on disentangling two different phenomena that seem at play in Paderewski puzzles. Kripke's (1979) conclusion that we are dealing with a paradox seems to us motivated by the interplay between the perspectival dimension introduced by the verb *believe* together with the *ambiguous* nature of the name *Paderewski* in the context of Peter's lexicon. In this case we not only have different perspectives regarding the interpretation of a term (the speaker's and Peter's), but the two interpretations also have different

8 To paraphrase Tina Turner, what's spelling got to do with it? (What's spelling but a second hand product of knowledge of language?) Surely illiterate people can fall prey to Paderewski puzzles.

cardinalities. Given that Peter can use the name Paderewski to refer to two different (from Peter's perspective) entities, in an example like (13) it is not possible to resolve whether we are talking about Peter's belief with regard to the pianist entity or the politician one. Therefore (13) seems to lack a determinate truth value: It is true with respect to Paderewski the politician, but false with respect to Paderewski the musician. We have competing interpretations, but each one is fully interpretable and can be assigned a truth value. Of course, this move itself only makes sense if the two instances of the name *Paderewski* in fact do not refer to one and the same entity for Peter, which is not possible for Kripke, and certainly not on a Naive Millian view (Salmon 1986), but is possible along the lines of our analysis in Section 4.[9] Furthermore, such an analysis easily generalizes to interpretations of any cardinality: It would be no more problematic if Peter thought there were three Paderewskis, or four, or more.

A similar example of conflicting interpretations of referential expressions is Quine's well-known Ortcutt story (Quine 1956). In this scenario the perspective of the speaker is contrasted with that of Ralph, who, after seeing Ortcutt in a shady context, wrongly believes in the existence of an additional person distinct from Ortcutt. In this scenario, rather than a case of ambiguity, in which the same name is used by Ralph for what he believes to be multiple entities, we seem to simply have a case where Ralph's interpretation of *Ortcutt* is in a sense only partial or indeterminate, as it attributes to the name *Ortcutt* only some of the properties included in the speaker's interpretation. The fact that both (21) and (22) have a true reading can be explained by assuming that *Ortcutt* is a contentious name.

(21) Ralph believes Ortcutt is a pillar of the community.

(22) Ralph believes Ortcutt is a spy.

According to our approach, (21) has two readings, both true, one in which the name *Ortcutt* is interpreted from Ralph's perspective and the other from the speaker's perspective. For (22), our approach would also generate two

9 In terms of the formalism that we will propose in what follows, this solution would require the introduction of additional technical details that concern how two monads can interoperate. The composition of two monads is discussed by Jones & Duponcheel (1993), Jaskelioff & Moggi (2010), Shan (2001), and Liang, Hudak & Jones (1995), the last of whom propose the use of *monad morphisms* to compose two monads for which composition is defined, which is also suggested by Shan (2001). A solution more in line with the logical calculus we will shortly introduce is the introduction of *distributive laws* as discussed by Jones & Duponcheel (1993), but we leave detailed discussion of this solution for future work.

readings, again one corresponding to Ralph's interpretation of the name *Ortcutt* and the other to the speaker's interpretation, but only the latter would result in a true proposition.[10]

2.3 Identity statements: Delusions and Mathematical Truths

The observations that homophonous terms and simple sentences can likewise lead to the substitutability puzzle and related puzzles is thus established in the literature. But it seems to us that we can drive the point home in an even simpler way, by starting with basic identity statements involving two homophonous tokens of the same name, avoiding accents and bypassing Paderewskis.

Statements such as the following are normally taken to be uninformative tautologies:

(24) Sandy is Sandy.

If this is true, then a statement like the following should mean that Kim does not believe a tautology:

(25) Kim doesn't believe Sandy is Sandy.

Let us call the reading where Kim does not believe a tautology an *unsatisfiable* reading.

However, sentences like (25) also have satisfiable readings in the right context:

(26) **Context:** Kim suffers from Capgras Syndrome, also known as the Capgras Delusion, a condition 'in which a person holds a delusion that a friend, spouse, parent, or other close family member has been replaced by an identical-looking impostor.'[11]

In this context, it is clear that one instance of *Sandy* is interpreted from the speaker's perspective, call this $\textbf{Sandy}_\sigma$ (where σ is the speaker index) and

10 A reviewer suggests that the following monad has the potential to serve as a generalized version of the indeterminacy approach (the ReaderT monad transformer applied to the Set monad):

(23) $\Diamond a : i \to a \to t$
$\eta(x) = \lambda i.\{x\}$
$m \star f = \lambda i.\bigcup_{x \in m(i)} f(x)(i)$

11 Wikipedia: https://en.wikipedia.org/wiki/Capgras_delusion

the other from Kim's, call this $\textbf{Impostor}_{\texttt{kim}}$. The speaker is then asserting that Kim does not believe that $\textbf{Impostor}_{\texttt{kim}} = \textbf{Sandy}_{\sigma}$.[12] In a sense, then, this is a linguistically simple, limiting case of the puzzles we have been looking at.

These kinds of expressions are not restricted to pathological cases. We can even construct similar examples involving mathematical terms, a domain that we would not expect to be open to interpretation in the same way. Consider the following piece of American history:

(27) **Context:** In 1897 Dr. Edwin J. Goodwin presented a bill to the Indiana General Assembly for '[...] introducing a new mathematical truth and offered as a contribution to education to be used only by the State of Indiana free of cost'. He had copyrighted that $\pi = 3.2$ and offered this 'new mathematical truth' for free use to the State of Indiana (but others would have to pay to use it).[13]

At the appropriate historical juncture, it is clear that the following sentence had a satisfiable reading:

(28) Dr. Goodwin doesn't believe that π is π.

Dr. Goodwin was clearly mathematically benighted, but given the context, it seems that (28) accurately reported his beliefs.

It may be tempting to explain these facts in terms of a *de re/de dicto* distinction based on compositional scope, as in Montague Semantics (Montague 1973). For standard examples, like the well known Hesperus and Phosphorus case, a number of convincing analyses based on a *de re/de dicto* ambiguity have been proposed in the literature. For instance, Aloni (2005) presents an appealing treatment of such cases in the context of an epistemic predicate

12 To clarify further: Kim, as someone who suffers from Capgras Syndrome, has two different mental representations of Sandy, which will be associated with different contexts of use. When pointing to the physical person, Kim will access the mental representation that corresponds to the double of Sandy who has replaced her (i.e., the impostor). Then when talking about Sandy in her absence Kim will still be able to access the previous mental representation. While this last representation is different from the speaker's representation (at least because they exist as entities in different minds), it will share a sufficient number of properties with the speaker's representation to allow them to talk about Sandy and to determine whether the entity in the world that is picked out by this representation is the same as the one picked out by the speaker's representation of Sandy. Also see footnote 37 below regarding the speaker's representation of Kim's denotation.

13 Wikipedia: https://en.wikipedia.org/wiki/Indiana_Pi_Bill

logic (Hintikka 1975). However if we try to apply the same type of analysis to the cases discussed here, we encounter some problems.

We could try to analyze an example like (28) such that one instance of the name π gets a *de re* interpretation (the real ratio of a circle's circumference to its diameter), and the other a *de dicto* one (the rational number that Dr. Goodwin calls 'π'). The first problem we encounter is that, if we assume that proper names get translated to constants in the epistemic predicate logic, then both instances are interpreted in the scope of the modal operator, and therefore get a *de dicto* interpretation.

A standard move to solve this problem would be to assume that proper names denote scopal operators (Montague 1973), such as quantifiers, built around their referents. This allows us to introduce two quantifiers that can scope either above or below the epistemic modality. We could therefore capture the meaning of (28) with the formula in (29), where $\Box_G$ is a doxastic belief operator relativized to Dr. Goodwin (Aloni 2005).

(29) $\exists x.\neg\Box_G\exists y.x = 3.1415926535... \wedge y = 3.2 \wedge x = y$

However, from a compositional point of view, arriving at this representation is not an easy task.

The formula in (29) assumes that the same name π gets instantiated with two different values (the irrational one and the rational one), but what would be the lexical representation of this difference? We could assume that π is ambiguous between these two meanings and that the expected reading emerges when each of the two meanings is picked. However, consider what would happen if Dr. Goodwin had had a rival — let's call him Dr. Badwin — in the Indiana General Assembly trying to push an alternative bill proposing that π equals 3.15. In that case, on the mooted approach we would be forced to also include 3.15 among the possible referents of π. But this would generate readings like (30):

(30) $\exists x.\neg\Box_G\exists y.x = 3.1415926535... \wedge y = 3.15 \wedge x = y$

This formula would be true given the circumstances, because it is in fact not the case that, in all doxastically accessible belief worlds of Dr. Goodwin's, 3.1415926535... = 3.15. But this is too weak an interpretation for Dr. Goodwin's actual beliefs: none of his belief worlds are such that he believes π is 3.15 — that's Dr. Badwin's belief. In other words, there is a stronger requirement on compositional interpretation than we would get, in the general case, by simply treating terms as ambiguous, *tout court*. Rather, they are potentially ambiguous in different ways for different speakers. One way to capture this is our method, explained below, of allowing interpretation to be anchored to different agents' potentially differing perspectives.

It is important to note that, like standard *de re/de dicto* approaches, our approach is also based on scope, but on the scope of operators in an enriched meaning language that represents perspectival semantics, not on the scope of quantifiers relative to attitudinal operators in some logical form or other representation of the syntax–semantics interface. This is what allows our system to deal with substitution in simple sentences, for which postulation of relevant attitudinal operators is not motivated (Saul 1997, 2007). We return to this distinction in Section 5, once we have laid out our proposal in detail.[14]

2.4 Summary: The space of explananda

Intuitively, what the Capgras and Indiana Pi Bill cases share is a mix of the speaker's perspective with some other perspective: that of the subject of the sentence. Thus, it seems to us that the key to these puzzles, as mentioned above, is a notion of *perspective*, which can also potentially explain the lack of substitutability in simple sentences involving verbs like *love* and *murder*, as well as the standard cases of non-substitutability of co-referential terms in embedded contexts. If we cross the factors of same/distinct terms with simple/embedded context, we obtain the space of explananda in Table 1, with cells filled by examples from the previous sections.

3 Formalization

In Section 3.2 we present our proposed formalization. But before proceeding with our proposal we present, in Section 3.1, an alternative formalization done in the style of the Logical Form semantics of Heim & Kratzer 1998. This will provide a baseline against which we argue that our solution is preferable.[15]

14 A reviewer points out that any scopal approach, including ours, seems likely to have trouble with the recently explored case of bound *de re* pronouns (Charlow & Sharvit 2014). However, the specific proposal of Charlow & Sharvit is couched in the descriptivist theory of *de re* ascription of Percus & Sauerland (2003), and we argue in Section 5 that such approaches are problematic in light of the full range of data that we consider here. This seems to present an intriguing impasse that deserves further attention in future work.

15 We would like to thank one of our anonymous reviewers for suggesting this alternative formalization.

	Simple	Embedded
Same term	#Dr. Octopus punched Spider-Man but he didn't punch Spider-Man.	Kim doesn't believe Sandy is Sandy.
Distinct term	Mary Jane loves Peter Parker but she doesn't love Spider-Man.	Kim doesn't believe Hesperus is Phosphorus.
	#Dr. Octopus killed Spider-Man but he didn't kill Peter Parker.	

Table 1 The space of explananda.

3.1 A non-monadic formalization

Our analysis depends crucially on the availability of different points of view during the interpretation process. One simple formalization of this idea is to make the interpretation function that maps expressions to meanings have an additional parameter representing a perspective. Therefore, in order to interpret an expression α, we will need both an assignment function (as is standard) and a *perspective index*. We represent the interpretation of an expression α as $[\![\alpha]\!]^{g,i}$, where g is the assignment function and i the perspective index. To get a compositional system we also need a way to represent *application* and *abstraction*. In both cases we simply want the perspective indices to be left untouched by the compositional process, as according to our analysis all changes in perspective are determined by the lexicon. The revised form of application (Heim & Kratzer 1998: 105) is defined in (31); the perspective index for the interpretation of the composed expression is the same as that of its subexpressions.

(31) *Revised Application Rule:* Let α be a branching node with daughters β and γ. Then for any assignment function g and perspective index i, $[\![\alpha]\!]^{g,i} = [\![\beta]\!]^{g,i}([\![\gamma]\!]^{g,i})$ or $[\![\alpha]\!]^{g,i} = [\![\gamma]\!]^{g,i}([\![\beta]\!]^{g,i})$, as determined by the semantic types of β and γ.

Similarly, in the case of the Predicate Abstraction rule, the interpretation index is carried over in the body of the lambda abstraction. In (32) we present a revised version of the rule as discussed in Heim & Kratzer 1998: 186.

(32) *Revised Predicate Abstraction Rule:* Let α be a branching node with daughters β and γ, where β dominates only a numerical index j. Then, for any variable assignment g and perspective index i, $[\![\alpha]\!]^{g,i} = \lambda x.[\![\gamma]\!]^{g^{x/j},i}$, where $g^{x/j}$ is the same assignment function as g except that it maps x to j.

In such a system all expressions are interpreted with respect to a perspective index. In most cases such indices are not used for determining the denotation of an expression. For instance, for a name that the speaker understands to be non-controversial, such as *Mary Jane* or *Peter Parker*, the interpretation is fixed and independent of a perspective:

(33) $[\![\text{Mary Jane}]\!]^{g,i} = \mathbf{mj}_\sigma$

(34) $[\![\text{Peter Parker}]\!]^{g,i} = \mathbf{pp}_\sigma$

On the other hand, in the case of a name whose interpretation is contentious between different speakers, the final denotation is based on the perspective index passed to the interpretation step. So the name *Spider-Man* will have a denotation that depends on the perspective taken during the interpretation process. For our speaker well-versed in the Spider-Man universe, the name will denote the same entity as *Peter Parker*, while for Mary Jane the same name will denote a different entity:[16]

(35) $[\![\text{Spider-Man}]\!]^{g,i} = \begin{cases} \mathbf{sm}_{\mathtt{mj}} \text{ if } i = \mathtt{mj} \\ \mathbf{pp}_\sigma \text{ if } i = \sigma \end{cases}$

The denotation for the verb *love* is slightly different, as it involves a direct manipulation of the perspective indices which are part of the interpretational meta-language.[17] In the case of *love* we want to be able to force the perspective

16 Note that it is not important for our account that the speaker's denotation for both *Peter Parker* and *Spider-Man* is "$\mathbf{pp}_\sigma$" as such, but rather just that 1. Mary Jane and the speaker's denotations are the same for *Peter Parker*; 2. are not the same for *Spider-Man*; and 3. the speaker's denotations are the same for *Peter Parker* and *Spider-Man*.

17 In principle we could add the indices to the target meaning language, and this is indeed the choice we make in our alternative monadic implementation below. However in the case of standard Heim & Kratzer-style semantics we would still need to modify the rules for functional application and predicate abstraction, as otherwise the types of the denotations would not match properly.

index of the expression in the object position to be the perspective of (the denotation of) the subject of the verb. Given that we can manipulate the perspective indices only at the level of the interpretational meta-language, the denotation for *love* needs to include as an argument the expression in the object position, rather than the denotation of the object itself (κ is a function that maps entities to perspective indices, which we discuss in more detail in Section 4):

(36) $[\![\text{loves DP}]\!]^{g,i} = \lambda s.\textbf{love}(s, [\![\text{DP}]\!]^{g,\kappa(s)})$

In contrast, in the case of a different transitive verb, like *punch*, which does not involve a potential switch in perspective, we provide a denotation that operates entirely at the level of the meaning language:

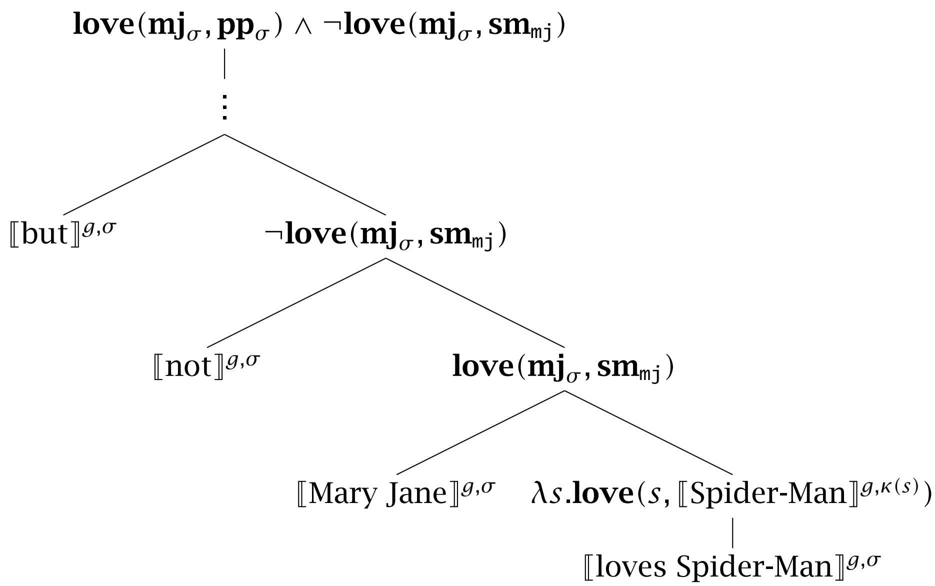
(37) $[\![\text{punch}]\!]^{g,i} = \lambda o.\lambda s.\textbf{punch}(s, o)$

Equipped with this mini lexicon, we can sketch a preliminary analysis of an example like (9), repeated here as (38)

(38) Mary Jane loves Peter Parker, but she doesn't love Spider-Man.

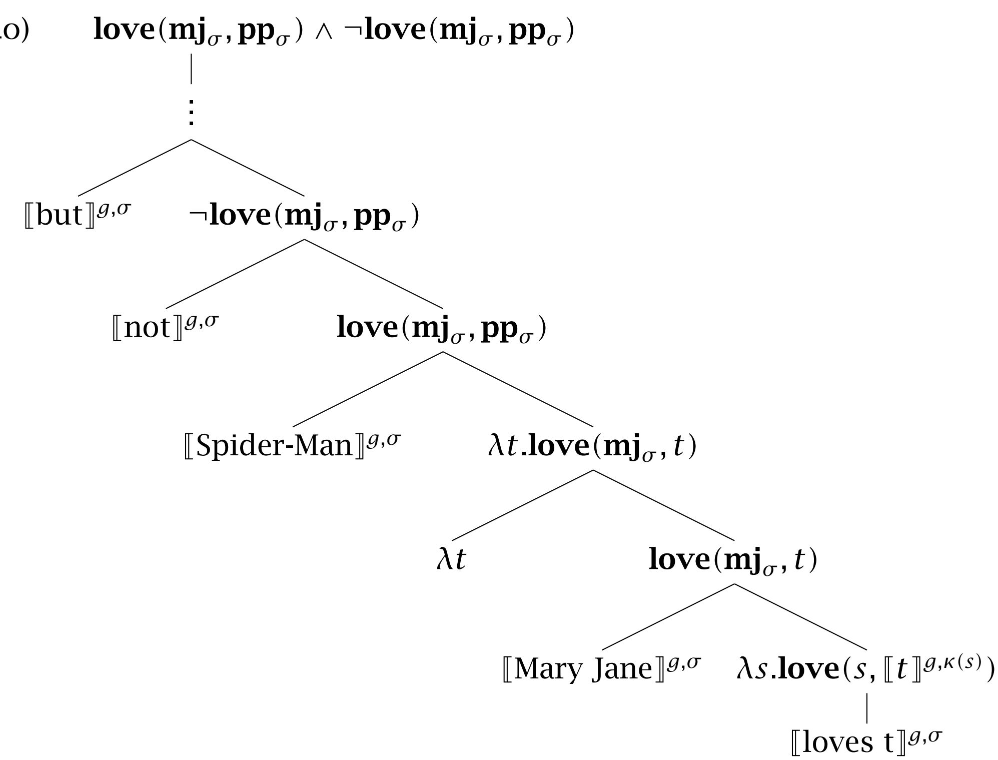
Our analysis is centred around the fact that (38) has a non-contradictory reading because the object of the second conjunct is not necessarily assigned the same denotation as the object of the first conjunct. We expect to have two readings, one contradictory and one instead consistent with what the enlightened know about the Spider-Man universe. The two readings correspond to two different scopal relationships between the proper names. In the case of the consistent reading, the name *Spider-Man* is evaluated in the scope of the verb *love* and therefore is interpreted from the perspective of Mary Jane:

(39)



In the case of the contradictory reading, the name *Spider-Man* is instead interpreted from the perspective of the speaker, who, according to our assumptions, knows his secret identity:

(40)



We make the standard assumption that traces are evaluated by applying the assignment function to their indices (which are not shown here).

There are a number of reasons why we think that the monadic approach we will introduce in the next section is preferable to the Logical Form semantics that we have just sketched. First, unlike the LF semantics, in our monadic account we are not forced to generalize the lexicon to the worst case, introducing perspective indices everywhere. Indices are introduced in the derivation only if needed and the process is entirely governed by the compositional logic, instead of being a generalized lifting of the lexicon. Second, in turn this means that we do not need to modify the rule for functional application: since indices are introduced in the derivation, their propagation is controlled by the specific part of the logic that deals with monads, together with special lexical specifications, such as the ones for verbs like *love* or *believe*. In the LF setting, we were forced to adopt syncategorematic rules for interpreting special expressions like the verb *love*. Thus, the third reason to prefer the monadic lexicalist approach is that it allows us to treat these expressions categorematically, avoiding undesirable syncategorematicity. In the LF semantics, we could obtain a categorematic treatment of perspectival expressions like *love* by lifting all lexical meanings to be functions from perspectives to extensions,[18] but this once again comes at the cost of generalizing the lexicon to the worst case. In other words, on the LF approach there is a tension between categoremiticity of perspectival expressions and lexical parsimony. Fourth, although the distinction between the interpretational meta-language and the target language is still present in the monadic approach, we have a much more constrained way of bridging the two levels thanks to the monadic operations. Fifth, the monadic approach is preferable due to its generality. We can in fact reuse the same compositional mechanism to account for a variety of semantic phenomena, as pointed out by Shan (2001) and as further investigated in various other works (Giorgolo & Unger 2009, Unger 2012, Giorgolo & Asudeh 2011, 2012a,b, 2014a,b, Charlow 2014). For example, we have shown how the monadic approach can provide a principled formal account of multi-dimensional semantics (Giorgolo & Asudeh 2012a,b), prominent in analyses of conventional implicature (Potts 2005, Gutzmann 2015). In other words, the monadic approach makes more evident a general pattern of enhanced composition that is otherwise hard-wired in the system by generalized type lifts and alternative compositional rules.

18 We thank Kai von Fintel for discussion of this point.

3.2 Formalization with monads

Our actual formal proposal is a conservative extension of the simply-typed lambda calculus that allows us to model expressions that involve perspectives. Our extension is derived from previous work in the semantics of programming languages aimed at providing a mathematical characterization of computations that produce some kind of *side effect* or that are in some way characterized by some sort of context (the notion may take different forms, such as an environment, or additional information about the computation, e.g., whether there was some error, or the presence of multiple results; Moggi 1989), and is based on the notion of *monads* (Moggi 1989, Wadler 1992, 1994, 1995), which we have used in a number of previous papers to model analyses of natural language meaning (Giorgolo & Asudeh 2011, 2012a,b, 2014a,b), based on the pioneering work of Shan (2001); see also Charlow 2014.[19] Monads are a construction in category theory that defines a way to map a set of objects and functions that we may consider simple in some sense into a more complex space of objects and functions. They have been successfully used in the semantics of programming languages to characterize certain classes of computation (Moggi 1989, Wadler 1992, 1995); see Giorgolo & Asudeh (2014a) for some further discussion.

In the present case we will use the monad that describes values that are made dependent on some external parameter, commonly known in the functional programming literature as the Reader monad. This follows Shan 2001, who suggested the idea of using the Reader monad to model intensional phenomena in natural language. We will represent linguistic expressions that can be assigned potentially different interpretations as functions from perspective indices to values.[20] Effectively we will construct a kind of lexicon that not only represents the linguistic knowledge of a single speaker but also

19 Monads are related to *continuations* (Wadler 1994), which are the formal tool used in a rich body of work by Chris Barker and Ken Shan (see Barker & Shan 2014 and references to their antecedent work therein). Charlow (2014) provides a particularly insightful study of the interplay of monads and continuations as applied to natural language semantics. There is also a potential relationship between our work and recent work by Jim Pryor on *mental graphs* (Pryor 2015), even though the latter does not directly concern monads. It would be interesting to explore how our monad-based approach and its results relate to the work of Barker & Shan, Charlow, and Pryor, but this must await future work.

20 Our indices should not be confused with those of Fiengo & May (1998) or with the kinds of indices that are commonly used in Logical Form semantics (Heim & Kratzer 1998), as discussed in Section 3.1, or in binding theory (Büring 2005). We return to a comparison of our indices to those of Fiengo & May in Section 5.

her (possibly partial) knowledge of the language of other speakers. In other words, we construe lexicons to be aspects of the knowledge of language of *individuals*, and take standard circumlocutions like the "lexicon of English" to be atheoretical folk talk, if not simply incoherent. This is a well-established position in generative linguistics (Chomsky 1965, 1986, 2000, Jackendoff 1983, 1997, 2002, 2007).

So we claim that examples like the Capgras example (25) or the similar following example can be assigned non-contradictory readings:[21]

(41) Reza doesn't believe Jesus is Jesus.

The speaker's lexicon also includes the information regarding Reza's interpretation of the name *Jesus* and therefore makes it possible for the speaker to use the same expression, in combination with a verb such as *believe*, to actually refer to two different entities.[22] In one case we will argue that the name *Jesus* is interpreted using the speaker's perspective while in the other case it is Reza's perspective that is used.

3.2.1 A monad for perspectives

Our formalization is based on category theory, a mathematical formalism that has not yet been widely used in the field of formal linguistics. It is not the aim of this paper to provide a thorough introduction to this theory, as we will focus on the specific application of a single construction for a very concrete linguistic phenomenon, and we will try to do so avoiding the vast majority of the technicalities involved. However the reader interested in getting a deeper grip on what is going on behind the scenes may find it useful to consult one of the numerous textbooks aimed at introducing category theory. Two recommended textbooks that introduce category theory from a more logical and mathematical perspective are Awodey 2010 and Goldblatt 2014. For a more computationally minded introduction to the subject, Pierce 1991 is a short but approachable first read that can be complemented with the more complete Barr & Wells 1990. These last two may be the best way to get a better handle on the present work, since we will introduce monads as they are usually encountered in the computer science literature, which

21 This example is based on the controversy from the summer of 2013 in which the scholar Reza Aslan was taken to task by Fox News correspondent Lauren Green for his views about the historical figure of Jesus of Nazareth. It seems to us that (41) could have been said sincerely by Green in that context. https://en.wikipedia.org/wiki/Reza_Aslan

22 The nature of these entities is discussed in Section 4.

avoids some of the complexity of the categorical formalism. An interesting recent addition is also Spivak 2014.

Let us dive into the formal details of our analysis. A monad is defined as a triple $\langle \Diamond, \eta, \star \rangle$. $\Diamond$ is a *functor*.[23] In general a functor is a mapping between two categories that relates the objects that form the first category to the objects of the second one and also maps the morphisms that connects the objects of the first category into morphisms operating in the second category. So a functor is defined by two separate components: one for the objects and one for the morphisms. For the category of linguistic meanings we are working in, the objects are types (collections of linguistic meanings) and functions between these types. More precisely, $\Diamond$ is an *endofunctor*, given that it maps objects of our category to other objects of the same category. We call the component of $\Diamond$ that maps between types $\Diamond_1$ and the one that maps between functions $\Diamond_2$, which means that for any two types a and b, $\Diamond_2$ maps functions from a to b to functions from $\Diamond_1(a)$ to $\Diamond_1(b)$. For our monad, $\Diamond_1$ will map each type to a new type that corresponds to the original type with an added perspective index parameter. This means that in type-theoretical terms we can think of $\Diamond_1$ as a sort of type constructor that takes a type as input and yields a different type as output. However, strictly speaking we are not introducing a new kind of type: In the category we are working in, $\Diamond_1$ is defined in terms of the functional type constructor. Formally, if i is the type of perspective indices, then $\Diamond_1$ maps any type τ to $i \to \tau$. The functor $\Diamond_2$ maps any function $f : \tau \to \delta$ to a function $f' : (i \to \tau) \to i \to \delta$. $\Diamond_2$ corresponds to function composition:

(42) $\Diamond_2(f) = \lambda g.\lambda i.f(g(i))$

The component $\Diamond_2$ will not be used below, so we will use $\Diamond$ as an abbreviation for $\Diamond_1$. This means that we will write $\Diamond\tau$ for the type $i \to \tau$.

η (pronounced 'unit') is a polymorphic function that maps inhabitants of a type τ to inhabitants of its image under $\Diamond$, formally $\eta\colon\ \forall\tau.\tau \to \Diamond\tau$.[24] Using the computational metaphor, η should embed a value in a computation that

23 A small note about notation: We use the symbol $\Diamond$ because it is the one used by Benton, Bierman & de Paiva (1998) in the logical system that is the inspiration for our formalization. The reader should not confuse this symbol with the more familiar possibility modality, although as Benton, Bierman & de Paiva note their logic does indeed define a notion of possibility, although one that is somewhat different from the usual one, as already observed by Curry (1952).

24 In case the reader is not familiar with the concept of polymorphic functions, they can think of them in two ways: as functions that take one or more additional type arguments and return a function specialized for these types, or as a family of functions, each one specialized

returns that value without it being dependent in any way on the additional computational context. In other words, η takes a non-monadic value and maps it to a monadic value. η operates differently for different types of monads, but in many cases the effect is to associate additional information with the original value. Therefore in what follows we will often refer to this mapping as wrapping or embedding a term in a monadic layer, and the term can subsequently be extracted from this monadic layer, by respectively adding and removing the additional information. In the case of our monad η should simply add a vacuous parameter to the value:

(43) $\eta(x) = \lambda i.x$

$\star$ (pronounced 'bind') is a polymorphic function of type $\forall\tau.\forall\delta.\Diamond\tau \to (\tau \to \Diamond\delta) \to \Diamond\delta$, and acts as a sort of enhanced functional application.[25] Again using the computational metaphor, $\star$ takes care of combining the side effects of the argument and the function and returns the resulting computation. In the case of the monad that we are interested in, $\star$ is defined as in (44).

(44) $a \star f = \lambda i.f(a(i))(i)$

Notice that the definition for bind is similar to the *Revised Application Rule* given in (31) for the non-monadic formalization of our approach, with the difference that here the interpretation indices are part of the language used to represent meanings, while in the case of the non-monadic formalization these indices are part of the meta-language used to interpret the expressions. Again we think this as an advantage, because we are not required to introduce mixed expressions where the meaning language is mixed with the meta-language as we had to do in the case of a verb like *love* in the non-monadic approach (see interpretation rule (36)).

Another fundamental property of $\star$ is that, by imposing an order of evaluation, it provides us with an additional scoping mechanism distinct from standard functional application. This will allow us to correctly capture the multiple readings associated with the expressions under consideration.

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for specific types. In the case under consideration, we can think of unit as a function with an additional hidden type argument (so we would read the $\forall$ as a λ), so that if we pass it the type e the result would be a function of type $e \to \Diamond e$. Or we can think of it as a family of functions for all types, a family that would include a function of type $e \to \Diamond e$, but also one of type $t \to \Diamond t$, $(e \to t) \to \Diamond(e \to t)$, etc.

25 We use the argument order for $\star$ that is normally used in functional programming, rather than swapping the arguments to make it look more like standard functional application, which would be an alternative, equivalent notational choice. We write $\star$ in infix notation.

Every monad defined in terms of unit and bind must satisfy the following three axioms:[26]

(45) $\eta(x) \star f = f(x)$

(46) $m \star \lambda x.\eta(x) = m$

(47) $(m \star f) \star g = m \star \lambda x.(f(x) \star g)$

The first two axioms guarantee that unit behaves as a “multiplicative unit” with respect to bind, in the sense that if we have a lambda expression where unit appears to the left or the right of bind the presence of unit does not change the result of evaluating the given lambda expression. The first axiom can be paraphrased as saying that if we have a value x which is lifted in a given monad using unit, and then the same value is again extracted from the monad to be passed as an argument to a monad producing function f, then we can directly pass x as an argument to the function f without lifting it to the monadic level. We can interpret this axiom as stating that all the additional information/effects should be produced by f and not by unit and bind or their interaction. Axiom (46) requires this to also be the case when unit appears to the right of bind. The axiom says that if we have a monadic value m and then we extract its core value and re-lift it to the monadic level using unit the final result should be equal to just the initial monadic value m. Again unit, bind and their interaction should not add anything to the existing monadic layer. Axiom (47) states that bind should be an associative operation, i.e., only the linear order of the monads combined with bind matters, not their grouping into a tree structure. The reader can check that these axioms are satisfied by our monad by substituting the definitions for unit and bind given in (43) and (44) in the axiom equations.

In sum, we add two operators, η and $\star$, to the lambda calculus, as shown in (43) and (44), and the reductions work as expected. These reductions are implicit in our analyses in Section 4.

26 Following common practice, we take bind to be a right-associative operator: An expression such as $m \star \lambda x.(f(x) \star g)$ is equivalent to the fully parenthesized expression $m \star (\lambda x.(f(x) \star g))$.

3.2.2 Composition

For semantic composition we use a logical calculus adapted for the linear case[27] from the one introduced by Benton, Bierman & de Paiva (1998). The calculus is based on a language with two connectives corresponding to our type constructors: ⊸, a binary connective, that corresponds to (linear) functional types, and ◊, a unary connective, that represents monadic types.

The logical calculus is described by the proof rules in Figure 1.[28] The terms in the rules are reminiscent of terms in Glue Semantics (Dalrymple 1999, 2001, Asudeh 2012), which consist of a pairing of a term from a meaning language, formalized in the lambda calculus, and a term from a logic of composition, formalized in linear logic (Girard 1987). The correspondence between the linear logic terms and meaning terms is characterized by the Curry-Howard correspondence between proofs and terms (Curry & Feys 1958, Howard 1980, de Groote 1995). For example, the linear implication, ⊸, functions like an undirected slash, |, in Categorial Grammar (Ades & Steedman 1982, Carpenter 1998, Morrill 2011): *modus ponens*/elimination for the implication corresponds to functional application. It is assumed that the logical calculus is associated with some syntactic representation, but the exact nature of the underlying syntactic formalism is not strictly relevant, so long as it can instantiate the terms for semantic composition.[29]

The axiom rule, the Cut rule and the right- and left-introduction rules for the linear implication ⊸ are those of standard linear logic, but the rules for the monadic connective ◊ deserve some comment. The right-introduction rule for ◊, when read from top to bottom, basically states that if we have a derivation that proves that from a set of hypotheses Γ we can derive a proposition A, then we are also able to derive from the same set of hypotheses the same proposition "encapsulated" into a monadic layer. If we interpret ◊ as representing an operation that allows us to lift a value into a new space of

27 Various researchers have argued for linearity as a property of composition in natural language semantics (Moortgat 1999, Moortgat 2011, Asudeh 2012, among others). Asudeh (2012) discusses it under the rubric of 'resource sensitivity'.

28 We can prove that the *Cut* rule is admissible, therefore the calculus becomes an effective (although inefficient) way of computing the meaning of a linguistic expression.

29 Glue Semantics is most closely associated with Lexical-Functional Grammar (Kaplan & Bresnan 1982, Dalrymple 2001, Bresnan et al. 2016), but Asudeh & Crouch (2002) have paired it with Head-Driven Phrase Structure Grammar (Pollard & Sag 1994) and Burke (2015) has recently paired it with the sorts of syntactic representations assumed in Tree-Adjoining Grammar (Joshi, Levy & Takahashi 1975) and the Minimalist Program (Chomsky 1995b).

$$\frac{}{x : A \vdash x : A}\ id \qquad \frac{\Gamma \vdash B \qquad B, \Delta \vdash C}{\Gamma, \Delta \vdash C}\ Cut$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x.t : A \multimap B} \multimap R \qquad \frac{\Delta \vdash t : A \qquad \Gamma, x : B \vdash u : C}{\Gamma, \Delta, y : A \multimap B \vdash u[y(t)/x] : C} \multimap L$$

$$\frac{\Gamma \vdash x : A}{\Gamma \vdash \eta(x) : \Diamond A}\ \Diamond R \qquad \frac{\Gamma, x : A \vdash t : \Diamond B}{\Gamma, y : \Diamond A \vdash y \star \lambda x.t : \Diamond B}\ \Diamond L$$

Figure 1 Sequent calculus for a fragment of multiplicative linear logic enriched with a monadic modality, together with a Curry-Howard correspondence between formulae and meaning terms. (Giorgolo & Asudeh 2014a)

values, the rule allows us to lift a value into this new space. If we interpret the monadic connective in terms of computations, the rule states that we can create a computation that has as a result the resource that we started with.

The nature of this computation is perhaps better understood if we look at the lambda term associated with the rule. The new computation is produced by wrapping the original result with the unit η. Recall that the unit acts as a lifting operator that actually does not do anything besides taking the original value to a different value space. In this sense the right-hand rule for $\Diamond$ is safe as it does not introduce any additional information in the system.[30]

The left-introduction rule for $\Diamond$ is slightly more complicated. First of all notice that, despite being a rule governing the behaviour of a connective on the left-hand side of the turnstyle, the rule imposes some restrictions on what is on the right-hand side of the sequent, namely that the resulting resource/proposition is a monadic one (but notice that with the right-hand rule for $\Diamond$ we can always construct a “dummy” monadic resource). The rule thus controls how we can introduce a monadic resource in the set of hypotheses. Again reading it top-down, if we have a proof that from a set of hypotheses containing a certain term A we can produce a monadic conclusion $\Diamond B$, then we can produce the same conclusion if we take the same set of hypotheses and replace A with a monadic term that encapsulates A.

Again, looking at the lambda term associated with the rule gives us an idea of how this replacement is performed. We assume that the original proof is encoded in the term t associated with $\Diamond B$, and that the original resource A

30 To be precise, the unit may introduce only neutral information that acts as a multiplicative unit with respect to bind.

contributes a term to t that we represent with variable x. Now, if the lambda term associated with the monadic resource $\Diamond A$ is y, we can extract from it the encapsulated value that corresponds to A and give it the name x so that it is substituted in t for the original x. In terms of proof search (i.e., looking at the rule bottom-up), the left rule ensures that a monadic resource on the left which is not consumed directly by some negative context (such as those set up on the left-hand side of an implication) is matched by a monadic layer in the result formula. This is because we do not in general have a function that maps from the monadic layer to the non-monadic one. However, for the specific monad that we are interested in (and many others) we can define such a function; in our case the function represents taking the default perspective, that of the speaker.

For the interested reader, we present in Appendix A three short derivations that exemplify some of the main derivation schemata at the core of our analysis. Although we include only these short derivations here, for reasons of space, the interested reader may test our analysis using the online automated theorem prover available at this address: http://www.sas.rochester.edu/lin/sites/asudeh/tp.html. For each of our key examples, we provide the input string to generate the relevant derivations, in Appendix B.

A key advantage of the monadic approach is that we are not forced to generalize all lexical entries to the "worst case", or richest type (as in, e.g., standard Montague Semantics or the semantics sketched in Section 3.1). With the logical setup we have just described we can freely mix monadic and non-monadic terms. For example, we can combine a pure version of a binary function with arguments that are either pure or monadic, as the following are all provable theorems in our logic.

(48) $A \multimap B \multimap C, A, B \vdash \Diamond C$

(49) $A \multimap B \multimap C, \Diamond A, B \vdash \Diamond C$

(50) $A \multimap B \multimap C, A, \Diamond B \vdash \Diamond C$

(51) $A \multimap B \multimap C, \Diamond A, \Diamond B \vdash \Diamond C$

In contrast, the following is not a theorem in the logic:

(52) $A \multimap B \multimap C, I \multimap A, I \multimap B \nvdash I \multimap C$

In short, if we were to instead simply lift the type of the lexical terms whose interpretation may be dependent on a specific perspective, we would be forced to lift all linguistic expressions that may combine with them, thus generalizing to the worst case. We do not have to do this, given our logic.

The monadic machinery also achieves a higher level of compositionality. In principle we could directly encode our monad using the → type constructor. However this alternative encoding wouldn't have the same deductive properties. Compare the pattern of inferences we have for the monadic type, in (48)–(51), with the corresponding pattern for the mooted simple type:[31]

(53) $A \multimap B \multimap C, A, B \vdash C$

(54) $A \multimap B \multimap C, I \multimap A, B \vdash I \multimap C$

(55) $A \multimap B \multimap C, A, I \multimap B \vdash I \multimap C$

(56) $A \multimap B \multimap C, I \multimap A, I \multimap B \vdash I \multimap I \multimap C$

For the simple types (53)–(56), the final formula we derive depends in some non-trivial way on the entire collection of terms on the left-hand side of the sequent. In contrast, for the monadic types (48)–(51), the same result type can be derived for all configurations. What is important is that we can predict the final formula without having to consider the entire set of terms available. This shows that the compositionality of our monadic approach cannot be equivalently recapitulated in a simple type theory.

4 Analysis

We will exemplify our approach with analyses of a selection of the examples discussed above, repeated here for convenience:

(57) Kim doesn't believe Hesperus is Phosphorus.

(58) #Dr. Octopus punched Spider-Man but he didn't punch Spider-Man.

(59) Mary Jane loves Peter Parker but she doesn't love Spider-Man.

(60) Kim doesn't believe Sandy is Sandy.

Example (59) is to be understood given the context that MJ does not know Peter Parker's secret and example (60) is to be understood in a Capgras context. The starting point for our analysis of these examples is the lexicon in Table 2. The lexicon represents the linguistic knowledge of the speaker, including her knowledge of other individuals' grammars.[32]

31 Rather than writing the types with →, we write them with linear implication, ⊸, for better parity with the types above and to ensure that all other aspects of the logic are kept constant.

32 We have simplified some entries in Table 2 by writing, e.g., '$\mathbf{ms}_{\mathsf{k}}$ if $i = \mathsf{k}$' instead of '$\mathbf{ms}_i$ if $i = \mathsf{k}$', where there are not multiple options for i. For example, contrast the entry of *Phosphorus* with that of *Spider-Man*.

Word	Denotation	Type
Reza	$\mathbf{r}_\sigma$	e
Kim	$\mathbf{k}_\sigma$	e
Dr. Octopus	$\mathbf{o}_\sigma$	e
Mary Jane	$\mathbf{mj}_\sigma$	e
Peter Parker	$\mathbf{pp}_\sigma$	e
not	$\lambda p.\neg p$	$t \to t$
but	$\lambda p.\lambda q.p \wedge q$	$t \to t \to t$
is	$\lambda x.\lambda y.x = y$	$e \to e \to t$
punch	$\lambda o.\lambda s.\mathbf{punch}(s, o)$	$e \to e \to t$
believe	$\lambda c.\lambda s.\mathbf{B}(s, c(\kappa(s)))$	$\Diamond t \to e \to t$
love	$\lambda o.\lambda s.\mathbf{love}(s, o(\kappa(s)))$	$\Diamond e \to e \to t$
Hesperus	$\lambda i.\begin{cases}\mathbf{es}_{\mathsf{k}} & \text{if } i = \mathsf{k},\\ \mathbf{v}_\sigma & \text{if } i = \sigma\end{cases}$	$\Diamond e$
Phosphorus	$\lambda i.\begin{cases}\mathbf{ms}_{\mathsf{k}} & \text{if } i = \mathsf{k},\\ \mathbf{v}_\sigma & \text{if } i = \sigma\end{cases}$	$\Diamond e$
Spider-Man	$\lambda i.\begin{cases}\mathbf{sm}_i & \text{if } i = \mathtt{o} \text{ or } i = \mathtt{mj},\\ \mathbf{pp}_\sigma & \text{if } i = \sigma\end{cases}$	$\Diamond e$
Jesus	$\lambda i.\begin{cases}\mathbf{j}_{\mathsf{r}} & \text{if } i = \mathsf{r},\\ \mathbf{j}_\sigma & \text{if } i = \sigma\end{cases}$	$\Diamond e$
Sandy	$\lambda i.\begin{cases}\mathbf{imp}_{\mathsf{k}} & \text{if } i = \mathsf{k},\\ \mathbf{s}_\sigma & \text{if } i = \sigma\end{cases}$	$\Diamond e$

Table 2 Speaker's lexicon.

Most lexical entries are standard, since we do not have to generalize to the worst case. So we do not need to change the type and denotation of lexical items that are not involved in the phenomena under discussion. For instance, logical operators such as *not* and *but* are interpreted in the standard way, as is a verb like *punch* or *kill*. Referring expressions that are possibly contentious, in the sense that they can be interpreted differently by the speaker and other individuals, instead have the monadic type $\Diamond e$.[33] This is reflected in their denotation by the fact that their value varies according to a perspective index. We use a special index σ for the speaker's own perspective, and assume that this is the default index used whenever no other index is specifically introduced. For example, in the case of the name *Spider-Man*, we are assuming that the speaker is aware of his secret identity and therefore interprets it as another name for the individual Peter Parker,[34] while Mary Jane and Dr. Octopus consider Spider-Man to be a different entity from Peter Parker.

We assume an internalist semantics such that sentences are interpreted in a model in which all entities are mental entities, i.e., that there is no direct reference to entities in the world, but only to mental representations.[35] This stance is more consistent with standard assumptions in cognitive science (Pitt 2013) and with standard generative views about the nature of language (Chomsky 1995a, 2000, Larson & Segal 1995, Jackendoff 2002, 2007, Ludlow 2003) than the externalist view in the philosophy of language (Lau & Deutsch 2014), which is often adopted tacitly in linguistic semantics.[36] Entities are therefore relativized with respect to the individual that mentally represents them, where entities that the speaker believes to be non-contentious are

33 It may be that there is an equivalence between these sorts of contentious expressions in our system and the *restricted names* of Zimmermann (2005) and between our non-contentious expressions and his *neutral names*, but the formal details are sufficiently different that the equivalence is not immediately obvious. Moreover, Zimmermann's distinction is restricted to names, but we show in Section 5 that our solution is more general than this.

34 See footnote 16 for some further clarification of this point.

35 A standard objection to this kind of move is that it makes semantics inherently "subjective", as discussed notably by Haas-Spohn (1995), which is sometimes taken to make communication impossible. This is too strong a conclusion: our stance makes communication about the same entity *less direct*, but does not render it impossible. Two agents succeed in communicating about the same thing in the world if the relevant mental representations that their grammars makes reference to are representations of that same thing. In short, the argument against internalism is a simplicity argument (it posits an extra layer of mental representations), but simplicity arguments only cut ice if all else is held constant. However, we have argued that our account captures cases that other accounts do not.

36 Some philosophers and linguists consider the externalism–internalism debate "settled" by the Twin Earth argument (Putnam 1975, Burge 1979), but this is too strong a conclusion. Crane (1991) has argued that the Twin Earth argument in fact does not provide a basis for choosing between internalism and externalism. Moreover, the question is really an empirical one about how a natural object (the brain) works, so the issue could never be settled entirely by thought experiments alone (Cummins 1991, Chomsky 1995a).

always relativized according to the speaker. This allows us to represent the fact that different individuals may have distinct equivalencies between entities. For example, Kim in our model does not equate the evening star and the morning star, but the speaker equates them with each other and with Venus. Therefore, the speaker's lexicon in Table 2 represents the fact that the speaker's epistemic model includes what the speaker knows about other individuals' models, for example, that Kim has a distinct denotation (from the speaker) for Hesperus, that Mary Jane has a distinct representation for Spider-Man, that Kim has a distinct representation for Sandy, etc.[37]

We should stress that this internalist stance is not a *necessary* stance for our formal theory, but we think it is a *sensible* one, despite its potentially controversial nature. With respect to our formal theory, it does not matter *what* the model for interpretation is a model *of*, whether mental representations or reality. However, the representational layer that internalism offers us provides a way to make sense of the notion of distinct denotations, which is especially relevant to the Capgras and Aslan cases. For example, in the Reza Aslan case, Aslan and Lauren Green were not in disagreement about which actual historical figure they were referring to, but rather about which properties that *very same person* had (see footnote 21 above for relevant details of this incident).[38]

The other special lexical entries in our lexicon are those for verbs like *believe* and *love*. The two entries are similar in the sense that they both take an already monadic resource and actively supply a specific perspective index that corresponds to the subject of the verb. The function κ maps each entity to the corresponding perspective index, i.e.:[39]

(61) $\kappa : e \rightarrow i$

κ is defined for the relevant cases under consideration as follows:

37 The speaker's Kim-denotation of *Sandy* is then not plausibly Kim's actual denotation — a mental representation that would seem privileged to Kim — but rather the speaker's representation of that representation.

38 It is of course possible that other, non-internalist alternatives to our view of the model could be worked up, for example based on Neo-Meinongian theories of reference (Parsons 1980, Castañeda 1989) or relativist theories of truth (MacFarlane 2014).

39 In the context of the examples discussed here we could equate the type of perspective indices with the type of entities, and therefore reduce the κ function to the identity function. We nevertheless use separate perspective indices for reasons of extensibility: In certain cases it may be necessary to use perspective indices that have a more complex internal structure (they might include, for example, temporal and spatial parameters) and in those cases the use of an explicit distinct type for interpretation indices would be necessary.

(62) $\kappa(\mathbf{r}_{\sigma}) = \mathtt{r}$

(63) $\kappa(\mathbf{k}_{\sigma}) = \mathtt{k}$

(64) $\kappa(\mathbf{o}_{\sigma}) = \mathtt{o}$

(65) $\kappa(\mathbf{mj}_{\sigma}) = \mathtt{mj}$

In the lexical entries for *believe* and *love*, κ maps the subject to the perspective index of the subject. Thus, the entry for *believe* uses the subject's point of view as the perspective used to evaluate its entire complement, while *love* changes the interpretation of its object relative to the perspective of its subject. However we will see that the interaction of these lexical entries and the evaluation order imposed by $\star$ will allow us to let the complement of a verb like *believe* and the object of a verb like *love* escape the specific effect of forcing the subject perspective, and instead we will be able to derive readings in which the arguments of the verb are interpreted using the speaker's perspective.

In all examples, the different readings will be generated by different orders in which the monadic arguments to verbs like *believe* and *love* are introduced in the proofs. In the proof terms that we associate with the proofs and that corresponds to the different readings, this order is translated in the left-to-right order of appearance of the corresponding meaning terms. More specifically, if an argument to such a verb appears to the left of the verb meaning (i.e., outside of its scope) it will give rise to a reading where its interpretation will be the default one (generally the speaker's point of view), while if it appears directly to the right of the verb meaning (and so in its scope), then it will be interpreted from the perspective of the subject of the verb. To give readers a more concrete example of how the different readings are generated, we show two complete derivations for the sentence *Mary Jane loves Spider-Man*. According to our analysis and the lexicon just introduced should have two different readings, one where *Spider-Man* is interpreted from the speaker perspective and another where *Spider-Man* is interpreted from Mary Jane's perspective. The derivation are shown in Figure 2. For reason of space we shorten the two names *Mary Jane* and *Spider-Man* respectively to *MJ* and *SM*. The sequent to be proven is $mj, \Diamond sm \multimap mj \multimap l, \Diamond sm \vdash \Diamond l$ (here shown without the meaning terms), which corresponds to the statement that the linguistic material that makes up the sentence (the formulae on the left-hand side of the turnstile) generates a proposition whose interpretation depends on a perspective, here called $\Diamond l$.

$$\dfrac{\dfrac{}{\llbracket \mathrm{SM} \rrbracket : \Diamond sm \vdash \llbracket \mathrm{SM} \rrbracket : \Diamond sm}\ id \qquad \dfrac{\dfrac{}{\llbracket \mathrm{MJ} \rrbracket : mj \vdash \llbracket \mathrm{MJ} \rrbracket : mj}\ id \qquad \dfrac{}{y : l \vdash y : l}\ id}{x : mj \multimap l, \llbracket \mathrm{MJ} \rrbracket : mj \vdash x(\llbracket \mathrm{MJ} \rrbracket) : l}\ {\multimap}L}{\dfrac{\llbracket \mathrm{love} \rrbracket : \Diamond sm \multimap mj \multimap l, \llbracket \mathrm{MJ} \rrbracket : mj, \llbracket \mathrm{SM} \rrbracket : \Diamond sm \vdash \llbracket \mathrm{love} \rrbracket(\llbracket \mathrm{SM} \rrbracket)(\llbracket \mathrm{MJ} \rrbracket) : l}{\llbracket \mathrm{MJ} \rrbracket : mj, \llbracket \mathrm{love} \rrbracket : \Diamond sm \multimap mj \multimap l, \llbracket \mathrm{SM} \rrbracket : \Diamond sm \vdash \eta(\llbracket \mathrm{love} \rrbracket(\llbracket \mathrm{SM} \rrbracket)(\llbracket \mathrm{MJ} \rrbracket)) : \Diamond l}\ \Diamond R}\ {\multimap}L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{z : sm \vdash z : sm}\ id}{z : sm \vdash \eta(z) : \Diamond sm}\ \Diamond R \qquad \dfrac{\dfrac{}{\llbracket \mathrm{MJ} \rrbracket : mj \vdash \llbracket \mathrm{MJ} \rrbracket : mj}\ id \qquad \dfrac{}{y : l \vdash y : l}\ id}{x : mj \multimap l, \llbracket \mathrm{MJ} \rrbracket : mj \vdash x(\llbracket \mathrm{MJ} \rrbracket) : l}\ {\multimap}L}{\llbracket \mathrm{love} \rrbracket : \Diamond sm \multimap mj \multimap l, \llbracket \mathrm{MJ} \rrbracket : mj, z : sm \vdash \llbracket \mathrm{love} \rrbracket(\eta(z))(\llbracket \mathrm{MJ} \rrbracket) : l}\ {\multimap}L}{\llbracket \mathrm{love} \rrbracket : \Diamond sm \multimap mj \multimap l, \llbracket \mathrm{MJ} \rrbracket : mj, z : sm \vdash \eta(\llbracket \mathrm{love} \rrbracket(\eta(z))(\llbracket \mathrm{MJ} \rrbracket)) : \Diamond l}\ \Diamond R}{\llbracket \mathrm{MJ} \rrbracket : mj, \llbracket \mathrm{love} \rrbracket : \Diamond sm \multimap mj \multimap l, \llbracket \mathrm{SM} \rrbracket : \Diamond sm \vdash \llbracket \mathrm{SM} \rrbracket \star \lambda z.\eta(\llbracket \mathrm{love} \rrbracket(\eta(z))(\llbracket \mathrm{MJ} \rrbracket)) : \Diamond l}\ \Diamond L$$

Figure 2 Two witness derivations for the two non-equivalent readings of *Mary Jane loves Spider-Man.*

The specific way in which the left-hand side formulae are generated is beyond the scope of this paper (in previous works we have used the framework of Lexical-Functional Grammar (Kaplan & Bresnan 1982, Dalrymple 2001, Bresnan et al. 2016), although other frameworks may be used), but the general idea is that the formulae are generated from template-formulae specified in the lexicon and a syntactic derivation that determines the grammatical relations between the relevant components (Dalrymple 1999, 2001, Asudeh 2012). The target goal is in general the proposition corresponding to the application of the main predicate of the sentence to all its arguments, and lifted to the monadic level. In this case the main predicate is *love* which, when combined with its object and subject, generates a proposition that we call l.

The first derivation is associated with the meaning term $\eta(\llbracket \mathrm{love} \rrbracket(\llbracket \mathrm{SM} \rrbracket)(\llbracket \mathrm{MJ} \rrbracket))$. Here the interpretation of the name *Spider-Man* is to the right of and hence under the scope of the verb *love*, which means that it is going to be interpreted from the perspective of the subject of the verb, Mary Jane. By substituting the meanings defined in the lexicon we obtain the reading $\mathbf{love}(\mathbf{mj}_\sigma, \mathbf{sm}_{\mathtt{mj}})$. If we read the derivation bottom-up and follow the left branch of the first application of the $\multimap L$ rule, we can see that the interpretation of *Spider-Man* is consumed by *love* as is. In the second derivation, the interpretation of *Spider-Man* is instead first the target of the $\Diamond L$ rule, which “extracts” its non-monadic core meaning and binds it to the name z. It is this new resource that is then consumed by *love* after being re-lifted to the monadic layer by unit, but the use of unit means that its value is basically shielded from the change of perspective that *love* tries to enforce.

In the meaning term that corresponds to the entire derivation we can see that the meaning term for *Spider-Man* appears to the left of the meaning term for *love* and thus escapes its scope and gets interpreted from the perspective of the speaker. After substituting the meaning terms for the lexical items we obtain the reading $\mathbf{love}(\mathbf{mj}_\sigma, \mathbf{pp}_\sigma)$.

Figure 3 shows the four non-equivalent readings that we derive in our system for example (57), repeated here as (66).[40]

(66) Kim doesn't believe that Hesperus is Phosphorus.

Reading (71) assigns to both *Hesperus* and *Phosphorus* the subject Kim's interpretation and results, after contextualising the sentence by applying it to the standard σ perspective index, in the truth conditions in (67), i.e., that Kim does not believe that Hesperus *qua* the evening star is Phosphorus *qua* the morning star. This reading would not be contradictory in an epistemic model (such as Kim's model) where the evening star and the morning star are not the same entity.

(67) $\neg\mathbf{B}(\mathbf{k}_\sigma, \mathbf{es}_\mathsf{k} = \mathbf{ms}_\mathsf{k})$

In the case of readings (72) and (73), we get a similar effect, although here we mix the epistemic models of the speaker and Kim: one of the referring expressions is interpreted from the speaker's perspective while the other is again interpreted from Kim's perspective. For these two readings we obtain respectively the truth conditions in (68) and (69).

(68) $\neg\mathbf{B}(\mathbf{k}_\sigma, \mathbf{v}_\sigma = \mathbf{ms}_\mathsf{k})$

(69) $\neg\mathbf{B}(\mathbf{k}_\sigma, \mathbf{es}_\mathsf{k} = \mathbf{v}_\sigma)$

Finally for reading (74) we get the contradictory reading that Kim does not believe that Venus is Venus, as both referring expressions are evaluated using the speaker's perspective index.

(70) $\neg\mathbf{B}(\mathbf{k}_\sigma, \mathbf{v}_\sigma = \mathbf{v}_\sigma)$

40 The system generates six possible readings, as there are two possible orders of evaluation for the meaning of *Hesperus* and *Phosphorus* when they are both outside or inside the scope of *believe*. However, for our specific monad we have the following equality if x does not appear free in n and y does not appear free in m:

(i) $m \star \lambda x.n \star \lambda y.p = n \star \lambda y.m \star \lambda x.p$

This captures the intuition that the interpretation value of independent expressions does not depend on the order of evaluation.

(71) $\eta([\![\text{not}]\!]([\![\text{believe}]\!]([\![\text{Hesperus}]\!] \star \lambda x.[\![\text{Phosphorus}]\!] \star \lambda y.\eta([\![\text{is}]\!](x)(y)))([\![\text{Kim}]\!])))$

(72) $[\![\text{Hesperus}]\!] \star \lambda x.\eta([\![\text{not}]\!]([\![\text{believe}]\!]([\![\text{Phosphorus}]\!] \star \lambda y.\eta([\![\text{is}]\!](x)(y)))([\![\text{Kim}]\!])))$

(73) $[\![\text{Phosphorus}]\!] \star \lambda y.\eta([\![\text{not}]\!]([\![\text{believe}]\!]([\![\text{Hesperus}]\!] \star \lambda x.\eta([\![\text{is}]\!](x)(y)))([\![\text{Kim}]\!])))$

(74) $[\![\text{Hesperus}]\!] \star \lambda x.[\![\text{Phosphorus}]\!] \star \lambda y.\eta([\![\text{not}]\!]([\![\text{believe}]\!](\eta([\![\text{is}]\!](x)(y)))([\![\text{Kim}]\!])))$

Figure 3 Non-equivalent readings for *Kim doesn't believe Hesperus is Phosphorus.*

The different contexts for the interpretation of referring expressions are completely determined by the order in which we evaluate monadic terms. This means that, just by looking at the linear order of the lambda term, we can check whether a referring expression is evaluated inside the scope of a potentially perspective-changing operator such as *believe*, or if it is interpreted using the standard/speaker's interpretation.

Notice that, given our internalist assumption about the nature of the model, our analysis of a sentence like (66) does not specify what the actual case is with respect to the mind-external reality of any of the readings. Our system is based on the idea that the lexicon of a speaker is connected to her model of reality. The speaker's model, which is not necessarily representationally correct, also represents information that the speaker knows about the knowledge of other language users. For instance, in the case of the satisfiable readings for sentence (66), Kim's model will contain different axioms regarding the identities of the celestial bodies than the model of the speaker. In the scenario under consideration, the speaker knows facts about the world that Kim does not. Kim's mental model is not a completely accurate representation of reality, because Kim is unaware of an identity that should hold. But it is equally possible for the speaker's model to not adhere to reality. Before the discovery that Hesperus and Phosphorus are the same planet, a sentence like *Lysippus falsely believes that Hesperus is Phosphorus* would have been considered true.[41]

41 We operate under the assumption that the adverb *falsely* presupposes that the complement of the modified doxastic verb is false for the speaker of the sentence.

If we consider a case like sentence (58), repeated in (75), we ought to get only a contradictory reading as there is no intuitively non-contradictory reading of the sentence (in the absence of focal stress on the second occurrence of *punch* or *Spider-Man*).

(75) #Dr. Octopus punched Spider-Man but he didn't punch Spider-Man.

Our analysis produces a single reading that indeed corresponds to a contradictory interpretation:

(76) $[\![\text{Spider-Man}]\!] \star \lambda x.[\![\text{Spider-Man}]\!] \star$
$\lambda y.\eta([\![\text{but}]\!]([\![\text{punch}]\!]([\![\text{Dr. Octopus}]\!])(x))$
$([\![\text{not}]\!]([\![\text{punch}]\!]([\![\text{Dr. Octopus}]\!])(y))))$

The verb *punch* is not a verb that can change the interpretation perspective and therefore the potentially controversial name *Spider-Man* is interpreted in both instances using the speaker's perspective index. The result is unsatisfiable truth conditions, as expected:

(77) $\mathbf{punch}(\mathbf{o}_\sigma, \mathbf{pp}_\sigma) \wedge \neg\mathbf{punch}(\mathbf{o}_\sigma, \mathbf{pp}_\sigma)$

In contrast a verb like *love* is defined in the lexicon in Table 2 as possibly changing the interpretation perspective about its object to that of its subject. Therefore in the case of a sentence like (59), repeated in (78), we expect one reading where the potentially contentious name *Spider-Man* is interpreted according to the subject of *love*, Mary Jane.

(78) Mary Jane loves Peter Parker but she doesn't love Spider-Man.

This is in fact the result we obtain. Figure 4 reports the two readings that our framework generates for (78).

Reading (80), corresponds to the non-contradictory interpretation of sentence (78), where *Spider-Man* is interpreted according to Mary Jane's perspective and therefore is assigned an entity different from Peter Parker:[42]

(79) $\mathbf{love}(\mathbf{mj}_\sigma, \mathbf{pp}_\sigma) \wedge \neg\mathbf{love}(\mathbf{mj}_\sigma, \mathbf{sm}_{\mathtt{mj}})$

42 We use the term $\mathbf{pp}_\sigma$ rather than $\mathbf{pp}_{\mathtt{mj}}$ in (79), simply because of the notational choice we made in the lexicon in Table 2 (page 29), which notates non-contentious names with the speaker's index. Similarly, we notate the term that is the speaker's denotation of *Spider-Man* in (82) as $\mathbf{pp}_\sigma$ rather than as $\mathbf{sm}_\sigma$, again because of the notational choice we made in the lexicon in Table 2.

(80) $\eta(\llbracket\text{but}\rrbracket(\llbracket\text{love}\rrbracket(\eta(\llbracket\text{Peter Parker}\rrbracket))(\llbracket\text{Mary Jane}\rrbracket))$
$(\llbracket\text{not}\rrbracket(\llbracket\text{love}\rrbracket(\llbracket\text{Spider-Man}\rrbracket)(\llbracket\text{Mary Jane}\rrbracket))))$

(81) $\llbracket\text{Spider-Man}\rrbracket \star$
$\lambda x.\eta(\llbracket\text{but}\rrbracket(\llbracket\text{love}\rrbracket(\eta(\llbracket\text{Peter Parker}\rrbracket))(\llbracket\text{Mary Jane}\rrbracket))$
$(\llbracket\text{not}\rrbracket(\llbracket\text{love}\rrbracket(\eta(x))(\llbracket\text{Mary Jane}\rrbracket))))$

Figure 4 Non-equivalent readings for *Mary Jane loves Peter Parker but she doesn't love Spider-Man.*

Reading (81) instead generates unsatisfiable truth conditions, as Spider-Man is identified with Peter Parker according to the speaker's interpretation:

(82) $\mathbf{love}(\mathbf{mj}_\sigma, \mathbf{pp}_\sigma) \wedge \neg\mathbf{love}(\mathbf{mj}_\sigma, \mathbf{pp}_\sigma)$

A reviewer points out that our system likewise derives a non-contradictory reading for (83), so long as one instance of *Spider-Man* is evaluated from the speaker's perspective and the other from Mary-Jane's.

(83) Mary Jane loves Spider-Man but she doesn't love Spider-Man.

We believe this to be a desirable prediction, since this is a valid reading of the sentence and the sentence construed as such is not contradictory.[43] The reviewer also points out, though, that this way of expressing the relevant proposition is somewhat odd compared to (78) (*Mary Jane loves Peter Parker, but she doesn't love Spider-Man*). We agree with the reviewer, but believe that this is a pragmatic effect: The second sentence is a clearer way of expressing the relevant proposition and so is preferred over the version with two instances of the same name. But the semantics should still generate the reading in both cases. We do not discuss here how to rank preferences for different readings of a given sentence or how to rank preferences for how a given proposition is expressed, because we restrict ourselves to the semantic aspects of the relevant expressions.

Our last example, the Capgras example (60), repeated here as (84), is particularly interesting as the embedded clause is just a simple identity statement with two tokens of the same name. We are not aware of formal analysis

43 We note in passing that the non-contradictory reading of this sentence would seem to pose another challenge to the Saulian theory discussed in Section 2.1 (Saul 1997, 2007, Braun & Saul 2002).

of this kind of example in the literature, the closest being the Hecdnett example in Castañeda 1989.[44] The non-contradictory reading that this sentence has seems to be connected specifically to two different interpretations of the same name, *Sandy*, both syntactically embedded under the propositional attitude verb *believe*.

(84) Kim doesn't believe Sandy is Sandy.

Our system generates three non-equivalent readings, reported here in Figure 5.[45]

(85) $\eta(\llbracket\text{not}\rrbracket(\llbracket\text{believe}\rrbracket(\llbracket\text{Sandy}\rrbracket \star \lambda x.\llbracket\text{Sandy}\rrbracket \star \lambda y.\eta(\llbracket\text{is}\rrbracket(x)(y)))(\llbracket\text{Kim}\rrbracket)))$

(86) $\llbracket\text{Sandy}\rrbracket \star \lambda x.\llbracket\text{Sandy}\rrbracket \star \lambda y.\eta(\llbracket\text{not}\rrbracket(\llbracket\text{believe}\rrbracket(\eta(\llbracket\text{is}\rrbracket(x)(y)))(\llbracket\text{Kim}\rrbracket)))$

(87) $\llbracket\text{Sandy}\rrbracket \star \lambda x.\eta(\llbracket\text{not}\rrbracket(\llbracket\text{believe}\rrbracket(\llbracket\text{Sandy}\rrbracket \star \lambda y.\eta(\llbracket\text{is}\rrbracket(x)(y)))(\llbracket\text{Kim}\rrbracket)))$

Figure 5 Non-equivalent readings for *Kim doesn't believe Sandy is Sandy.*

Readings (85) and (86) are two contradictory readings of the sentence. In the first case, both instances of the name *Sandy* are interpreted from the subject's perspective and therefore a lack of belief in a tautology is attributed to Kim. In the second case, both instances of the name *Sandy* are interpreted from the speaker's perspective, again resulting in an assertion that Kim does

44 The "Masked Ball" scenario discussed by Cumming (2008) is perhaps also a close analogue. We have not applied our system to masked ball examples yet, but we see no principled barrier to a successful analysis at this point. The claim by Cumming (2008) that names are variables seems at least superficially similar to our analysis of contentious names like *Spider-Man*, but a careful comparison would deserve a paper in its own right. Cumming (2008) contends that the scenario is a problem for standard Millianism, but recent work by Rieppel (2015) gives reason to doubt this conclusion.

45 Again the system generates six non-equivalent readings (see footnote 40), which are further reduced in this case as we have the same linguistic term appearing twice and combined with a commutative predicate (*is*). Some work recently reported by Percus & Sharvit (2014) contends that the structure of copular sentences is not symmetrical — a fact that is well-known in the linguistics literature (see, e.g., Mikkelsen 2005 and relevant references therein) — and that this has consequence for the semantics of belief reports about identity. Their work seems congruent with ours, but along the lines of the analysis in Section 3.1 rather than the monadic analysis. Further careful comparison might prove fruitful.

not believe a tautology. In contrast the reading in (87) corresponds to the interpretation that assigns two different referents to the two instances of the name *Sandy*, producing the truth conditions in (88) which are satisfiable in a suitable model.

(88) $\neg\mathbf{B}(\mathbf{k}_\sigma, \mathbf{s}_\sigma = \mathbf{imp}_\mathsf{k})$

We use $\mathbf{imp}_\mathsf{k}$ as the speaker's representation of the "impostor" that Kim thinks has taken the place of Sandy.

The analysis of the Aslan/Jesus example (41), repeated in (89), is equivalent; the non-contradictory reading is shown in (90).

(89) Reza doesn't believe Jesus is Jesus.

(90) $\neg\mathbf{B}(\mathbf{r}_\sigma, \mathbf{j}_\sigma = \mathbf{j}_\mathsf{r})$

There are again three non-equivalent readings, including the one above, which are just those in Figure 5, with ⟦Sandy⟧ replaced by ⟦Jesus⟧ and ⟦Kim⟧ replaced by ⟦Reza⟧.

5 Comparison with previous approaches

Our approach seems superficially similar to Fiengo & May's, since both approaches are concerned with substitutability and use indices, but it is in fact quite distinct. First, the use of indices here resides entirely in the model. We do not require that the expression '[$_{\mathrm{NP}}$ Sandy]' bear an index in order to address the puzzles above.[46]

Second, we are not forced to agree with Fiengo & May (1998: 381) that 'there is no value associated with the name 'Max' qua lexical item', despite agreeing with their claim that 'if we are to determine the identity conditions for words . . . we should determine the identity conditions on words as *they appear in the lexicon of an individual*.' (emphasis in the original; Fiengo & May 1998: 379).[47] Fiengo & May thus claim that the name *Max* in the lexicon does not refer, only the syntactic structure [$_{\mathrm{NP}_i}$ Max] refers. This claim is not unreasonable, although it's certainly hard to see how to test it empirically. However, it does not seem to fit our folk understanding of names, which

46 This does not mean that there could not be other, independent reasons for expressions to bear indices, although we are most sympathetic to the spirit of David Beaver's question, 'What do those little numbers mean, and who put them there anyway?' (Beaver 1999).

47 The full quote has 'in particular, names' where we have the ellipsis, but we are not in fact convinced that names are special in this regard, for reasons that will become clear shortly.

do seem referential in their own right, as words. Now, we are perfectly comfortable with folk understanding giving way to theory, but if two theories are equal in all other respects, it would seem to us slightly capricious, if not outright perverse, to pick the one that rejects folk understanding of its phenomena. On our theory, a name like *Max* does have its (range of) reference determined in the (speaker's) lexicon. There is some sense, then, in which our theory is more natural.

Third, it is well known that these puzzles are not just about names, but also about natural kinds and other predicates. It is hard to see how the kinds of indices that Fiengo & May (1998) use could be generalized to cover such cases; they seem too specialized to do the job, since they represent reference to individuals and this is not appropriate for predicates. In contrast, a virtue of our analysis is that we can apply it to not just names and referring expressions, but to any natural language expression that may have different perspectival interpretations. This means that we can extend our analysis to other cases, such as the standard examples involving synonymous natural kind terms like *groundhog* and *woodchuck* (see, e.g., Fox & Lappin 2005) or *furze* and *gorse* (Kripke 1979) or of synonymous verbs, such as *masticate* and *chew* (Mates 1950) or *photocopy* and *xerox* (Larson & Ludlow 1993).

As an illustration, consider the following example:

(91) Elena loves dolphins, but she doesn't love marine mammals.

Suppose Elena thinks that Flipper is a dolphin and Hoover is a seal, but she thinks only Hoover is a marine mammal; i.e., she thinks seals are marine mammals, but dolphins are not. Suppose also that the speaker and Elena are in agreement about which entities the names *Flipper* and *Hoover* refer to, so the names are not controversial. Table 3 sketches (the relevant part of) the lexicon for the speaker of (91), which shows how the perspective index on *marine mammal* allows Elena and the speaker to treat the predicate distinctly despite agreeing on the predicates *dolphin* and *seal* and on the reference of the relevant names. We do not mean to imply that this extension of our approach is trivial, since matters of compositionality of, for example, *marine mammal*, have not been addressed here, but the extension is at least a natural candidate for further exploration.

Lastly, let us return to the matter of traditional approaches to substitutability/opacity based on *de re*/*de dicto* ambiguities derived from differential compositional scopings. In Section 2.3, we discussed certain problems

Word	Denotation	Type
dolphin	$\{\textbf{flipper}_\sigma\}$	$e \to t$
seal	$\{\textbf{hoover}_\sigma\}$	$e \to t$
marine mammal	$\lambda i. \begin{cases} \{\textbf{hoover}_\sigma\} & \text{if } i = \mathsf{e}, \\ \{\textbf{flipper}_\sigma, \textbf{hoover}_\sigma\} & \text{if } i = \sigma \end{cases}$	$\Diamond(e \to t)$

Table 3 (Relevant portion of) speaker's lexicon for (91)

that such an approach may have for ordinary proper names in Capgras and Indiana Pi Bill examples, repeated here:

(92) Kim doesn't believe Sandy is Sandy.

(93) Dr. Goodwin doesn't believe π is π.

Here we outline some further issues for standard scope approaches.

First, substitutability puzzles in simple sentences, as discussed in Section 2.1, pose a *prima facie* problem, since there does not seem to be a relevant scope operator present in the case of the composition of a simple predicate like *love* or *murder* with its object argument. Of course, many linguists would be perfectly willing to postulate null operators in such cases, but it is not clear that this derives the right result. That would be tantamount to treating *love* or *murder* as an 'opaque transitive verb' like *owe*, but the former have existential entailments that the latter lack (Zimmermann 2006):

(94) Frodo owes Sam a horse.
$\not\to$ There is a horse that Frodo owes Sam.

(95) Saruman murdered a horse.
$\to$ There was a horse that Saruman murdered.

(96) Princess Caroline loves a horse.
$\to$ There is a horse that Princess Caroline loves.

An account that postulates a hidden operator for *murder* or *love* would have to explain this contrast.

Second, even in the case of embedded contexts, which offer a scopal operator in composition, in the form of a modal or propositional attitude verb, a scope-based *de re/de dicto* approach faces some problems, at least in the case of our most challenging examples, such as the Indiana Pi Bill example, the Capgras example, and the Aslan/Jesus example (repeated and discussed further below). To try to explain the two readings in the context of a standard possible worlds semantics, we could take the Capgras example (92) to be ambiguous with respect to a *de re/de dicto* reading. In the case of the *de dicto* reading (which corresponds to the unsatisfiable reading) the two names are evaluated under the scope of the doxastic operator *believe*, i.e., they both refer to the same entity that is assigned to the name *Sandy* in each accessible world.[48] Clearly this is always the case, and so (92) is not satisfiable. In the case of the *de re* reading, we assume that the two names are evaluated at different worlds that assign different referents to the two names. One of these two worlds will be the actual world and the other world one of the accessible worlds. The reading is satisfiable if the doxastic modality links the actual world with one in which the name *Sandy* refers to a different entity. Notice that for this analysis to work we need to assume that names behave like quantifiers with respect to scoping both over and under modal and propositional attitude operators, as discussed in Section 2.3.

However, it has been argued that even if we model names as generalized quantifiers, they are scopeless (Zimmermann 1993). But this is problematic for a scopal approach to the Capgras example. It would predict that both instances of the name *Sandy* escape the scope of *believe*. The resulting reading would bind the quantified individual to the interpretation of *Sandy* in the actual world. This would capture only an unsatisfiable reading. To save the scopal approach, we would need to assume that names in fact are sometimes interpreted in the scope of operators.

Even assuming that we find a satisfactory solution for these inconsistencies, the standard *de re/de dicto* scopal approach cannot really capture the intuitions behind opacity in all contexts. Consider again our Aslan/Jesus example, repeated here:

(97) Reza doesn't believe Jesus is Jesus.

Assume that there are two views about Jesus: Jesus as a divine being and Jesus as a non-divine, simply human being. Assume that Jesus is non-divine in the actual world and that Reza is an atheist; then the only possible reading is the unsatisfiable one, as the referent for Jesus will be the same in the

48 In order to put the discussion on firmer footing, we have adopted the language of a Hintikka-like analysis of propositional attitudes (Hintikka 1969, 1975), since this view is currently influential in linguistics, but our points should follow without a substantive commitment to that sort of analysis, so long as the propositional attitude verb is assumed to provide a scope point, as is commonly assumed.

actual world and all accessible Reza-belief-worlds. The problem is that the scopal approach assumes a single modal model, while in this case it seems that there are two doxastic models necessary, Reza's model and the speaker's model. In contrast, in our approach the relevant part of Reza's model is embedded inside the speaker's model and perspective indices indicate which interpretation belongs to Reza and which to the speaker.

A subset of approaches to *de re* ascription are often collectively referred to as *descriptivist* approaches (among others, Kaplan 1968, Lewis 1979, Cresswell & von Stechow 1982, Percus & Sauerland 2003). These approaches share the assumption that what is believed is something more structured than a simple proposition, such as a pairing of an individual (the *res*, what the belief is about) and a property. As Lewis (1979: 521) puts it, '[S]ometimes property objects will do and propositional objects won't.' It does not strike us as likely that a descriptivist approach to *de re* ascription would give a satisfactory account of the Aslan/Jesus case (or the Capgras or Indiana Pi Bill cases), for similar reasons to the ones given in the previous paragraph. The property at stake is that of *being Jesus* (or *being Sandy* or *being* π). But this property contains the contentious name that is at issue. It seems this kind of approach equally needs some way of mixing models/perspectives.

Such approaches would also inherit some of the general problems of *de re* approaches. First, if a scope point were postulated for verbs like *love* or *murder*, their lack of opacity with respect to existential entailments would be unexplained, as discussed above. Moreover, there seems to be no relevant structural difference in the object that could explain the distinction between these verbs and, e.g., *punch* or *kill*. Lastly, such approaches would still be challenged by lack of substitutability in simple sentences, as discussed above.

We have thus far assumed that the relevant perspectives are the subject's and the speaker's, but this likely needs further refinement, in ways that seem straightforward for our system. In particular, we have treated the speaker's perspective as the default, but there may be circumstances in which the speaker is purposefully adopting an alternative perspective. This may give us some purchase on examples like the *Barbara Vine* example in Zimmermann (2005: 69), which hinges on a bookshop owner purposefully using a pseudonym of author Ruth Rendell to refer to only a subset of her books:

(98) I've read all of Barbara Vine's books.

Crucially, in this scenario, the bookshop owner and Z himself are both enlightened about the multiple identity. Zimmermann characterizes the bookshop owner as adopting the language of a novice, although he is in fact an expert. In our system this could be captured by the bookshop owner modifying his lexicon such that Barbara Vine is (at least temporarily) a contentious name, such that it refers to an entity that is distinct from Ruth Rendell so long as the index of evaluation is not the bookshop owner or Z. The details remain to be worked out, though.

6 Conclusion

We have offered a semantics of perspective that offers a solution to the substitutability puzzle in both simple and embedded contexts. Our solution extends to cases of distinct interpretations of tokens of the same name, which gives rise to a related puzzle. We exemplified this case with respect to simple identity cases, as in the Capgras, Indiana Pi Bill, and Aslan/Jesus examples. Our solution to these puzzles rests on an analysis in terms of a combination of different perspectives. We have claimed that the switch to a different perspective is triggered by specific lexical items, such as propositional attitude verbs, but also verbs like *love* and *murder* which express some kind of perspective on the part of the subject of the verb towards its object, but which nevertheless cannot easily be argued to be opaque in their object position. The context switch is not obligatory, as witnessed by the multiple readings that the sentences discussed seem to have.

The formalization of our analysis is based on monads. The main idea of our formal implementation is that referring expressions that have a potential perspectival dependency can be implemented as functions from perspective indices to fully interpreted values. Similarly, the linguistic triggers for context switch are implemented in the lexicon as functions that can modify the interpretation context of their arguments. Monads allow us to freely combine these "enriched" meanings with standard ones, avoiding unilluminating generalization to the worst case. We have also seen how more traditional approaches, while capable of dealing with some of the examples we discuss, are not capable of providing a general explanation of the totality of observed phenomena. We briefly explored how our approach could be extended to other types of natural language expressions, such as natural kind terms, nominal predicates, and verbs. Careful exploration of these extensions remains part of future work.

We have inevitably had to take positions on some issues that are far from settled, but we do not mean these positions themselves to be the main contribution of this paper. Rather, it seems to us that philosophers and linguists are in broad agreement that in some linguistic contexts there seems to be an "extra something" involved in interpreting names, and other expressions; we have made a formal proposal about what that extra something could be: perspectives.

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Appendix A

In this appendix we present some of the common derivation schemas that are working "behind the scenes" in our examples.

The first worked out proof of a theorem in our logic represents the combination of a resource that takes a certain argument, say A, consumes it and returns a combined resource B, something we represent in our system as $A \multimap B$, and a resource that is the required A but wrapped inside a monadic layer, which we represent as $\Diamond A$. We prove that from these two terms we can derive a B term wrapped in a monad: $A \multimap B, \Diamond A \vdash \Diamond B$. The proof is as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{y : A \vdash y : A}\,id \qquad \dfrac{}{x : B \vdash x : B}\,id}{f : A \multimap B, y : A \vdash f(y) : B}\multimap L}{f : A \multimap B, y : A \vdash \eta(f(y)) : \Diamond B}\Diamond R}{f : A \multimap B, m : \Diamond A \vdash m \star \lambda y.\eta(f(y)) : \Diamond B}\Diamond L$$

The lambda term associated with the right hand side of the sequent at the root of the tree gives us an idea of how the terms are combined: first the monadic argument, represented in the term by m, is evaluated according to the specific kind of monad we are dealing with, then the result is bound to the name y by the bind operator which also takes care of carrying over the monadic layer and whatever additional information/dependency was associated with m, and finally the value named y is passed to the argument consuming resource, f, and the result is wrapped in a neutral monadic layer by unit. Notice that this last step is necessary as in general we do not have a way to "exit" from a monad, also because the m resource may have introduced some additional semantic material that we do not want to lose.

The second theorem we discuss in detail is a simplified version of many of the examples presented in the paper. Here we combine an argument-consuming resource that requires a monadic argument, as in the case of our characterization of verbs like *believe* or *love*, with a monadic argument.[49] Assuming that the argument is represented as $\Diamond A$ and the functional resource as $\Diamond A \multimap B$ we can show that there are two different proofs for the following theorem: $\Diamond A \multimap B, \Diamond A \vdash \Diamond B$.

In the first proof the argument is passed directly to the functional resource and the final result is wrapped in a monadic layer by unit:

$$\dfrac{\dfrac{\dfrac{}{m : \Diamond A \vdash m : \Diamond A}\,id \qquad \dfrac{}{x : B \vdash x : B}\,id}{f : \Diamond A \multimap B, m : \Diamond A \vdash f(m) : B}\multimap L}{f : \Diamond A \multimap B, m : \Diamond A \vdash \eta(f(m)) : \Diamond B}\Diamond R$$

In the second proof the argument is first evaluated, the result is bound to the name y and then, after lifting it into a monad using unit, it is passed to the functional resource, and the result of their application is lifted in turn into a monad:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{y : A \vdash y : A}\,id}{y : A \vdash \eta(y) : \Diamond A}\Diamond R \qquad \dfrac{}{x : B \vdash x : B}\,id}{f : \Diamond A \multimap B, y : A \vdash f(\eta(y)) : B}\multimap L}{f : \Diamond A \multimap B, y : A \vdash \eta(f(\eta(y))) : \Diamond B}\Diamond R}{m : \Diamond A, f : \Diamond A \multimap B \vdash m \star \lambda y.\eta(f(\eta(y))) : \Diamond B}\Diamond L$$

The fact that we obtain two proofs is crucial for our analysis, as the first proof corresponds to the case where the argument is evaluated from a lexically defined perspective, while in the case of the second proof, the argument is evaluated from the default speaker's perspective.[50]

49 If the argument is not of a monadic type we can always lift it "for free" using the unit.

50 Notice that in case the argument is actually a non-monadic value the two readings are equivalent as expected: the first reading we obtain for a non-monadic resource x is $\eta(f(\eta(x)))$ and the second one, $\eta(x) \star \lambda y.\eta(f(\eta(y)))$, reduces to the first one due to axiom (45):

$$\begin{aligned}
&\eta(x) \star \lambda y.\eta(f(\eta(y))) = \\
&\lambda y.\eta(f(\eta(y)))(x) = \\
&\eta(f(\eta(x)))
\end{aligned}$$

Finally notice that the fact that we require generation of a monadic result from the combination of the two terms is crucial in obtaining two readings. If we try to generate a proof without the final monadic layer we obtain a single reading that is (roughly) equivalent to the first reading discussed above:

$$\dfrac{\dfrac{}{m : \Diamond A \vdash m : \Diamond A}\,id \qquad \dfrac{}{x : B \vdash x : B}\,id}{f : \Diamond A \multimap B, m : \Diamond A \vdash f(m) : B}\multimap L$$

Appendix B

In this appendix we provide a sequent for each of the key examples in Section 4, repeated below. By entering the sequent in the prover at http://llilab.carleton.ca/~giorgolo/tp.html, the reader can see the derivations generated by our system for each example.[51]

66 Kim doesn't believe Hesperus is Phosphorus.

```
kim:k.e, not:b.t->b.t, believe:<>i.t->k.e->b.t,
hesperus:<>h.e, is:h.e->p.e->i.t,
phosphorus:<>p.e => <>b.t
```

75 #Dr. Octopus punched Spider-Man but he didn't punch Spider-Man.

```
droctopus:d1.e, punch:sm1.e->d1.e->p1.t,
spiderman:<>sm1.e, but:p1.t->p2.t->b.t,
droctopus:d2.e, not:p2.t->p2.t,
punch:sm2.e->d2.e->p2.t, spiderman:<>sm2.e => <>b.t
```

78 Mary Jane loves Peter Parker but she doesn't love Spider-Man.

```
maryjane:mj1.e, love:<>pp.e->mj1.e->l1.t,
peterparker:pp.e, but: l1.t->l2.t->b.t,
maryjane:mj2.e, not: l2.t->l2.t,
love:<>sm.e->mj2.e->l2.t, spiderman:<>sm.e => <>b.t
```

84 Kim doesn't believe Sandy is Sandy.

```
kim:k.e, not:b.t->b.t, believe:<>i.t->k.e->b.t,
sandy:<>s1.e, is:s1.e->s2.e->i.t,
sandy:<>s2.e => <>b.t
```

51 Notice that due to limitations of the code that identifies equivalent proofs, the automatic theorem prover lists a slighlty larger number of proofs than expected, but many of these proofs are equivalent.

89 Reza doesn't believe Jesus is Jesus.

```
reza:r.e, not:b.t->b.t, believe:<>i.t->r.e->b.t,
jesus:<>j1.e, is:j1.e->j2.e->i.t,
jesus:<>j2.e => <>b.t
```